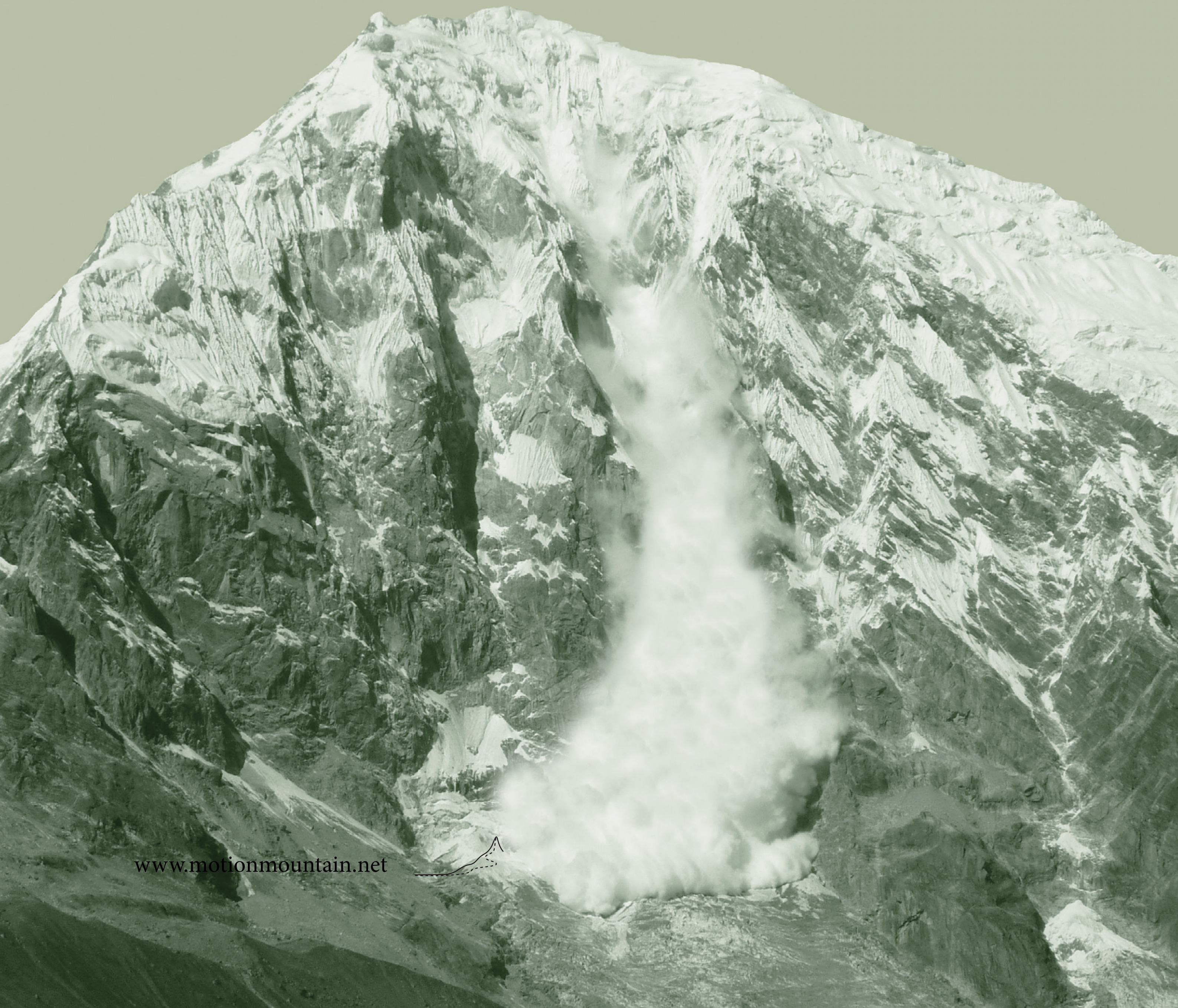
Christoph Schiller

MOTION MOUNTAIN

THE ADVENTURE OF PHYSICS – VOL. III

LIGHT, CHARGES AND BRAINS



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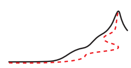

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Motion Mountain

The Adventure of Physics
Volume III

Light, Charges and Brains

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with films at www.motionmountain.net

Editio vicesima septima.

Twenty-seventh edition.

To Britta, Esther and Justus Aaron

τῷ ἐμοὶ δαὶμονι

Die Menschen stärken, die Sachen klären.

PREFACE

> Primum movere, deinde docere.*
>
> Antiquity

This book is written for anybody who is curious about nature and motion. Curiosity about how people, animals, things, images and empty space move leads to many adventures. This volume presents the adventures encountered when exploring everything electric. They lead from the weighing of electric current to the use of magnetic fields to heal bone fractures and up to the use of light to cut metals and the understanding of the human brain.

In the structure of physics, shown in Figure 1, motion due to electricity is the most fascinating aspect of the starting point at the bottom. Indeed, almost everything around us is due to electric processes. The present introduction to electricity, magnetism, light and the brain is the third of a six-volume overview of physics that arose from a threefold aim that I have pursued since 1990: to present motion in a way that is simple, up to date and captivating.

In order to be *simple*, the text focuses on concepts, while keeping mathematics to the necessary minimum. Understanding the concepts of physics is given precedence over using formulae in calculations. The whole text is within the reach of an undergraduate.

In order to be *up to date*, the text is enriched by the many gems – both theoretical and empirical – that are scattered throughout the scientific literature.

In order to be *captivating*, the text tries to startle the reader as much as possible. Reading a book on general physics should be like going to a magic show. We watch, we are astonished, we do not believe our eyes, we think, and finally we understand the trick. When we look at nature, we often have the same experience. Indeed, every page presents at least one surprise or provocation for the reader to think about.

The motto of the text, *die Menschen stärken, die Sachen klären*, a famous statement by Hartmut von Hentig on pedagogy, translates as: 'To fortify people, to clarify things.' Clarifying things – and adhering only to the truth – requires courage, as changing the habits of thought produces fear, often hidden by anger. But by overcoming our fears we grow in strength. And we experience intense and beautiful emotions. All great adventures in life allow this, and exploring motion is one of them. Enjoy it.

Munich, 15 December 2014.

* 'First move, then teach.' In modern languages, the mentioned type of *moving* (the heart) is called *motivating*; both terms go back to the same Latin root.

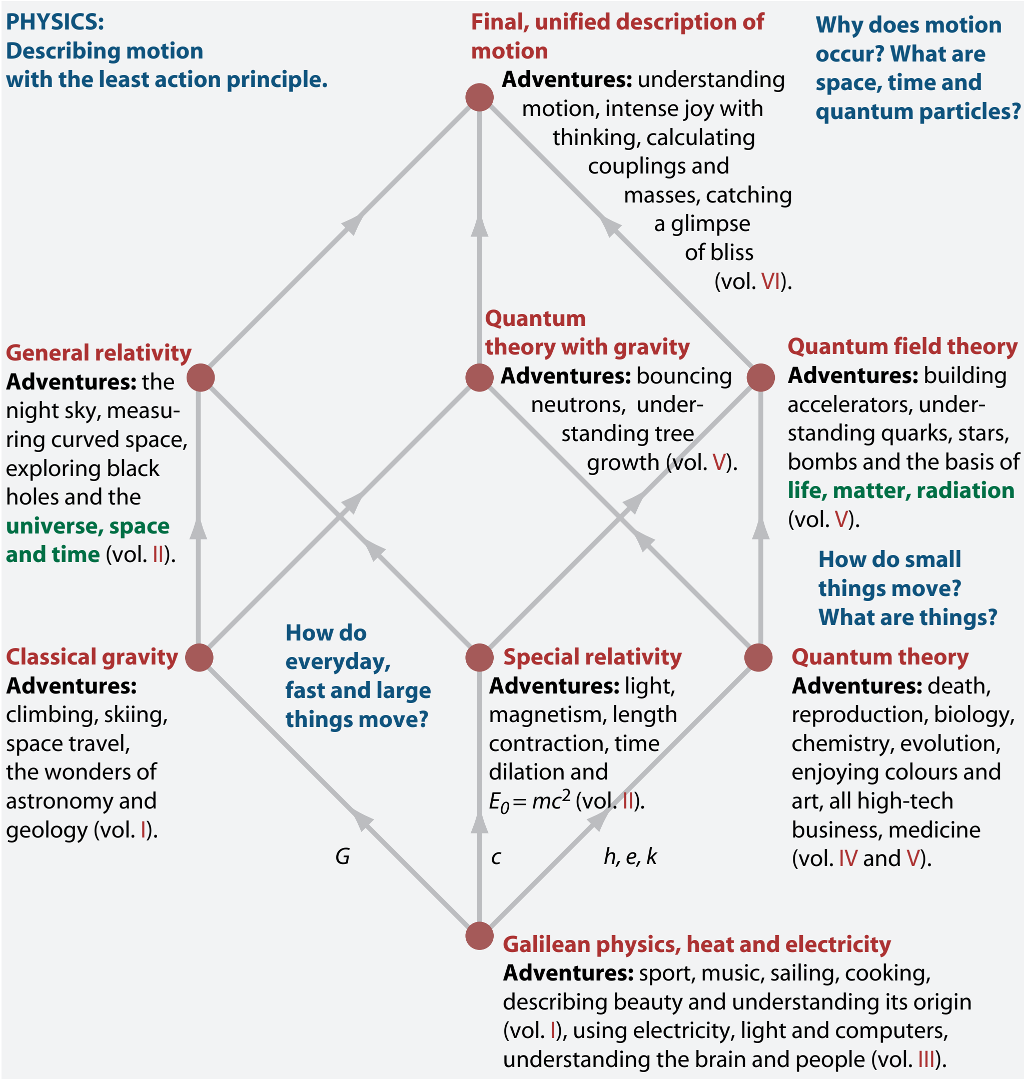

FIGURE 1 A complete map of physics: the connections are defined by the speed of light c, the gravitational constant G, the Planck constant h, the Boltzmann constant k and the elementary charge e.

Advice for learners

Learning widens knowledge, improves intelligence and allows us to discover what kind of person we can be. Learning from a book, especially one about nature, should be efficient and enjoyable. The most inefficient and the most tedious learning method is to use a marker to underline text: it is a waste of time, provides false comfort and makes the text unreadable. Nobody marking text is learning efficiently or is enjoying it.

In my experience as a student and teacher, one learning method never failed to transform unsuccessful pupils into successful ones: if you read a text for study, summarize every section you read, *in your own words and images, aloud*. If you are unable to do so, read the section again. Repeat this until you can clearly summarize what you read

in your own words and images, aloud. You can do this alone or with friends, in a room or while walking. If you do this with everything you read, you will reduce your learning and reading time significantly, enjoy learning from good texts much more and hate bad texts much less. Masters of the method can use it even while listening to a lecture, in a low voice, thus avoiding to ever take notes.

Advice for teachers

A teacher likes pupils and likes to lead them into exploring the field he chose. His or her enthusiasm for the job is the key to job satisfaction. If you are a teacher, before the start of a lesson, picture, feel and tell yourself how you enjoy the topic of the lesson; then picture, feel and tell yourself how you will lead each of your pupils into enjoying that topic as much as you do. Do this exercise consciously, every time. You will minimize trouble in your class and maximize your teaching success.

This book is not written with exams in mind; it is written to make teachers and students *understand* and *enjoy* physics, the science of motion.

Using this book

Marginal notes refer to bibliographic references, to other pages or to challenge solutions. In the colour edition, such notes and also the pointers to footnotes and to other websites are typeset in green. In the free pdf edition, all green links are clickable. The pdf edition also contains all films; they can be watched in Adobe Reader.

Solutions and hints for *challenges* are given in the appendix. Challenges are classified as research level (r), difficult (d), standard student level (s) and easy (e). Challenges for which no solution has yet been included in the book are marked (ny).

Links on the internet tend to disappear with time. Most links can be recovered via www.archive.org, which keeps a copy of old internet pages.

Feedback and support

This text is and will remain free to download from the internet. I would be delighted to receive an email from you at fb@motionmountain.net, especially on the following issues:

Challenge 1 s

— What was unclear and should be improved?
— What story, topic, riddle, picture or film did you miss?

In order to simplify annotations, the pdf file allows adding yellow sticker notes in Adobe Reader. Help on the specific points listed on the www.motionmountain.net/help.html web page are particularly welcome. All feedback will be used to improve the next edition. On behalf of all readers, thank you in advance for your input. For a particularly useful contribution you will be mentioned – if you want – in the acknowledgements, receive a reward, or both.

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Contents

Light, Charges and Brains

In our quest to learn how things move,
the experience of hiking and other motion
leads us to discover that images are produced by charges,
that charges move, accumulate and interact,
and that there is a smallest charge in nature.
We understand what love has to do with magnets and amber,
why the brain is such an interesting device,
and what distinguishes a good from a bad lie.

Chapter 1

LIQUID ELECTRICITY, INVISIBLE FIELDS AND MAXIMUM SPEED

What is *light*? The study of relativity left us completely in the dark, even though we had embarked in it precisely to find an answer to that question. True, we have learned how the motion of light compares with that of objects. We also learned that light is a moving entity that cannot be stopped, that light provides the speed limit for any type of energy, and that light is our measurement standard for speed. However, we haven't learned anything about the nature of light itself, nor about *colours*, nor about how rain drops,* and other matter produces them.

A second question is open: what is *contact*? We still do not know. In our exploration Vol. II, page 81 of relativity we only learned that truly mechanical interactions do not exist. We learned that all interactions, also contact, are due to exchange of particles. But which ones? And how can motion and levitation occur *without* any material contact?

A third question also arises: how do we *sense* contact or touch? What are *sensors* and how is their output, the data, processed in the brain or in machines? Not only the brain, also all other data processing systems use electricity. What is *data* and what is *electricity*?

The answer to the questions about the nature of light, contact and the brain is *not* Vol. I, page 216 related to gravitation. If we make a list of motors found in this world, we notice that gravitation hardly describes any of them. Neither the motion of sea waves, fire and earthquakes, nor that of a gentle breeze is caused by gravity. The same applies to the motion of Challenge 2 e muscles. Have you ever listened to your own heart beat with a stethoscope? (Or use, as many medical doctors do now, an MP3 player to record your heart beat.) Without having done so, you cannot claim to have experienced the mystery of motion. Your heart has about 3000 million beats in your lifetime. Then it stops.

It was one of the most astonishing discoveries of science that heart beats, sea waves and most other cases of everyday motion, as well as the nature of light and thought itself, are connected to observations made thousands of years ago using two strange stones. These stones show that all those examples of motion that are called *mechanical* in everyday life are, without exception, of *electrical* origin.

In particular, the solidity, the softness and the impenetrability of matter are due to internal electricity. But also the emission of light, the formation of colours and the work- Ref. 1 ing of our nerves and brains are due to electrical processes. As these aspects are part of everyday life, we can leave aside all complications due to gravity and curved space-time.

The most productive way to study electrical motion is to start, as in the case of gravity,

* The photograph of a circular rainbow on page 14 was taken in 2006 from the Telstra Tower in Canberra (© Oat Vaiyaboon).

FIGURE 2 Objects surrounded by fields: amber (c. 1 cm), lodestone (c. 1 cm) and mobile phone (c. 10 cm) (© Wikimedia, Philips).

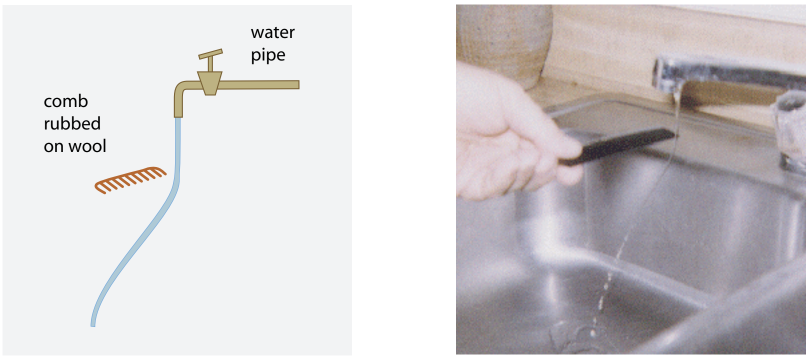


FIGURE 3 How to amaze kids, especially in dry weather (photo © Robert Fritzius).

with those types of motion which are generated without any contact between the bodies involved. This can happen in three ways.

Fields: amber, lodestone and mobile phones

You can always surprise children with the effect shown in Figure 3: a comb rubbed on wool deviates running tap water. The same effect can be produced with an air-filled rubber balloon rubbed on wool. Everybody can deviate water streams without any contact.

The Greeks had already observed this effect a long time age. In fact, the story of electricity starts with trees. Trees have a special relation to electricity. When a tree is cut, a viscous resin appears. With time it solidifies and, after millions of years, it forms *amber*. When amber is rubbed with a cat fur, it acquires the ability to attract small objects, such as saw dust or pieces of paper. This was already known to Thales of Miletus, one of the original seven sages, in the sixth century BCE. The same observation can be made with many other polymer combinations, for example with combs and hair, with soles of the shoe on carpets, and with dust and a cathode ray tube inside an old television. Another interesting effect can be observed when a rubbed comb is put near a burning candle. (Can you imagine what happens?)

Challenge 3 s

Another part of the story of electricity involves *lodestone*, an iron mineral found in certain caves around the world, e.g. in a region (still) called Magnesia in the Greek province

of Thessalia, and in some regions in central Asia. When two stones of this mineral are put near each other, they attract or repel each other, depending on their relative orientation. In addition, lodestone attracts objects made of cobalt, nickel or iron.

Today we also find various small objects in nature with more sophisticated properties, such as the one shown on the right of Figure 2. Some of these objects allow you to talk with far away friends, others unlock car doors, still others enable you to switch on a television.

In short, in nature there are situations where bodies exert influence on others *at a distance*. The space surrounding a body exerting such an influence is said to contain a field. A *(physical) field* is thus an entity that manifests itself by accelerating other bodies in a given region of space. *A field is space that changes momenta.* If you prefer, *a field is space that exerts forces.* Or again, a field is space with some extra structure. Despite this extra structure, fields, like space, are invisible.

The field surrounding the mineral found in Magnesia is called a *magnetic field* and the stones are called *magnets*. The field around amber – called ἤλεκτρον in Greek, from a root meaning 'brilliant, shining' – is called an *electric field*. The name is due to a proposal by the famous physician and part-time physicist William Gilbert (b. 1544 Colchester, d. 1603 London). Objects surrounded by a permanent electric field are called *electrets*. Electrets are far less common than magnets; among others, they are used in certain loudspeaker systems.

Ref. 2

The field around a mobile phone is called a *radio* field or, as we will see later, an *electromagnetic* field. In contrast to the previous fields, it oscillates over time. We will find out later that many other objects are surrounded by such fields, though these are often very weak. Objects that emit oscillating fields, such as mobile phones, are called radio transmitters or electromagnetic emitters. Certain radio transmitters, as we will see, are already familiar from everyday life: lamps and lasers.

Experiments show that fields have *no mass* and no material support. Fields influence bodies over a distance. Fields are invisible. To make them imaginable, we just need to colour them. Some ways to colour electric fields are shown in Figure 4. (Additional visualizations for magnetic and radio fields follow below.) These figures are the best way to *visualize* electric fields: also the man who first proposed the field concept, Michael Faraday, used such images.

Exploring such images, we note that one can visualize electric fields either as a tiny vector attached to every point of space, or as a bundle of lines. Both visualizations are useful. We will even encounter additional visualizations below.

For a long time, electric, magnetic and radio fields were rarely noticed in everyday life. Indeed, in the past, most countries had laws that did not allow producing such fields, nor building mobile phones or garage openers. Still today, laws severely restrict the properties of machines that use and produce such fields. These laws require that for any device that moves, produces sound, or creates moving pictures, fields need to remain *inside* the device. Also for this reason a magician moving an object on a table via a hidden magnet still surprises and entertains his audience. To feel the fascination of fields more strongly, a deeper look into a few experimental results is worthwhile.

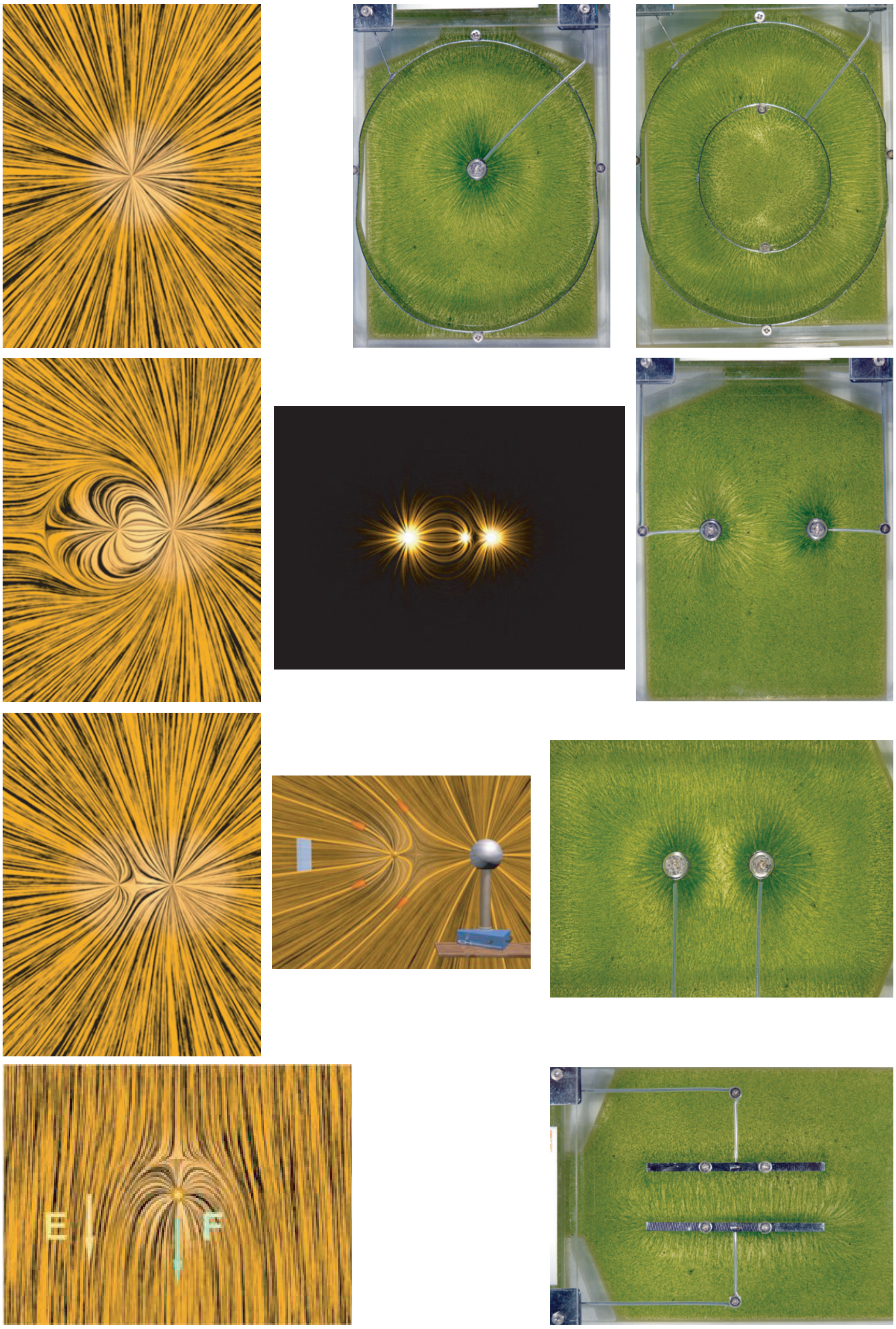


FIGURE 4 Visualizing what is invisible: a simple way to visualize electric fields as space with a structure, using computer graphics and using seeds in oil. Top: the field around a point or spherical charge; second row: two or three charges of different signs; third row: two charges of the same sign; bottom: a charge in an external field E, and the field between two plates. The charge will feel a force F directed along the so-called *electric field lines*; the density of the lines gives the intensity of the field and thus the strength of the force (© MIT, Eli Sidman, MIT).

FIGURE 5 Lightning: a picture taken with a moving camera, showing its multiple strokes (© Steven Horsburgh).

How can one make lightning?

Everybody has seen a lightning flash or has observed the effect it can have on striking a tree. Obviously lightning is a moving phenomenon. Photographs such as that of Figure 5 show that the tip of a lightning flash advance with an average speed of around 600 km/s. But *what* is moving? To find out, we have to find a way of making lightning for ourselves. In 1995, the car company Opel accidentally rediscovered an old and simple method of achieving this.

Opel engineers had inadvertently built a spark generating mechanism into their cars; when filling the petrol tank, sparks were generated, which sometimes lead to the explosion of the fuel at the petrol station. Opel had to recall 2 million vehicles. Ref. 3

What had the engineers done wrong? They had unwittingly copied the conditions for a electrical device which anyone can build at home and which was originally invented by William Thomson:* the *Kelvin generator*. Repeating his experiment today, we would take two water taps, four empty bean or coffee cans, of which two have been opened at both sides, some nylon rope and some metal wire. Putting this all together as shown in Figure 6, and letting the water flow, we find a strange effect: large sparks periodically jump between the two copper wires at the point where they are nearest to each other, Ref. 4

* William Thomson (b. 1824 Belfast, d. 1907 Largs), important physicist and professor at Glasgow University. He worked on the determination of the age of the Earth, showing that it was much older than 6000 years, as several sects believed, but also (falsely) maintained that the Earth was much younger than geologists and Darwin (correctly) had deduced. He strongly influenced the development of the theory of magnetism and electricity, the description of the aether, and thermodynamics. He propagated the use of the term 'energy' as it is used today, instead of the confusing older terms. He was one of the last scientists to propagate mechanical analogies for the explanation of phenomena, and thus strongly opposed Maxwell's description of electromagnetism. It was mainly for this reason that he did not receive a Nobel Prize. He was also one of the minds behind the laying of the first transatlantic telegraphic cable. Victorian and religious to his bones, when he was knighted, he chose the name of a small brook near his home as his new name; thus he became Baron Kelvin of Largs. Therefore the unit of temperature obtained its name from a small Scottish river.

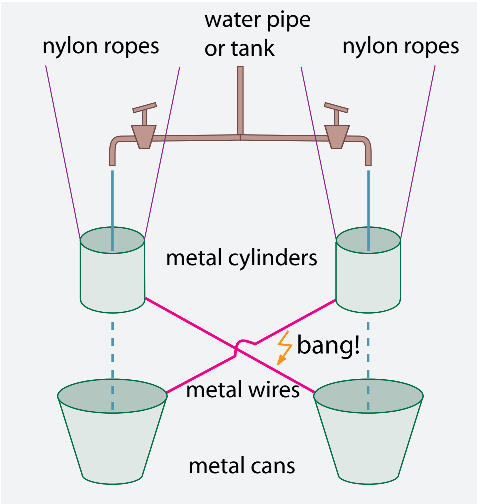


FIGURE 6 A simple Kelvin generator; the one on the right lights a fluorescent light bulb using dripping water (photograph © Harald Chmela).

giving out loud bangs. Can you guess what condition for the flow has to be realized for this to work? And what did Opel do to repair the cars they recalled? Challenge 4 s

If we stop the water flowing just before the next spark is due, we find that both buckets are able to attract sawdust and pieces of paper. The generator thus does the same that rubbing amber does, just with more bang for the buck(et). Both buckets, and the attached metal pieces, are thus surrounded by electric fields. The fields increase with time, until the spark jumps. Just after the spark, the buckets are (almost) without surrounding electric field. Obviously, the flow of water somehow collects something on each bucket; today we call this *electric charge*. We also say that such bodies are *electrically charged*. This and other experiments also show that charge can *flow* in metals. When the electric fields are high enough, charge can also flow through air, leading to sparks or lightning.

We also find that the two buckets are always surrounded by *two different types* of electric fields: bodies that are attracted by one bucket are repelled by the other. The discovery that there are *two different types* of electric charge is due to the universal genius Charles Dufay (b. 1698 Paris, d. 1739 Paris). In a long and careful series of experiments he confirmed that *all* materials he could get hold of can be charged electrically, and that all charges can be classified into two types. He was the first to show that charged bodies of the *same* charge *repel* each other, and that bodies of *different* charge *attract* each other. He showed in detail that all experiments on electricity can be explained with these statements. Dufay called the two types of charges 'vitreous' and 'resinous'. Unfortunately, Dufay died at a young age. Nevertheless, his results spread quickly. A few years later, Georg Bose used them to develop the first electrifying machine, which then made the exploration of sparks and the science of electricity fashionable across Europe.* Ref. 5

* In fact, the fashion still goes on. Today, there are many additional ways to produces sparks or even arcs, i.e., sustained sparks. There is a sizeable subculture of people who build such high voltage generators as a hobby at home; see, for example, the website www.kronjaeger.com/hv. There is also a sizeable subculture of people who do this professionally, paid by tax money: the people who build particle accelerators.

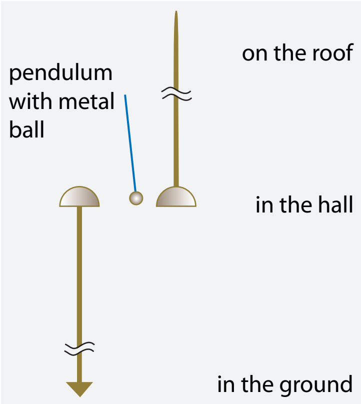


FIGURE 7 Franklin's personal lightning rod – a copy of Gordon's electric chime – is one of the many experiments that shows strikingly that charge can flow.

Twenty years after Dufay, in the 1750s, the politician and part-time physicist Benjamin Franklin (b. 1706 Boston, d. 1790 Philadelphia) proposed to call the electricity created on a glass rod rubbed with a dry cloth *positive* instead of vitreous, and that on a piece of amber *negative* instead of resinous. Thus, instead of two types of electricity, he proposed that there is really only *one* type, and that bodies can either have too much or too little of it. With the new terms, bodies with charges of the same sign repel each other, bodies with opposite charges attract each other; charges of opposite sign flowing together cancel each other out. Large absolute values of charge imply large charge effects. It took over a hundred years before these concepts were accepted unanimously.

In summary, *electric effects are due to the flow of charges.* Now, all flows take time. How fast is electricity? A simple way to measure the speed of electricity is to produce a small spark at one end of a long metal wire, and to observe how long it takes until the spark appears at the other end of the wire. In practice, the two sparks are almost simultaneous; the speed one measures is much higher than everything else we observe in our environment. How can we measure the time nevertheless? And why did different researchers get very different speed values in this experiment? The result of these experiments is that the speed of electricity is typically a large percentage of the speed of light.

Challenge 5 s

Sparks, electric arcs and lightning are similar. Of course, one has to check whether natural lightning is actually electrical in origin. In 1752, experiments performed in France, following a suggestion by Benjamin Franklin, published in London in 1751, showed that one can indeed draw electricity from a thunderstorm via a long rod.* Thunderstorm clouds are surrounded by electric fields. These French experiments made Franklin famous worldwide; they were also the start of the use of lightning rod all over the world. Later, Franklin had a lightning rod built through his own house, but of a somewhat unusual type, as shown in Figure 7. This device, invented by Andrew Gordon, is called an *electric chime*. Can you guess what it did in his hall during bad weather, all parts being made of metal, and why? (Do not repeat this experiment; any device attached to a lightning rod can kill.)

Ref. 6

Challenge 6 s

* The details of how lightning is generated and how it propagates are still a topic of research. An introduction is given on page 204.

FIGURE 8 A simple set-up to confirm electric charge conservation: if rubbed fur is moved from the first pot to the second, the charge taken away from the first pot is transferred to the second, as shown by the two electrometers (© Wolfgang Rueckner).

In summary, electric fields start at bodies, provided they are charged. Charging can be achieved by rubbing and other processes. There are two types of charge, negative and positive. Charge can flow: it is then called an electric *current*. The worst conductors of current are polymers; they are called *insulators* or *dielectrics*. A charge put on an insulator remains at the place where it was put. In contrast, metals are good conductors; a charge placed on a conductor spreads all over its surface. The best conductors are silver and copper. This is the reason that at present, after two hundred years of use of electricity, the highest concentration of copper in the world is below the surface of Manhattan. Charges can flow through air if the electric field is strong enough; this is a spark or, when the spark is large, a lightning bolt.

Electric charge

Because all experiments with electric charge can be explained by calling the two charges positive and negative, we deduce that some bodies have more, and some less charge than an uncharged, *neutral* body. Electric charges thus only flow when two differently charged bodies are brought into contact. Now, if charge can flow and accumulate, we must be able to somehow measure its amount. Obviously, the *amount* of electric charge on a body, usually abbreviated q, must be defined via the influence the body, say a piece of sawdust, feels when subjected to a field. Charge is thus defined by comparing it to a standard reference charge. For a charged body of mass m accelerated in a field, its charge q is determined by the relation

$$\frac{q}{q_{\text{ref}}} = \frac{\mathrm{d}\mathbf{p}/\mathrm{d}t}{\mathrm{d}\mathbf{p}_{\text{ref}}/\mathrm{d}t} , \qquad (1)$$

i.e., by comparing its momentum change with the momentum change of the reference charge. Charge thus determines the motion of bodies in electric fields in the same way that mass determines the motion of bodies in gravitational fields. Charge is therefore the second intrinsic property of bodies that we discover in our walk.

In practice, electric charge is measured with *electrometers*. A few such devices are

FIGURE 9 Various electrometers: a self-made electrometer based on a jam pot, an ancient (opened) high precision Dolezalek electrometer, the *Ampullae of Lorenzini* of a shark, and a modern digital electrometer (© Harald Chmela, Klaus Jost at www.jostimages.com, Advantest).

TABLE 1 Properties of *classical* electric charge: a scalar density.

Electric charges	Physical property	Mathematical name	Definition
Can be distinguished	distinguishability	element of set	Page 262
Can be ordered	sequence	order	Vol. IV, page 221
Can be compared	measurability	metricity	Vol. V, page 350
Can change gradually	continuity	completeness	Vol. V, page 359
Can be added	accumulability	additivity	Vol. I, page 78
Can be separated	separability	positive or negative	
Do not change	conservation	invariance	$q = \text{const}$

shown in Figure 9. The main experimental properties of electric charge that are discovered when experimenting with electrometers are listed in Table 1.

The unit of charge, the *coulomb*, is defined through a standard flow through metal wires, as explained in Appendix A. This is possible because all experiments show that charge is *conserved*, that it *flows*, and thus that it can *accumulate*. In other words, if the

TABLE 2 Values of electrical charge observed in nature.

Observation	Charge
Smallest measured non-vanishing charge	$1.6 \cdot 10^{-19}$ C
Charge per bit in computer memory	down to 10^{-15} C
Charge in small capacitor	10^{-7} C
Charge flow in average lightning stroke	1 C to 100 C
Charge stored in a fully charged car battery	0.2 MC
Charge of planet Earth	−1 MC
Charge separated by modern power station in one year	$3 \cdot 10^{11}$ C
Total charge of positive (or negative) sign observed in universe	$10^{60\pm1}$ C
Total charge observed in universe	0 C

electric charge of a physical system changes, the reason always is that charge is flowing into or out of the system. This can be checked easily with two metal pots connected to two electrometers, as shown in Figure 8. Charge thus behaves like a fluid substance. Therefore we are forced to use for its description a scalar quantity q, which can take positive, vanishing, or negative values on a physical body.

Ref. 7

Describing charge as a scalar quantity reproduces the behaviour of electrical charge in all everyday situations. However, as in the case of all previously encountered classical concepts, some of the experimental results for electrical charge in everyday situations from Table 1 will turn out to be only approximate. More precise experiments will require a revision of the idea of continuous change of charge value. Nevertheless, no counter-example to charge conservation has as yet been observed.

In summary, *electric charge is a scalar quantity that describes the origin of electric fields. Electric charge is conserved.* There is on way to destroy or create electric charge. We mentioned above that objects without electric charge are called *neutral*. Also neutral bodies are influenced by electric fields. This happens because a charged object that is brought near a neutral body polarizes it. *Electrical polarization* is the separation of the positive and negative charges onto different regions of a body. For this reason, neutral objects, such as hair or a water stream, are usually attracted to a charged body, such as a rubbed comb. Both insulators and conductors can be polarized; and polarization occurs for single molecules, everyday bodies and whole stars.

Electric field strength

Charges produce attraction and repulsion on other charges. Equivalently, charges change momenta; charges exert forces on other charges. This happens over large distances. Experiments that explore energy and momentum conservation show that the best description of these interactions is as told so far: a charge produces a field, the field then acts on a second charge.

Experiments show that the *electric field* forms lines in space. As a consequence, the electric field behaves like a small arrow fixed at each point $\boldsymbol{x}$ in space. Electric fields are described by a direction and a magnitude. The local direction of the field is given by the local direction of the field line – the tangent of the field line. The local magnitude of the

field is given by the local density of the field lines. The direction and the magnitude do not depend on the observer. In short, the electric field $\boldsymbol{E}(\boldsymbol{x})$ is a *vector* field. Experiments show that it is best defined by the relation

$$q\boldsymbol{E}(\boldsymbol{x}) = \frac{\mathrm{d}\boldsymbol{p}(\boldsymbol{x})}{\mathrm{d}t} \tag{2}$$

taken at every point in space $\boldsymbol{x}$. The definition of the electric field is thus based on how it *moves* charges. In general, the electric field is a vector

$$\boldsymbol{E}(\boldsymbol{x}) = (E_x, E_y, E_z) \tag{3}$$

Challenge 7 e and is measured in multiples of the unit N/C or V/m.

The definition of the electric field assumes that the test charge q is so small that it does not disturb the field E. We sweep this issue under the carpet for the time being. This is a drastic move: we ignore quantum theory and all quantum effects in this way; we come Page 231 back to it below.

The definition of the electric field also assumes that space-time is flat, and it ignores all issues due to space-time curvature.

By the way, does the definition of electric field just given assume a charge speed that Challenge 8 s is far less than that of light?

To describe the motion due to electricity completely, we need a relation explaining how charges *produce* electric fields. This relation was established with precision (but not for the first time) during the French Revolution by Charles-Augustin de Coulomb, on his private estate.* He found that around any small-sized or any spherical charge Q *at rest* there is an electric field. At a position $\boldsymbol{r}$, this electric field $\boldsymbol{E}$ is given by

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\frac{\boldsymbol{r}}{r} \quad \text{where} \quad \frac{1}{4\pi\varepsilon_0} = 9.0\,\mathrm{GV\,m/C}\,. \tag{4}$$

Later we will extend the relation for a charge in motion. The bizarre proportionality constant is universally valid. The constant is defined with the so-called *permittivity of free space* ε_0 and is due to the historical way the unit of charge was defined first.** The essential point of the formula is the decrease of the field with the square of the distance; can Challenge 10 s you imagine the origin of this dependence? A simple way to picture Coulomb's formula is illustrated in Figure 10.

The two previous equations allow us to write the interaction between two charged bodies as

$$\frac{\mathrm{d}\boldsymbol{p}_1}{\mathrm{d}t} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\frac{\boldsymbol{r}}{r} = -\frac{\mathrm{d}\boldsymbol{p}_2}{\mathrm{d}t}\,, \tag{5}$$

* Charles-Augustin de Coulomb (b. 1736 Angoulême, d. 1806 Paris), engineer and physicist, provided, with his careful experiments on electric charges, a firm basis for the study of electricity.

** Other definitions of this and other proportionality constants to be encountered later are possible, leading to *unit systems* different from the SI system used here. The SI system is presented in detail in Appendix A. Among the older competitors, the Gaussian unit system often used in theoretical calculations, the Heaviside–Lorentz unit system, the electrostatic unit system and the electromagnetic unit system are Ref. 8 the most important ones.

TABLE 3 Some observed electric fields.

Observation	Electric field
Field 1 m away from an electron in vacuum	Challenge 9 s
Field values sensed by sharks	down to 0.5 µV/m
Cosmic noise	10 µV/m
Field of a 100 W FM radio transmitter at 100 km distance	0.5 mV/m
Field inside conductors, such as copper wire	0.1 V/m
Field just beneath a high power line	0.1 to 1 V/m
Field of a GSM antenna at 90 m	0.5 V/m
Field inside a typical home	1 to 10 V/m
Field of a 100 W bulb at 1 m distance	50 V/m
Ground field in Earth's atmosphere	100 to 300 V/m
Field inside thunder clouds	up to over 100 kV/m
Maximum electric field in air before sparks appear	1 to 3 MV/m
Electric fields in biological membranes	10 MV/m
Electric fields inside capacitors	up to 1 GV/m
Electric fields in petawatt laser pulses	10 TV/m
Electric fields in U^{91+} ions, at nucleus	1 EV/m
Maximum practical electric field in vacuum, limited by electron pair production	1.3 EV/m
Maximum possible electric field in nature (corrected Planck electric field $c^4/4Ge$)	$1.9 \cdot 10^{62}$ V/m

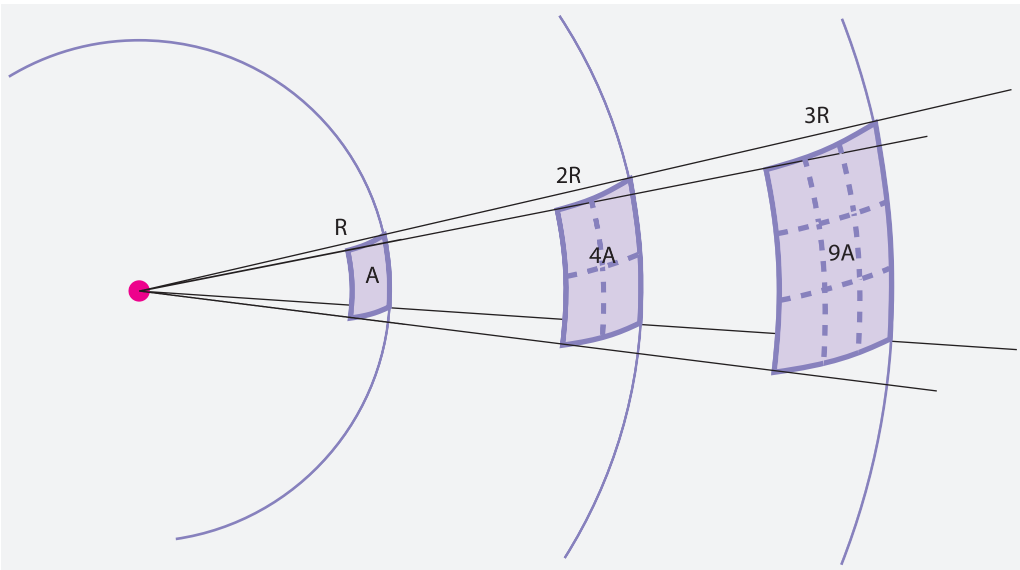


FIGURE 10 A visualization of Coulomb's formula and Gauss' law.

TABLE 4 Properties of the classical electric field: a (polar) vector at every point in space.

Electric fields can	Physical property	Mathematical name	Definition
Attract bodies	accelerate charges	coupling	equation (4)
Repel bodies	accelerate charges	coupling	equation (4)
Be distinguished	distinguishability	element of set	Page 262
Change gradually	continuum	real vector space	Vol. I, page 78, Vol. V, page 359
Point somewhere	direction	vector space, dimensionality	Vol. I, page 78
Be compared	measurability	metricity	Vol. V, page 350
Be added	additivity	vector space	Vol. I, page 78
Have defined angles	direction	Euclidean vector space	Vol. I, page 78
Exceed any limit	infinity	unboundedness	Page 263
Change direction under reflection	polarity	parity-odd vector	
Keep direction under time reversal		time-even vector	

where d$\boldsymbol{p}$ is the momentum change and $\boldsymbol{r}$ is the vector connecting the two centres of mass. This famous expression for electrostatic attraction and repulsion, also due to Coulomb, is valid only for charged bodies that are either of small size or spherical, and most of all, only for bodies that are *at rest* with respect to each other and to the observer. The exploration of interactions among charges at rest is called *electrostatics*.

Electric fields accelerate charges. As a result, in everyday life, electric fields have two main properties: they contain energy and they can polarize bodies. The energy content is due to the electrostatic interaction between charges. The strength of this interaction is considerable. For example, it is the basis for the force of our muscles. Muscular force is a macroscopic effect of Coulomb's relation (5). Another example is the material strength of steel or diamond. As we will discover, all atoms are held together by electrostatic attraction. To convince yourself of the strength of electrostatic attraction, answer the following: What is the force between two boxes with a gram of protons each, located on the two poles of the Earth? Try to guess the result before you calculate the astonishing value.

Challenge 11 s

Coulomb's relation for the field around a charge can be rephrased in a way that helps to generalize it to non-spherical bodies. Take a closed surface, i.e., a surface than encloses a certain volume. Then the integral of the electric field over this surface, the electric flux, is the enclosed charge Q divided by ε_0:

$$\oint_{\text{closed surface}} \boldsymbol{E}\, \mathrm{d}\boldsymbol{A} = \frac{Q}{\varepsilon_0} . \tag{6}$$

Challenge 12 s This mathematical relation, called *Gauss's 'law'*,* from the result of Coulomb. (Note that in the simplified form given here, it is valid only for static situations.) Since inside conductors the electrical field is zero, Gauss's 'law' implies, for example, that if a charge q is surrounded by an uncharged metal sphere, the *outer* surface of the metal sphere shows the same charge q. Challenge 13 e

Do uncharged, neutral bodies attract one other? In first approximation they do not. But when the question is investigated more precisely, we will find that they can attract one other. Can you find the conditions for this to happen? In fact, the conditions are quite important, as our own bodies, which are made of neutral molecules, are held together in this way. Vol. V, page 116 Challenge 14 s

Pumping charge

Owing to the high strength of electromagnetic interactions, separating charges is not an easy task. This is the reason that electrical effects have only been commonly used for about a hundred years. Humanity had to wait for practical and efficient devices to be invented for separating charges and putting them into motion: to use electric effects, we need *charge pumps*. Some devices are shown in Figure 11. Can you explain whether batteries or any other of these devices are sources of charges? Challenge 15 s

Of course, every charge pump requires energy. Batteries in mobile phones and the ion channels in living cells use chemical energy to do the trick. Thermoelectric elements, as used in some watches, use the temperature difference between the wrist and the air to separate charges; solar cells use light, piezoelectric elements use stress and dynamos or Kelvin generators use kinetic energy.

What is electricity?

The answer to this question is: *Electricity* is more the name for a field of inquiry, and less the name for any specific observation or effect. Electricity is not a specific term; the term is used to refer to the effects of electric charges, of their motion and their fields.

In fact the vocabulary issue hides a deeper question: what is the nature of electric charge? In order to solve this issue, we start with the following question.

FIGURE 11 Various types of charge pumps: a bicycle dynamo, an alternator in a power station, a Wimshurst machine, an electric eel, a voltaic cell, a leaf and a solar cell (© Wikimedia, Q-Cells).

Can we detect the inertia of electricity?

If electric charge really is something *flowing* through metals, we should be able to observe the effects shown in Figure 12: electric charge should fall, should have inertia and should

* Carl-Friedrich Gauß (b. 1777 Braunschweig, d. 1855 Göttingen) was, together with the Leonhard Euler, the most important mathematician of all times. A famous child prodigy, when he was 19 years old, he constructed the regular heptadecagon with compass and ruler (see www.mathworld.wolfram.com/Heptadecagon.html). He was so proud of this result that he put a drawing of the figure on his tomb. Gauss produced many results in number theory, topology, statistics, algebra, complex numbers and differential geometry which are part of modern mathematics and bear his name. Among his many accomplishments, he produced a theory of curvature and developed non-Euclidean geometry. He also worked on electromagnetism and astronomy.

Gauss was a difficult character, worked always for himself, and did not found a school. He published little, as his motto was: pauca sed matura. As a consequence, when another mathematician published a new result, he regularly produced a notebook in which he had noted the very same result already years before. These notebooks are now available online, at www.sub.uni-goettingen.de.

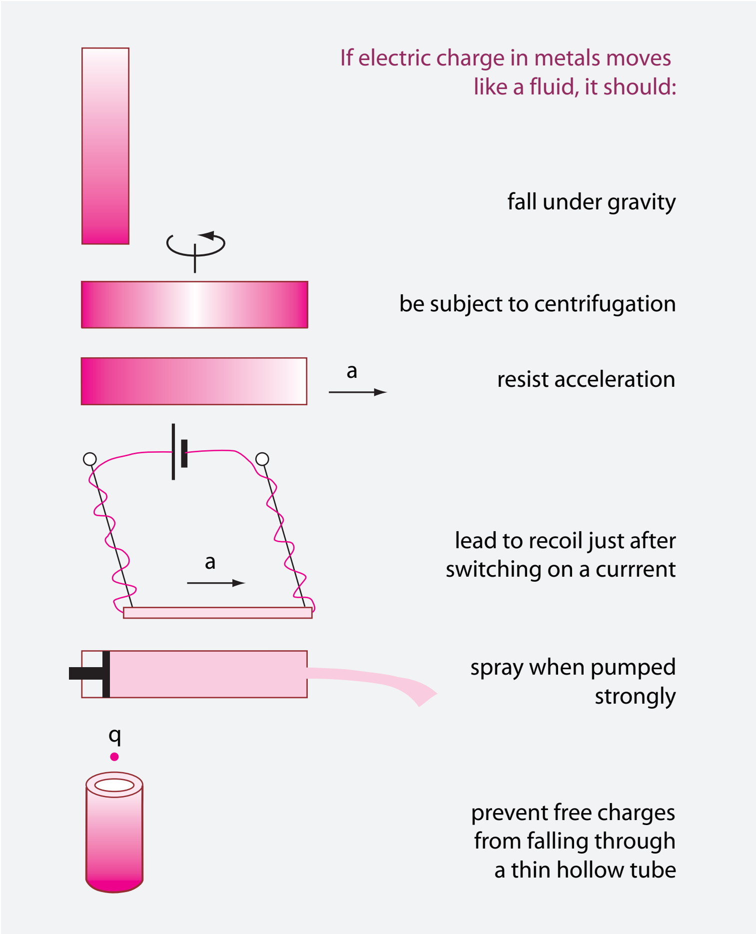


FIGURE 12 Consequences of the flow of electricity.

be separable from matter. And indeed, each of these effects has been observed. For example, when a long metal rod is kept vertically, we can measure an electrical potential difference, a *voltage*, between the top and the bottom. In other words, we can measure the *weight* of electricity in this way. Similarly, we can measure the potential difference between the ends of an accelerated rod. Alternatively, we can measure the potential difference between the centre and the rim of a rotating metal disc. The last experiment was, in fact, the way in which the ratio q/m for currents in metals was first measured with precision. The result is Ref. 9

$$q/m \approx -1.8(2) \cdot 10^{11}\ \mathrm{C/kg} \qquad (7)$$

for all metals, with small variations in the second digit. The minus sign is due to the definition of charge. In short, electrical charge in metals has mass, though a very small one.

If electric charge has mass, whenever we switch on an electrical current, we get a *recoil*. This simple effect can easily be measured and confirms the mass to charge ratio just given. Also, the emission of current into air or into vacuum is observed; in fact, every cathode Ref. 10

ray tube inside an old television uses this principle to generate the beam producing the picture. The emission works best for metal objects with sharp, pointed tips. The rays created this way – we could say that they are 'free' electricity – are called *cathode rays*. Within a few per cent, they show the same mass to charge ratio as expression (7). This correspondence thus shows that charges move almost as freely in metals as in air; this is the reason that metals are such good conductors of electric current.

Ref. 11

If electric charge *falls* inside vertical metal rods, we can make the astonishing deduction that cathode rays should not be able to fall through a vertical metal tube. As we will see later, cathode rays consist of free electrons. The name 'electron' is due to George Stoney. Electrons are the smallest and lightest charges moving in metals; they are, usually – but not always – the 'atoms' of electricity. In particular, electrons conduct electric current in metals. The charge of an electron is small, 0.16 aC, so that flows of charge typical of everyday life consist of huge numbers of electrons; as a result, electrical charge effectively behaves like a continuous fluid. The particle itself was discovered and presented in 1897 by Johann Emil Wiechert (b. 1861 Tilsit, d. 1928 Göttingen) and, independently, three months later, by Joseph John Thomson (b. 1856 Cheetham Hill, d. 1940 Cambridge).

Cathode rays should not be able to fall through a vertical metal tube because the acceleration by the electrical field generated by the displaced electricity in the metal tube and the gravitational acceleration cancel. Thus electrons should not be able to fall through a long thin cylinder. This would not be the case if electricity in metals did not behave like a fluid. The experiment has indeed been performed, and a reduction of the acceleration of free fall for electrons of 90 % has been observed. Can you imagine why the ideal value of 100 % is not achieved?

Challenge 16 e
Ref. 12
Challenge 17 s

Precision experiments with charges ejected from metals show that they have a charge to mass ratio of

$$q/m = -1.758\,820\,150(44) \cdot 10^{11}\ \mathrm{C/kg} \tag{8}$$

The particles with this property are called *electrons*. Other types of charges, with different charge-to-mass ratio, also exist in nature. Examples are the *ions* found in batteries and leaves, the *muons* found in cosmic rays, and the *mesons* produced in particle accelerators. We will meet these particles later in our adventure.

Since electric current behaves like a liquid, we should be able to measure its speed. The first to do so, in 1834, was Charles Wheatstone. In a famous experiment, he used a wire of a quarter of a mile length to produce three sparks: one at the start, one at the middle, and one at the end. He then mounted a rapidly moving mirror on a mechanical watch. By noting how much the three spark images were shifted against each other on a screen, he determined the speed to be 0.45 Gm/s, though with a large measurement error. Latter, more precise measurements showed that the speed is always below 0.3 Gm/s, and that it depends on the metal and the type of insulation of the wire. The high value of the speed convinced many people to use electricity for transmitting messages. In fact, these experiments measure the *signal speed* of electromagnetic waves carried by metal wires. The actual speed of electric charges is much lower, as shown below. A modern version of the signal speed experiment, for computer fans, uses the 'ping' command from the UNIX operating system. The 'ping' command measures the time for a computer signal to reach another computer and return back. If the cable length between two computers is known, the signal speed can be deduced. Just try.

Page 232
Ref. 13
Challenge 18 e

TABLE 5 Some observed electric current values.

Observation	Current
Smallest current ever measured (for one moving electron)	3 aA
Human nerve signals	20 μA
Lethal current for humans	as low as 20 mA, typically 100 mA
Current drawn by a train engine	600 A
Current in a lightning bolt	10 to 100 kA
Highest current produced by humans	20 MA
Current inside the Earth, at the origin of its magnetic field	*c.* 100 MA
Maximum possible current in nature (corrected Planck electric current $e\sqrt{c^5/4\hbar G}$)	1.5 YA

TABLE 6 Some sensors for electrical current.

Measurement	Sensor	Range
Conventional 20 euro multimeter	voltage drop over resistor	up to *c.* 3 A
Feeling threshold	human nerve	felt from 0.1 mA upwards
Reversible muscle contraction without danger	human nerve	up to 10 mA over long times, or up to 200 mA for at most 10 ms
Rhythm change	human heart	heart stops when about 20 mA flow through it
Strong muscle contraction with some damage	human nerve	up to 100 mA over long times, or up to 1 A for at most 200 ms
Smoke emission, strong burns	human flesh	from 1 A
Fire	trees	from 1 kA
Electric eel *Electrophorus electricus*	bult-in	up to 1 A and 500 V

The speed of electricity is *too slow* for many people. Computer chips could be faster if it were higher. And computers that are connected to stock exchanges are located as near as possible to the stock exchange, because the time advantage the short communication distance (including the delay inside switching chips) provides is essential for getting a good financial performance in certain trading markets. Ref. 14

In summary, experiments show that *all charges have mass*. And like all massive bodies, *charges move slower than light*. Charge is a property of matter; images and light have no charge.

Feeling electric fields

Why is electricity dangerous to humans? The main reason is that the human body is controlled by 'electric wires' itself. As a result, electricity applied to human bodies from the outside interferes with the internal signals. This has been known since 1789. In that year the medical doctor Luigi Galvani (b. 1737 Bologna, d. 1798 Bologna) discovered that electrical current makes the muscles of a dead animal contract. The famous first experiment used frog legs: when electricity was applied to them, they twitched violently. Subsequent investigations confirmed that all nerves make use of electrical signals. Using electricity, one can make fresh corpses move, for example. Nerves are the 'control wires' of animals.
Page 50 We will explore nerves in more detail below.

Being electrically controlled, all mammals can sense strong electric fields. Humans can sense fields as low as 10 kV/m, when hair stands on end. In contrast, several animals can sense much weaker electric (and magnetic) fields. Sharks, for example, can detect fields down to 0.5 μV/m using special sensors, the *Ampullae of Lorenzini*, which are found around their mouth. Sharks use them to detect the field created by prey moving in water; this allows them to catch their prey even in the dark.

The muscles in living prey generate electric fields. Various water animals have developed electric field sensors to detect prey in water which is too muddy to see through. The salamander and the platypus (*Ornithorhyncus anatinus*), the famous duck-billed mammal, can also sense electric fields; but they achieve only sensitivities of the order of mV/m. Numerous fish, the so-called strongly and *weakly-electric fish*, even generate electric fields in order to achieve better prey detection.* This approach is used, for example, by the elephantnose fish (*Gnathonemus petersii*). The achieved sensitivity is below 2 mV/m. In fact, various electric fish use time-varying electric dipole fields to communicate! They tell each other their species, their sex, their identity, and communicate about courtship, aggression, appeasement and dangers. The frequencies they use are in the range between a few and 200 Hz, and the fields are dipole fields created between the anterior and posterior sections of their bodies. Ref. 15 Ref. 16

No land animal has special sensors for electric fields, because any electric field in air is strongly damped when it encounters a water-filled animal body.** Indeed, the usual atmosphere has a low, vertical electric field of around 100 V/m; inside the human body this field is damped to the μV/m range, which is far less than an animal's internal electric fields. In other words, humans do not have sensors for low electric fields because they are land animals. (Do humans have the ability to sense electric fields in water? Nobody seems to know.) However, there a few exceptions. You might know that some older people can sense approaching thunderstorms in their joints. This is due the coincidence between the electromagnetic field frequency emitted by thunderclouds – around 100 kHz – and the resonant frequency of nerve cell membranes. Challenge 19 r Page 104

The water content of the human body also means that the electric fields in air that are found in nature are rarely dangerous to humans. But whenever humans consciously sense

* It took until the year 2000 for technology to make use of the same effect. Nowadays, airbag sensors in cars often use electric fields to sense whether the person sitting in the seat is a child or an adult, thus changing the way that the airbag behaves in an accident.
** Though a few land animals that swim a lot under water have electric field sensors.

TABLE 7 Searches for magnetic monopoles, i.e., for magnetic charges, in over 140 experiments.

Search	Magnetic charge
Smallest magnetic charge suggested by quantum theory	$g = \frac{h}{e} = \frac{eZ_0}{2\alpha} = 4.1\,\mathrm{pWb}$
Search in minerals, from mountains to the deep ocean	none, only dipoles Ref. 17
Search in meteorites and moon minerals	none, only dipoles Ref. 17
Search in cosmic rays	none (one false alarm in the 1970s), only dipoles Ref. 17
Search with particle accelerators	none, only dipoles Ref. 17

electric fields, such as when high voltage makes their hair stand on end, the situation is potentially dangerous.

The high impedance of air also means that, in the case of time-varying electromagnetic fields, humans are much more prone to be affected by the magnetic component than by the electric component.

Magnets and other magnetic materials

The study of magnetism progressed across the world independently of the study of electricity. Towards the end of the twelfth century, the compass came into use in Europe. At that time, there still were heated debates on whether it pointed to the north or the south. Then, in 1269, the military engineer Pierre de Maricourt (b. 1219 Maricourt, d. 1292 unknown) published his study of magnetic materials. He found that every magnet has *two* points of highest magnetization, and he called them *poles*. He found that even after a magnet is cut, the resulting pieces always retain two poles: when the stone is left free to rotate, one points to the north and the other to the south. Ref. 18

▷ All magnets are dipoles.

The two poles are called the *north pole* and the *south pole*. Maricourt also found that

▷ Like poles repel, and unlike poles attract.

As a consequence, the magnetic north pole of the Earth is the one near the south pole, and vice versa.

Magnets are surrounded by magnetic fields. Magnetic fields, like electric fields, can be visualized with field lines. Figure 14 shows some ways to do this. We directly note the main difference between magnetic and electric field lines: magnetic field lines have no beginning and no ends, whereas electric field lines do. (However, magnetic field lines are usually not closed; this only happens in very special cases.) The direction of the field lines gives the direction of the magnetic field, and the density of the lines gives the magnitude of the field.

Many systems in nature are magnets, as shown in Figure 13. The existence of two magnetic poles is valid for all magnets in nature: molecules, atoms and elementary particles are either dipoles or non-magnetic.

FIGURE 13 Various types of magnets and effective magnets: the needle in a compass, some horseshoe magnets, two galaxies, the magnetic organ of a dove, the Earth, a lifting magnet, and the Sun. (© Wikimedia, Shambhavi, Anthony Ayiomamitis, NASA).

TABLE 8 Some observed magnetic fields.

Observation	Magnetic field
Lowest measured magnetic field (e.g., fields of the Schumann resonances)	1 fT
Magnetic field produced by brain currents	0.1 pT to 3 pT
Magnetic field produced by single muscle action	1 pT
Intergalactic magnetic fields	1 pT to 10 pT
Magnetic field in the human chest, due to heart currents	100 pT
Magnetic field of our galaxy	0.5 nT
Magnetic field due to solar wind	0.2 to 80 nT
Magnetic field directly below high voltage power line	0.1 to 1 μT
Magnetic field of Earth	20 to 70 μT
Magnetic field inside home with electricity	0.1 to 100 μT
Magnetic field near mobile phone	100 μT
Magnetic field that influences visual image quality in the dark	100 μT
Magnetic field near iron magnet	100 mT
Solar spots	1 T
Magnetic fields near high technology permanent magnet	max 1.3 T
Magnetic fields that produces sense of coldness in humans	5 T or more
Magnetic fields in particle accelerator	10 T
Maximum static magnetic field produced with superconducting coils	22 T
Highest static magnetic fields produced in laboratory, using hybrid magnets	45 T
Highest *pulsed* magnetic fields produced without coil destruction	76 T
Pulsed magnetic fields produced, lasting about 1 μs, using imploding coils	1000 T
Field of white dwarf	10^4 T
Fields in petawatt laser pulses	30 kT
Field of neutron star	from 10^6 T to 10^{11} T
Quantum critical magnetic field	4.4 GT
Highest field ever measured, on magnetar and soft gamma repeater SGR-1806-20	0.8 to $1 \cdot 10^{11}$ T
Estimated magnetic field near atomic nucleus	1 TT
Maximum possible magnetic field in nature (corrected Planck magnetic field $c^3/4Ge$)	$6.3 \cdot 10^{53}$ T

▷ There are no magnetic monopoles.

Magnetic field lines could start or end at a magnetic monopole – if one existed. Despite the promise of eternal fame, no magnetic monopole has ever been found. The searches are summarized in Table 7.

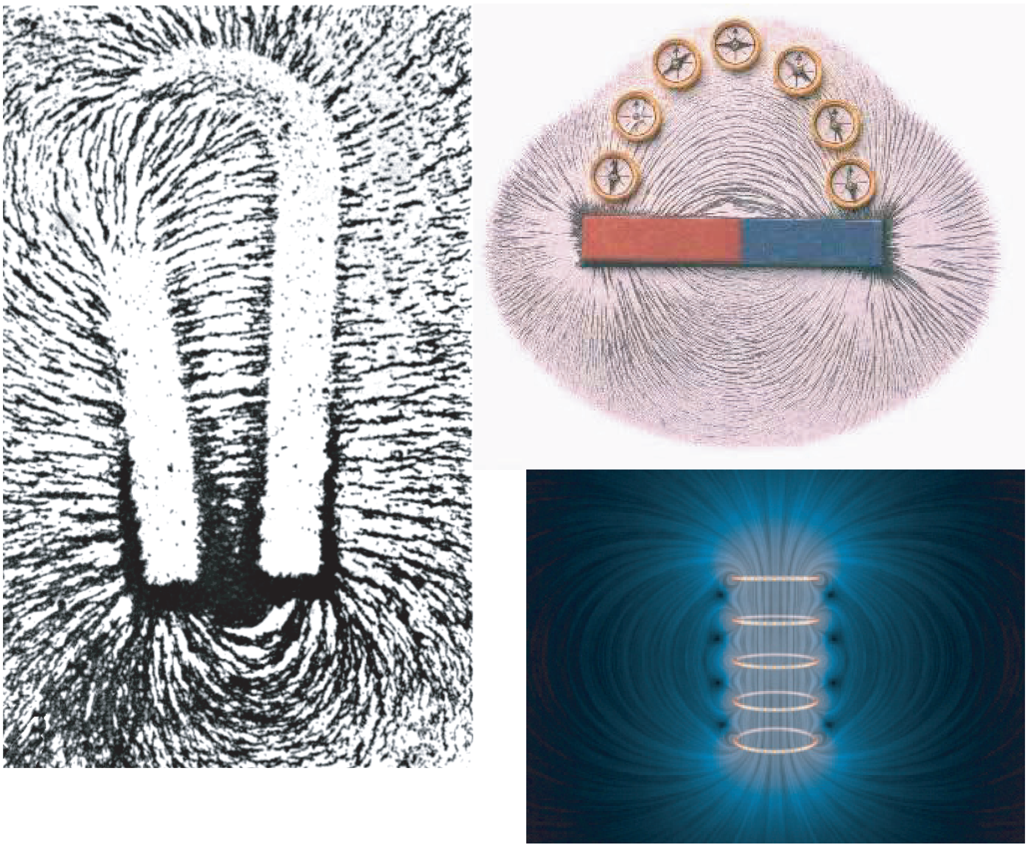


FIGURE 14 Visualizing magnetic fields around magnets and coils – with iron fillings, with compass needles and with computer graphics and with iron filings (© Wikimedia, MIT).

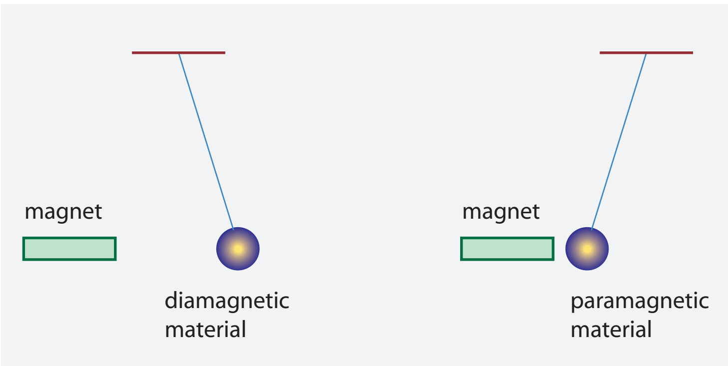


FIGURE 15 The two basic types of magnetic material behaviour (tested in an *inhomogeneous* field): diamagnetism and paramagnetism.

Magnets have a second important property, shown in Figure 15: magnets, through their magnetic field, transform non-magnetic materials into magnetic ones. There is thus a *magnetic polarization*, similar to the electric polarization. The amount of polarization depends on the material; some values are given in Table 9.

— Certain materials, the so-called *diamagnetic materials*, are *repelled* by magnets,

TABLE 9 The magnetic properties of materials.

Material	Relative magnetic permeability μ_r
Diamagnetic materials $\mu_r < 1$, repelled by magnets	
Type I superconductors	0
Highly oriented pyrolitic graphite	0.999 55
Bismuth	0.999 83
Graphite	0.999 84
Gold	0.999 966
Copper	0.999 9936
Water	0.999 9912
Usual animals and plants	like water
Paramagnetic materials $\mu_r > 1$, attracted by magnets	
Air, oxygen	1.000 0019
Biomagnetic particles in living organisms	1.000 006
Aluminium	1.000 022
Platinum	1.000 26
Ferromagnetic materials $\mu_r \gg 1$, able to form magnets	
SmCo	*c.* 1.04
NdFeB	*c.* 1.15
Cobalt	80 to 200
Nickel	100
Iron	300 to 10 000
Permalloy	*c.* 8 000
Ferrites	up to 15 000
μ-metal	up to 140 000
Amorphous metals	up to 500 000

though usually only by weak forces.
— Others, the so-called *paramagnetic* materials, are *attracted* to magnets.
— Some important materials, the *ferromagnetic materials*, such as steel, *retain* the induced magnetic polarization: they become permanently magnetized. This happens when the atoms in the material get aligned by an external magnet. Ferromagnetic materials are used to produce permanent *magnets* – thus artificial lodestone.

Magnetic materials are essential for the industrial production of electric current and are part of most devices that use electricity.

TABLE 10 The dielectric properties of materials.

Material	Relative electric permittivity ε_r
Dielectric materials	
Vacuum	1
Air	1.0006
Teflon	2.1
Graphite	10 to 15
Silicon dioxide	3.9
Silicon	11.7
Methanol	30
Water	80.1
Titanium dioxide	86-173
Paraelectric materials	
Strontium titanate (a perovskite)	310
Barium strontium titanate (a perovskite)	500
Ferroelectric materials $\varepsilon_r \gg 1$**, able to form electrets**	
Lithium niobate (below 1430 K)	...
Barium titanate	1 250 to 10 000
Ferroelectric polymers	up to 100 000
Calcium copper titanate	over 250 000

Note: the values of the electric permittivity depend on the frequency of the applied field and on the temperature. The values given here are only for static electric fields at room temperature. Values for higher frequencies Page 71 or other temperatures show strong variations.

How do animals feel magnetic fields?

> Any fool can ask more questions than seven sages can answer.
>
> Antiquity

It is known that honey bees, sharks, pigeons, the sandhill crane, various other birds, salmon, trout, sea turtles, dolphins and certain bacteria can feel magnetic fields. One Ref. 19 speaks of the ability for *magnetoreception*. All these life forms use this ability for navigation. The most common detection method is the use of small magnetic particles inside a cell; the cell then senses how these small built-in magnets move in a magnetic field. The magnets are tiny, typically around 50 nm in size. These small magnets are used to navigate along the magnetic field of the Earth. For higher animals, the variations of the magnetic field of the Earth, 20 to 70 μT, produce a landscape that is similar to the visible landscape for humans. They can remember it and use it for navigation.

In fact, migrating birds like the sandhill crane (*Grus canadensis*) seem to have two ways to sense magnetic fields. First of all, they have small iron crystals located inside

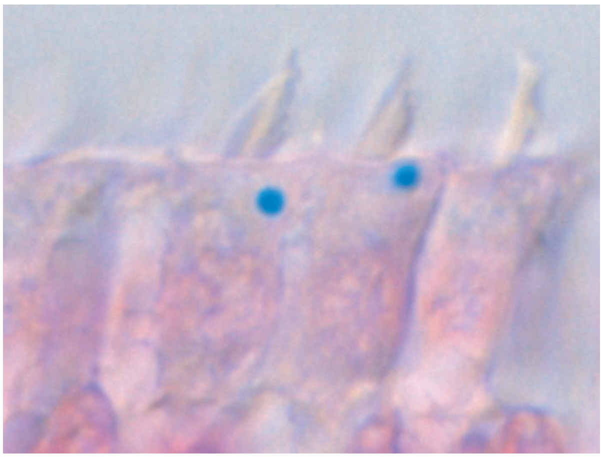


FIGURE 16 Stained cells from the inner ear of pigeons; the used chemical gives iron particles a blue colour. The magnetic particles, one in each cell, lie just beneath the hairs (© Institute of Molecular Pathology, Vienna).

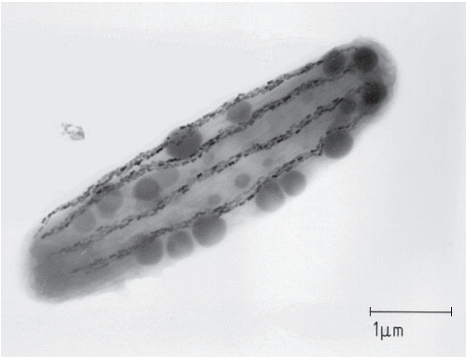


FIGURE 17 The magnetotactic bacterium *Magnetobacterium bavaricum* with its magnetosomes (© Marianne Hanzlik).

neurons that provide a magnetic map that is used for local navigation. For a long time, it was thought that these neurons were located in the skin above the beak. In recent years, it finally appeared that this often-cited 'fact' was a collective mistake; the true magnetic sensor particles are probably located in the neurons inside the ears of the birds, just below the hairs, as shown in Figure 16. The second magnetic sense of migrating birds is an inclination compass that tell them the angle between the magnetic field lines and the vertical. This system is based on magnetically sensitive protein molecules, so-called *cryptochromes*. The mechanism is located in the eye and is based on blue light. This second magnetic sense, which is still not properly understood, is used by birds to decide the general direction in which to fly.

Ref. 20

Ref. 21

Can humans feel static magnetic fields? So far, there is no definite answer. Magnetic microcrystals are present in the human brain, but whether humans can feel magnetic fields is still an open issue. Maybe you can devise a way to test the this possibility?

Challenge 20 r

In contrast, oscillating or pulsed magnetic fields can be felt by humans. There is anecdotal evidence that 0.2 T oscillating at 170 kHz leads to numbness in fingers for a few days. Beneficial effects of pulsed fields on well-being are also claimed, but are questionable; on the other hand, oscillating magnetic fields have positive effect on bone fracture healing.

Magnetism and electricity

Are magnetism and electricity related? In the early 19th century, the François Arago* discovered that they were. He explored a ship that had survived a bad thunderstorm. At that time, ships where made of wood. The ship had been struck by lightning; as a result, the ship needed a new compass. Thus lightning has the ability to demagnetize compasses. Arago knew that lightning is an electrical phenomenon. He concluded that magnetism and electricity must be related.

In short, magnetism must be related to the *motion* of electric charges. If magnetism is related to motion, it must be possible to use magnetism and electricity to move matter.

How can one make a motor?

> “Communism is the power of the local councils plus electricification of the whole country.”
>
> Lenin.**

The reason for Lenin's famous statement were two discoveries. One was made in 1820 by Hans Christian Oersted*** and the other in 1831 by Michael Faraday.**** The consequences of these experiments changed the world completely in less than one century.

On the 21st of July of 1821, Hans Christian Oersted published a leaflet, in Latin, which took Europe by storm. Oersted had found – during a lecture demonstration to his students – that when a current is sent through a wire, a nearby magnet is put into motion. In other words, he found

▷ The flow of electricity can *move* bodies.

Due to Oersted's leaflet, everybody in Europe with a bit of dexterity started to experiment with electricity. Further experiments show that *two* wires in which charges flow attract or repel each other, depending on whether the currents are parallel or antiparallel. These and other experiments show that

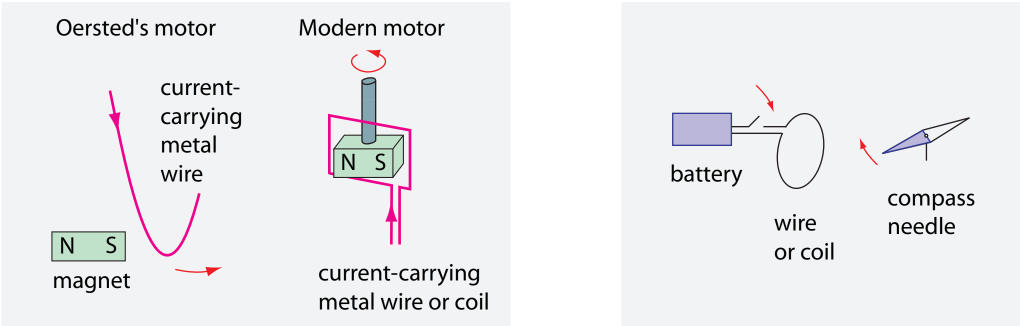
▷ Wires that carry an electric current behave like magnets.

* François Arago (b. 1786 Estagel, d. 1853 Paris) was physicist and politician; he was a friend of Alexander von Humboldt.

** Lenin (b. 1870 Simbirsk, d. 1924 Gorki), founder of the Union of Soviet Socialist Republics, in 1920 stated this as the centre of his development plan for the country. In Russian, the local councils of that time were called soviets.

*** Hans Christian Oersted (b. 1777 Rudkøbing, d. 1851 Copenhagen) physicist and professor, founded the school that later became the Technical University Denmark.

**** Michael Faraday (b. 1791 Newington Butts, d. 1867 London) was born to a simple family, without schooling, and of deep and naive religious ideas. As a boy he became assistant to the most famous chemist of his time, Humphry Davy (b. 1778 Penzance, d. 1829 Geneva). Faraday had no mathematical training, but become an influential physicist and late in his life he even became member of the Royal Society. A modest man, he refused all other honours in his life. He worked on chemical topics, the atomic structure of matter and, most of all, he developed the idea of (magnetic) fields and field lines. He used fields to describe all his numerous experimental discoveries about electromagnetism, such as the Faraday effect. Fields were later described mathematically by Maxwell, who at that time was the only person in Britain to take over Faraday's field concept.



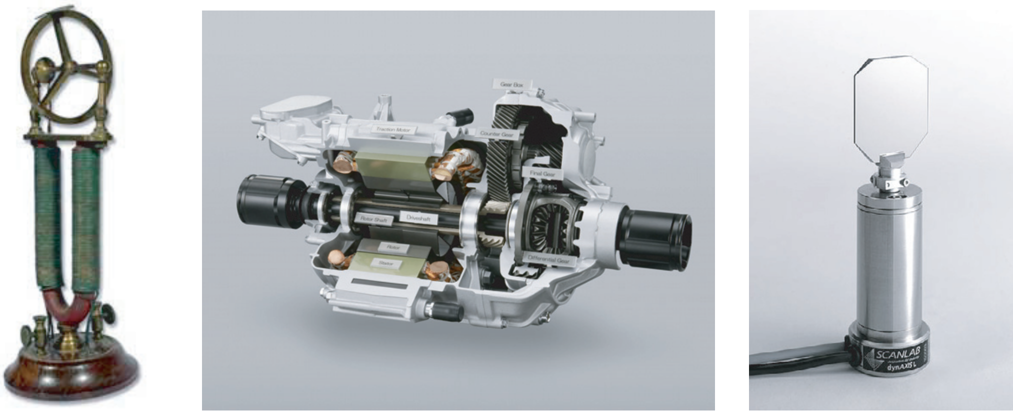


FIGURE 18 An old and a modern version of electric motor, and a mirror galvanometer with limited rotation range used for steering laser beams. Sizes are approximately 20 cm, 50 cm and 15 cm (© Wikimedia, Honda, Wikimedia).

In fact, the opposite is also true: if we imagine tiny currents moving in circles inside magnets, we get a unique description for all magnetic fields observed in nature. In other words, Oersted had found the definite proof that electricity can be turned into magnetism.

Shortly afterwards, Ampère* found that *coils* increase these effects dramatically compared to wires.

▷ Coils behave like small magnets.

* André-Marie Ampère (b. 1775 Lyon, d. 1836 Marseille), physicist and mathematician. Autodidact, he read the famous *Encyclopédie* as a child; in a life full of personal tragedies, he wandered from maths to chemistry and physics, worked as a school teacher, and published nothing of importance until 1820. Then the discovery of Oersted reached all over Europe: electrical current can deviate magnetic needles. Ampère worked for years on the problem, and in 1826 published the summary of his findings, which lead Maxwell to call him the 'Newton of electricity'. Ampère named and developed many areas of electrodynamics. In 1832, he and his technician also built the first *dynamo*, or rotative current generator. Of course, the unit of electrical current is named after him.

Ampère had two cats, which he liked dearly, a large one and a small one. When he was doing his experiments in his laboratory, they wanted to come in, and when they were in, they soon wanted to go out. One day he was fed up. He made two holes in his door, a large one and a small one.

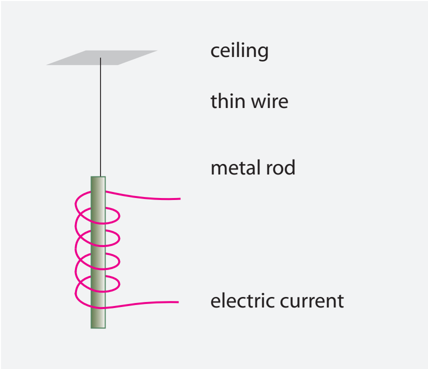


FIGURE 19 Current makes a metal rod rotate.

In particular, current-carrying coils, like magnets, always have two poles, usually called the north and the south pole. Opposite poles attract, like poles repel each other. As is well known, the Earth is itself a large magnet, with its magnetic north pole near the geographic south pole, and vice versa. However, the magnetic field of the Earth is *not* due to a solid permanent magnet inside it. The Earth's solid core is too hot to be a permanent magnet; instead, the magnetic field is due to circulating currents in the outer, liquid core. By the way, the power to keep the geodynamo running is estimated to be between 200 and 500 GW and is due to the heat in the centre of the Earth.

All the relations between electricity and magnetism can be used to make *electric motors*. First, electric current in a coil is used to generate a magnetic field; then the field is used to move a magnet attached to the motor axis. The details on how to do this effectively depend on the size of the motor one is building; they form a science on its own: electric engineering. Figure 18 shows some examples of electric motors.

Which currents flow inside magnets?

Magnetic monopoles do not exist. Therefore, all magnetic fields in nature are due to moving electric charges. But that is strange; if all magnetic fields are due to the motion of charges, this must be also the case inside lodestone, or inside a usual permanent magnet. Can this be shown?

In 1915, two men in the Netherlands found a simple way to prove that in any permanent magnet, charges are moving. They suspended a metal rod from the ceiling by a thin thread and then put a coil around the rod, as shown in Figure 19. They predicted that the tiny currents inside the rod would become aligned by the magnetic field of the coil. As a result, they expected that a current passing through the coil would make the rod turn around its axis. Indeed, when they sent a strong current through the coil, the rod rotated. (As a result of the current, the rod was magnetized.) Today, this effect is called the *Einstein–de Haas effect* after the two men who imagined, measured and explained it.*

Ref. 22

* Wander Johannes de Haas (b. 1878 Lisse, d. 1960 Bilthoven) was a physicist who is most known for two additional magneto-electric effects named after him, the *Shubnikov–de Haas effect* (the strong increase of the magnetic resistance of bismuth at low temperatures and high magnetic fields) and the *de Haas–van Alphen effect* (the diamagnetic susceptibility of bismuth at low temperatures is a periodic function of the magnetic field).

The effect thus shows that even in the case of a permanent magnet, the magnetic field is due to the internal motion of charges. The magnitude of the Einstein–de Haas effect also shows that the moving particles are electrons. Twelve years later, in 1927, it became clear that the angular momentum responsible for the effect is a mixture of orbital and spin angular momentum; in fact, the electron spin plays a central role in the effect. We will explore electron spin in the volumes on quantum theory.

Permanent magnets are made from ferromagnetic materials. Their permanent magnetization is due to the alignment of microscopic rotational motions. Due to this connection, an even more surprising effect can be predicted: Rotating a piece of non-magnetized ferromagnetic material should magnetize it, because the tiny rotating currents would
Ref. 23 then be aligned along the axis of rotation. This effect has indeed been observed; it is called the *Barnett effect* after its discoverer. Like the Einstein–de Haas effect, the magnitude of the Barnett effect can also be used to determine the gyromagnetic ratio of the
Vol. IV, page 104 electron. In short, also the Barnett effect proves that the spins of electrons (usually) play a larger role in magnetism than their orbital angular momentum.

Describing magnetic fields

All experiments show that the magnetic field has a given direction in space, and a magnitude common to all (resting) observers, whatever their orientation. We are thus tempted to describe the magnetic field by a vector. However, this would be wrong, since a magnetic field does not behave like an arrow when placed before a mirror. Imagine that a system produces a magnetic field directed to the right. You can take any system, a coil, a machine, etc. Now build or imagine a second system that is the exact mirror version of the first: a mirror coil, a mirror machine, etc. The magnetic system produced by the mirror system does not point to the left, as maybe you expected: it still points to the right.
Challenge 21 e (Check by yourself.) In simple words, magnetic fields do *not* fully behave like arrows.

In other words, it is *not* completely correct to describe a magnetic field by a vector $\boldsymbol{B} = (B_x, B_y, B_z)$, as vectors behave like arrows. The magnetic field is a *pseudovector*; angular momentum and torque are also examples of such quantities. The precise way is to describe the magnetic field by the quantity*

$$\boldsymbol{B} = \begin{pmatrix} 0 & -B_z & B_y \\ B_z & 0 & -B_x \\ -B_y & B_x & 0 \end{pmatrix} , \tag{9}$$

called an *antisymmetric tensor*.

The *magnetic field* is defined by the acceleration it imparts on moving charges. This acceleration is observed to follow

$$\boldsymbol{a} = \frac{q}{m} \boldsymbol{v} \times \boldsymbol{B} \tag{10}$$

* The quantity $\boldsymbol{B}$ was not called the 'magnetic field' until recently. We follow here the modern, logical definition, which supersedes the traditional one, where $\boldsymbol{B}$ was called the 'magnetic flux density' or 'magnetic induction' and another quantity, $\boldsymbol{H}$, was called – incorrectly, but for over a century – the magnetic field. This quantity $\boldsymbol{H}$ will not appear in this walk, but it is important for the description of magnetism in materials.

TABLE 11 Properties of the classical magnetic field: an axial vector.

Magnetic fields can	Physical property	Mathematical name	Definition
Attract currents	deflect charges	coupling	equation (10)
Repel currents	deflect charges	coupling	equation (10)
Be distinguished	distinguishability	element of set	Page 262
Change gradually	continuum	real vector space	Vol. I, page 78, Vol. V, page 359
Point somewhere	direction	vector space, dimensionality	Vol. I, page 78
Be compared	measurability	metricity	Vol. V, page 350
Be added	additivity	vector space	Vol. I, page 78
Have defined angles	direction	Euclidean vector space	Vol. I, page 78
Exceed any limit	infinity	unboundedness	Page 263
Keep direction under reflection	axiality	parity-even vector, pseudovector	
Change direction under time reversal	axiality	time-odd vector	

for a charge q with mass m. The relation is often called *Lorentz acceleration*, after the important physicist Hendrik A. Lorentz* who first stated it clearly.** The Lorentz acceleration, also called the *Laplace acceleration*, defines the magnitude and the direction of the magnetic field $\boldsymbol{B}$. The unit of the magnetic field is called tesla and is abbreviated T. We have $1\,\mathrm{T} = 1\,\mathrm{N\,s/C\,m} = 1\,\mathrm{V\,s/m^2} = 1\,\mathrm{V\,s^2/A\,m}$.

The magnetic field is defined and measured by its influence moving charges. Let us explore the definition. Does the definition of magnetic field given here assume a charge speed much lower than that of light?

Challenge 22 s

The definition of the magnetic field assumes, like that of the electric field, that the test charge q is so small that it does not disturb the field B to be measured. Again, we ignore this issue, which means that we ignore all quantum effects, until later in our adventure.

Page 231

The definition of the magnetic field also assumes that space-time is flat, and it ignores all issues due to space-time curvature.

Vol. II, page 38

* For more details about Hendrik A. Lorentz (b. 1853 Arnhem, d. 1928 Haarlem), see the volume on relativity.

Vol. I, page 108

** The expression $\boldsymbol{v} \times \boldsymbol{B}$ is the vector product of the two vectors. The most practical way to calculate the vector product $\boldsymbol{v} \times \boldsymbol{B}$ component by component is given by the determinant

$$\boldsymbol{v} \times \boldsymbol{B} = \begin{vmatrix} \boldsymbol{e}_x & v_x & B_x \\ \boldsymbol{e}_y & v_y & B_y \\ \boldsymbol{e}_z & v_z & B_z \end{vmatrix} \quad \text{or, more sloppily} \quad \boldsymbol{v} \times \boldsymbol{B} = \begin{vmatrix} + & - & + \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} . \qquad (11)$$

This is easy to remember and easy to perform, both with letters and with numerical values. (Here, $\boldsymbol{e}_x$ is the unit basis vector in the x direction.) Written out, it is equivalent to the relation

$$\boldsymbol{v} \times \boldsymbol{B} = (v_y B_z - B_y v_z, B_x v_z - v_x B_z, v_x B_y - B_x v_y) \qquad (12)$$

which is harder to remember.

TABLE 12 Some sensors for static and quasistatic magnetic fields.

Measurement	Sensor	Range
Voltage	Hall probe	up to many T
Induced electromotive force (voltage)	doves	from a few nT
Bone growth stimulation	piezoelectricity and magnetostriction of bones	from 50 mT
Induced electromotive force (voltage)	human nerves	from a few T
Sensations in thorax and shoulders	human nerves	strong switched gradients
Sharks	induced voltage when waving left to right	a few nT
Plants	unclear	small effects on growth

The Lorentz acceleration is the fundamental effect that a magnetic field has on a moving charge. The Lorentz acceleration is the effect at the root of any electric motor. An electric motor is a device that uses a magnetic field as efficiently as possible to accelerate charges flowing in a wire. Through the motion of the charges, the wire is then also moved. In an electric motor, electricity is thus transformed into magnetism and then into motion. The first efficient electric motors were built already in the 1830s.

Moving charges produce magnetic fields. Like for the electric field, we need to know how the *strength* of a magnetic field is determined by a moving charge. Experiments such as Oersted's show that the magnetic field of a point-like charge moving with velocity $\boldsymbol{v}$ produces a field $\boldsymbol{B}$ given by

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} q \frac{\boldsymbol{v} \times \boldsymbol{r}}{r^3} \quad \text{where} \quad \frac{\mu_0}{4\pi} = 10^{-7}\,\mathrm{N/A^2}\,. \tag{13}$$

This is called *Ampère's 'law'*. Again, the strange factor $\mu_0/4\pi$ is due to the historical way in which the electrical units were defined. The constant μ_0 is called the *permeability of the vacuum* and is defined by the fraction of newton per ampere squared given in the formula. It is easy to see that the magnetic field has an intensity given by $\boldsymbol{v}\boldsymbol{E}/c^2$, where $\boldsymbol{E}$

Challenge 23 e is the electric field measured by an observer moving *with* the charge. This is one of the many hints that magnetism is a relativistic effect.

We note that equation (13) is valid only for small velocities and accelerations. Can you

Challenge 24 s find the general relation?

Electromagnetism

In 1831, Michael Faraday discovered an additional piece of the jigsaw puzzle formed by electricity and magnetism, one that even the great Ampère had overlooked. He found that

▷ A *moving* magnet causes a current flow in an electrical circuit.

Magnetism can thus be turned into electricity. This important discovery allowed the production of electrical current flow by generators, so-called *dynamos*, using water power, wind power or steam power. In fact, the first dynamo was already built in 1832 by Ampère and his technician. Dynamos jump-started the use of electricity throughout the world. Behind every electrical wall plug there is a dynamo somewhere.

Oersted had found that electric current can produce magnetic fields. Faraday had found that magnetic fields could produce electric currents and electric fields. Electric and magnetic fields are thus two aspects of the same phenomenon: *electromagnetism*. It took another thirty years to unravel the full description.

Additional experiments show that magnetic fields also lead to electric fields when one changes to a moving viewpoint. You might check this on any of the examples of Figures 18 to 43.

▷ Magnetism is relativistic electricity.

Electric and magnetic fields are partly transformed into each other when switching from one inertial reference frame to the other. Magnetic and electrical fields thus behave like space and time, which are also mixed up when changing from one inertial frame to the other. In such a case, the theory of special relativity thus tells us that there must be a single concept, an *electromagnetic field*, describing them both. Investigating the details, one finds that the electromagnetic field $\boldsymbol{F}$ surrounding charged bodies has to be described by an antisymmetric 4-*tensor*

$$\boldsymbol{F}^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix} \text{ or } \boldsymbol{F}_{\mu\nu} = \begin{pmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{pmatrix} . \tag{14}$$

Obviously, the electromagnetic field $\boldsymbol{F}$, and thus every component of these matrices, depends on space and time. Above all, the matrices show that electricity and magnetism are two faces of the same effect.* In addition, since electric fields appear only in the topmost row and leftmost column, the expressions show that in everyday life, for small speeds, electricity and magnetism *can* be separated. (Why?) Challenge 25 s

Using relativistic notation, the electromagnetic field is thus defined through the 4-acceleration $\boldsymbol{b}$ that it produces on a charge q of mass m and 4-velocity $\boldsymbol{u}$:

$$m\boldsymbol{b} = q\boldsymbol{F}\boldsymbol{u}$$

or, in 3-vector notation

$$\mathrm{d}E/\mathrm{d}t = q\boldsymbol{E}\boldsymbol{v} \quad \text{and} \quad \mathrm{d}\boldsymbol{p}/\mathrm{d}t = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) . \tag{15}$$

The expressions show how the power $\mathrm{d}E/\mathrm{d}t$ (the letter E denotes energy, whereas $\boldsymbol{E}$ denotes the electric field) and the three-force $\mathrm{d}\boldsymbol{p}/\mathrm{d}t$ depend on the electric and magnetic

* Actually, the expression for the field contains everywhere the expression $1/\sqrt{\mu_o\varepsilon_0}$ instead of the speed of light c. We will explain the reason for this substitution shortly.

fields.* The 4-vector expression and the 3-vector expression describe the same content; the simplicity of the first one is the reason for the involved matrices (14) describing the electromagnetic field $\boldsymbol{F}$.

We stress that the extended *Lorentz relation* (15) is the *definition* of the electromagnetic field $\boldsymbol{F}$, since the field is defined as that 'stuff' which accelerates charges. In particular, all devices that put charges into motion, such as batteries and dynamos, as well as all devices that are put into motion by flowing charges, such as electric motors and muscles, are described by this relation. That is why this relation is usually studied, in the 3-vector form, already in secondary school. The Lorentz relation describes all cases in which the motion of objects can be seen by the naked eye or felt by our senses, such as the movement of an electrical motor in a high speed train, in a lift and in a dental drill, the motion of the picture generating electron beam in a cathode ray tube inside an old television, or the travelling of an electrical signal in a cable and in the nerves of the body. Ref. 24, Ref. 25

In summary, we found that the interaction between charges can be described in two statements: First, charges *produce* electric and magnetic fields; second, charges *are affected* by electric and magnetic fields.

The invariants and the Lagrangian of electromagnetic fields

The electromagnetic field tensor $\boldsymbol{F}$ is an *antisymmetric* 4-tensor. (Can you write down the relation between $\boldsymbol{F}^{\mu\nu}$, $\boldsymbol{F}_{\mu\nu}$ and $\boldsymbol{F}^{\mu}{}_{\nu}$?) Like any antisymmetric tensor, the electromagnetic field has two *invariants*, i.e., two deduced properties that are the same for every observer. The first invariant is the expression Challenge 26 e

$$B^2 - E^2/c^2 = \frac{1}{2}\mathrm{tr}\,\boldsymbol{F}^2 \qquad (17)$$

and the second invariant is the product

$$4\boldsymbol{EB} = -c\ \mathrm{tr}\,\boldsymbol{F}^*\boldsymbol{F}\ . \qquad (18)$$

Can you confirm the two invariants, using the definition of trace as the sum of the diagonal elements? Challenge 27 s

The first invariant expression, $B^2 - E^2/c^2 = \frac{1}{2}\mathrm{tr}\,\boldsymbol{F}^2$, turns out to be (proportional to) the Lagrangian density of the electromagnetic field. In particular, this first invariant is a scalar. This first invariant implies that if E is larger, smaller or equal to cB for one observer, it also is for all other observers. Like for all intensive quantities that evolve, the

* In component notation, using the convention to sum over Greek indices that appear twice, the definition of the Lorentz force is

$$mb^\mu = m\frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} = q\boldsymbol{F}^\mu{}_\nu u^\nu \quad \text{or}$$

$$m\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix}\gamma c\\ \gamma v_x\\ \gamma v_y\\ \gamma v_z\end{pmatrix} = q\begin{pmatrix}0 & E_x/c & E_y/c & E_z/c\\ E_x/c & 0 & B_z & -B_y\\ E_y/c & -B_z & 0 & B_x\\ E_z/c & B_y & -B_x & 0\end{pmatrix}\begin{pmatrix}\gamma c\\ \gamma v_x\\ \gamma v_y\\ \gamma v_z\end{pmatrix}. \qquad (16)$$

Lagrangian is proportional to the *square* of the intensive quantity. The minus sign in the expression is the same minus sign that appears also in $c^2t^2-x^2$: it results from the mixing of electric and magnetic fields that is due to boosts.

The Lagrangian density can be used to define the classical action of the electromagnetic field:

$$S = \int \frac{\varepsilon_0}{2}E^2 - \frac{1}{2\mu_0}B^2 \mathrm{d}t\mathrm{d}V . \tag{19}$$

As usual, the action measures the *change* occurring in a system; it thus defines the amount Vol. IV, page 47 of change that occurs when an electromagnetic field moves. (The expression for the change, or action, of a moving light beam reduces to the product of its intensity and total phase change.) The action of an electromagnetic field thus increases with its intensity and with its frequency. As usual, this expression for the action can be used to describe the motion of the electromagnetic field by using the *principle of least action*. Indeed, the principle implies the evolution equations of the electromagnetic field, which are called *Maxwell's field equations of electrodynamics*. This approach is the simplest way to deduce Page 74 them. We will discuss the field equations in detail shortly.

The second invariant of the electromagnetic field tensor, $4\boldsymbol{EB} = -c \operatorname{tr} \boldsymbol{F}^*\boldsymbol{F}$, is a pseudoscalar; it describes whether the angle between the electric and the magnetic field is acute or obtuse for all observers.*

The uses of electromagnetic effects

The application of electromagnetic effects to daily life has changed the world. For example, the installation of electric lighting in city streets has almost eliminated the previously so common night assaults. These and all other electrical devices exploit the fact that charges can flow in metals and, in particular, that electromagnetic energy can be transformed

— into mechanical energy – as done in loudspeakers, motors and muscles;
— into light – as in lamps, lasers, glass fibres, glow worms, giant squids and various deep ocean animals;
— into heat – as in electric ovens, blankets, tea pots and by electric eels to stun and kill prey;
— into chemical effects – as in hydrolysis, battery charging, electroplating and the brain;

* There is in fact a third Lorentz invariant, far less known. It is specific to the electromagnetic field and is a combination of the field and its vector potential:

$$\begin{aligned}\kappa_3 &= \frac{1}{2}A_\mu A^\mu \boldsymbol{F}_{\rho\nu}\boldsymbol{F}^{\nu\rho} - 2A_\rho \boldsymbol{F}^{\rho\nu}\boldsymbol{F}_{\nu\mu}A^\mu \\ &= (\boldsymbol{A}\cdot\boldsymbol{E})^2 + (\boldsymbol{A}\cdot\boldsymbol{B})^2 - |\boldsymbol{A}\times\boldsymbol{E}|^2 - |\boldsymbol{A}\times\boldsymbol{B}|^2 + 4\frac{\varphi}{c}(\boldsymbol{A}\cdot\boldsymbol{E}\times\boldsymbol{B}) - \left(\frac{\varphi}{c}\right)^2(E^2+B^2) .\end{aligned} \tag{20}$$

Ref. 26 This expression is Lorentz (but not gauge) invariant; knowing it can help clarify unclear issues, such as the lack of existence of waves in which the electric and magnetic fields are parallel. Indeed, for plane monochromatic waves all three invariants *vanish* in the Lorentz gauge. Also the quantities $\partial_\mu j^\mu$, $j_\mu A^\mu$ – j being the Challenge 84 · Page 284 electric current – and $\partial_\mu A^\mu$ are Lorentz invariants. (Why?) The last one, the frame independence of the divergence of the four-potential, reflects the invariance of gauge choice. The gauge in which the expression is set to zero is called the *Lorentz gauge*.

— into coldness – as in refrigerators and Peltier elements, but in no known living system;
— into radio wave signals – as in radio and television, but in no known living system;
— into stored information – as in magnetic records, computers, animal and human memory.

Due to all these options, electrical light, lasers, batteries, electric motors, refrigerators, radio, telephones, X-rays, television and computers have changed human life completely in less than one century.

Electromagnetic effects are thus useful to perform something at a specific place and time, thus to realize actuators. In addition, electromagnetic effects are useful so capture Page 216 information from the environment, thus to realize sensors.

Many of these uses of electromagnetism also occur in biological systems. However, Challenge 29 s no biological system makes use of X-rays, though. (Why?) No living being seems to use electric cooling. (Why?) And could there be biological systems that communicate via Challenge 30 s radio waves?

How do nerves work?

Nerves are wonders. Without nerves, we would not experience pleasure, we would not experience pain, we would not see and we would not hear. Without nerves, we would not live. But how do nerves transport signals?

Page 33 In 1789, as mentioned above, Luigi Galvani discovered that nerves transport electric signals, by doing experiments with frog legs. Are nerves wires? One and a half centuries after Galvani it became clear that nerves, more precisely, nerve axons, do not conduct electricity using electrons, as metal wires do, but by using *ions*. Nerve signals propagate using the motion of sodium Na^+ and potassium K^+ ions through the cell membrane of the nerve. The resulting signal speed is between 0.5 m/s and 120 m/s, depending on the type of nerve. (Nerve axons coated with myelin, a protein that acts as an electric insulator, are faster than uncoated axons.) The signal speed is sufficient for the survival of most species – it helps the body to run away in case of danger.

Nerves differ from wires in another aspect: they cannot transmit constant voltage signals, but only signal *pulses*. The first, approximate model for this behaviour was presented Ref. 27 in 1952 by Hodgkin and Huxley. Using observations about the behaviour of potassium and sodium ions, they deduced an elaborate evolution equation that describes the voltage V in nerves, and thus the way the signals propagate. The equation reproduces the characteristic voltage spikes measured in nerves, shown in Figure 20.

The precise mechanism with which ions cross the membranes, using so-called *channel proteins*, was elucidated only twenty years later. Despite this huge body of work, and even though Hodgkin and Huxley received the Nobel Prize for Medicine for their work, the model cannot be correct. The model does not explain the reversibility of the propagation process, the observed thickness change of the nerve during propagation nor the excitation of nerves by simple deformation or temperature changes; most of all, the model does not explain the working of anaesthetics. The detailed working of nerves remained unknown.

Only around the year 2000 did Thomas Heimburg and his team discover the way sigRef. 28 nals propagate in nerves. He showed that a nerve pulse is an electromechanical solitonic

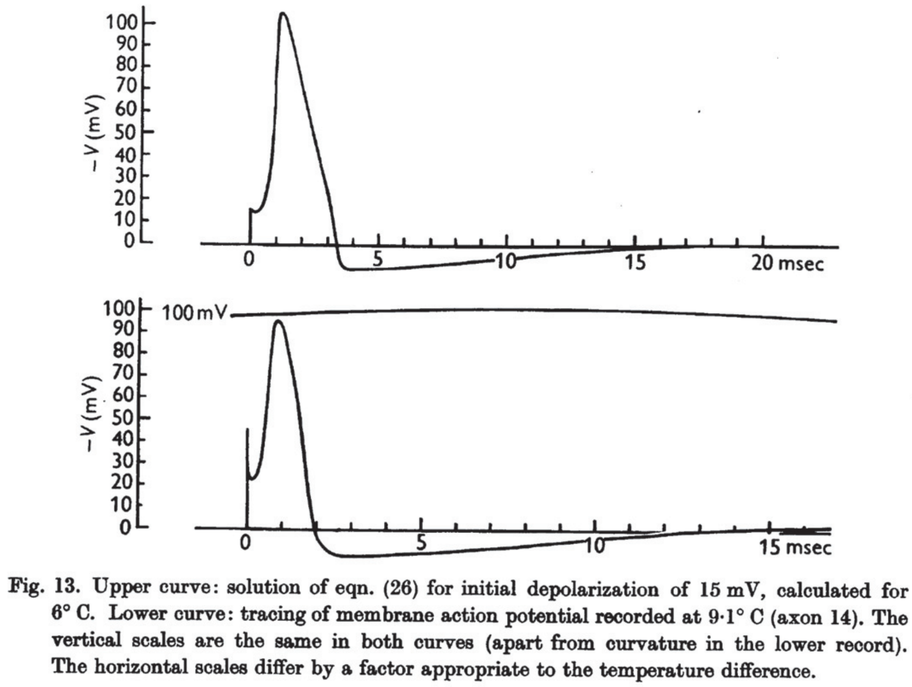


FIGURE 20 The electrical signals calculated (above) and measured (below) in a nerve, following Hodgkin and Huxley.

wave of the cylindrical membrane. In the cylindrical membrane, the protein structure changes from liquid to solid and back to liquid. A short, slightly thicker ring of solid proteins propagates along the cylinder: that is the nerve pulse. The model is shown in Figure 21. (The term 'solid' has a precise technical meaning in two-dimensional systems and describes a specific ordered state of the molecules.) This model explains all the properties of nerve pulses that were unexplained before. In particular, it explains that anaesthetics work because they dissolve in the membrane and thus block the formation and the propagation of the rings. All quantitative predictions of the model match observations.

In summary, nerve signals are electromechanical pulses; they are a mixture of current and sound waves. The electromechanical model of nerves explains how signals propagate, how pain is felt and why no pain is felt during anaesthesia.

Interestingly, the electromechanical model of nerve pulse propagation does not (yet) explain why we loose consciousness during anaesthesia. This is an additional process that takes place in the brain. It is known that loss of consciousness is related to the change of brain waves, but the details are still a topic of research. Brains still have wonderful properties to be explored.

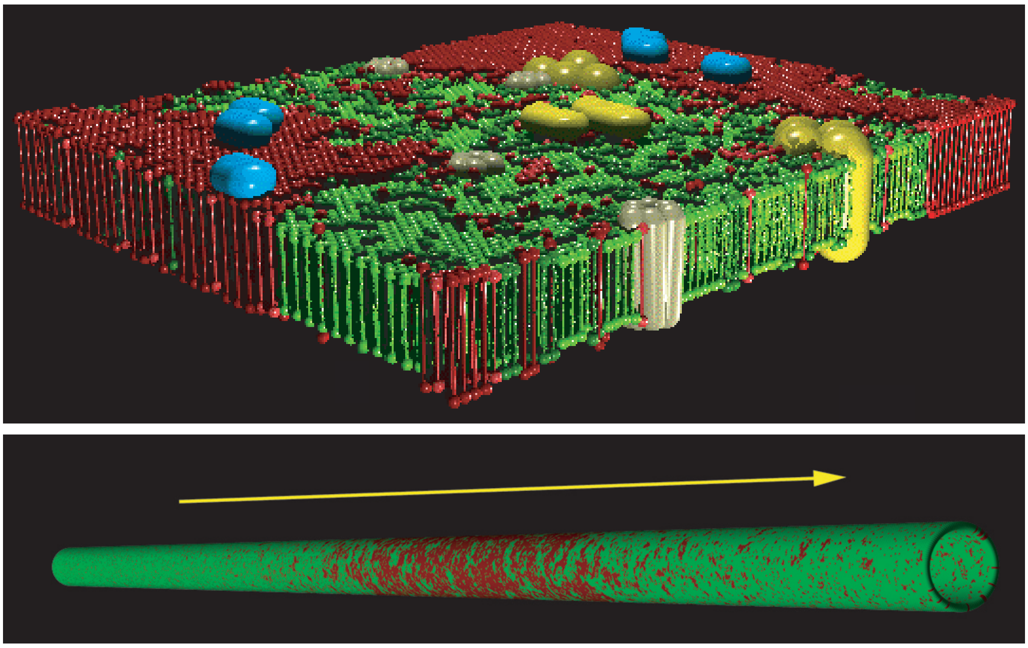


FIGURE 21 Top: A biomembrane, with solid-type lipids (red), liquid lipids (green) and various dissolved proteins (yellow, blue, white). Bottom: a nerve pulse propagating as a two-dimensional phase transformation liquid/solid/liquid along a cylindrical nerve membrane (© Thomas Heimburg/Wiley-VCH).

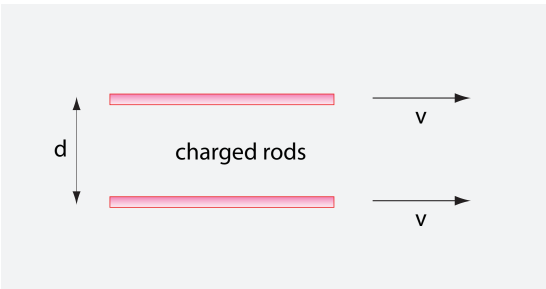


FIGURE 22 The relativistic aspect of magnetism.

How motors prove relativity to be right

> "The only mathematical operation I performed in my life was to turn the handle of a calculator."
> Michael Faraday

All electric motors are based on the result that electric currents interact with magnetic fields. The simplest example is the attraction of two wires carrying parallel currents. This observation alone, made in 1820 by Ampère, is sufficient to make motion larger than a certain maximal speed impossible. The argument is beautifully simple. Ref. 29

We change the original experiment and imagine two long, electrically charged rods of mass m, moving in the same direction with velocity v and separation d. An observer

Challenge 31 e moving with the rods would see an electrostatic repulsion between the rods given by

$$ma_e = -\frac{1}{4\pi\varepsilon_0}\frac{2\lambda^2}{d} \tag{21}$$

where λ is the charge per length of the rods. A second, *resting* observer sees two effects: the electrostatic repulsion and the attraction discovered by Ampère. The second observer Challenge 32 e therefore observes

$$ma_{em} = -\frac{1}{4\pi\varepsilon_0}\frac{2\lambda^2}{d} + \frac{\mu_0}{2\pi}\frac{\lambda^2 v^2}{d} \,. \tag{22}$$

This expression must be consistent with the observation of the first observer. This is the case only if both observers find repulsions. It is easy to check that the second observer sees a repulsion, as does the first one, only if

$$v^2 < \frac{1}{\varepsilon_0\mu_0} = c^2 \,. \tag{23}$$

This maximum speed c, with a value of 0.3 GM/s, is thus valid for any object carrying charges. But *all* everyday objects contain charges: there is thus a maximum speed for matter.

Are you able to extend the argument for a maximum speed to neutral particles as well? Challenge 33 d We will find out more on this limit velocity, which we know already, in a minute.

Another argument for magnetism as a relativistic effect is the following. In a wire with electrical current, the charge is zero for an observer at rest with respect to the wire: the wire is *neutral* for that observer. The reason is that the charges enter and exit the wire at the same time for that observer. Now imagine an observer who flies along the wire. The entrance and exit events do not occur simultaneously any more; the wire is *charged* for a moving observer. (The charge depends on the direction of the observer's motion.) Now imagine that the moving observer is electrically charged. He will be attracted or repelled by the wire, because for him, the wire is charged. The moving observer will say that the attraction is due to the *electric* field of the wire. The observer at rest will also note the attraction or repulsion of the moving observer, but since for him, the wire is neutral, he will deduce that moving charges experience a force – possibly with a slightly different value, but this is a technicality – due to the electric current in the wire; the observer at rest will thus say that a wire with a current is surrounded by a *magnetic* field which only produces an effect on charges that move.

In summary, electric effects are due to more or less static electric charges and to their electric fields; magnetism, magnetic effects and magnetic fields are due to *moving* electric charges.* The existence of magnetic fields is a relativistic consequence of the existence of electric fields. In particular, magnetism is *not* due to particles with magnetic charges. Such particles, called magnetic monopoles, do not exist. (Magnetic charges can be intro-

Page 232 * 'Electrons move in metal with a speed of about 1 μm/s; thus if I walk with the same speed along a cable carrying a constant current, I should not be able to sense any magnetic field.' What is wrong with this Challenge 34 d argument?

TABLE 13 Voltage values observed in nature.

Observation	Voltage
Smallest measured voltage	*c.* 10 fV
Human nerves	70 mV
Volta cell	1 V
Voltaic cell ('battery')	1.5 V
Mains in households	230 V or 110 V
Electric eel	100 to 600 V
Tramway supply	500 V
Sparks when rubbing a polymer pullover	1 kV
Electric fence	0.7 to 10 kV
Train supply	15 kV
Ignition plug in cars	15 kV
Colour television cathode ray tube	30 kV
X-ray tube	30 to 200 kV
Electron microscopes	0.5 kV to 3 MV
Stun gun	65 to 600 kV
Lightning stroke	10 to 100 MV
Record accelerator voltage	1 TV
Maximum possible voltage in nature (corrected Planck voltage $\sqrt{\hbar c^5/4G}/e$)	$6.1 \cdot 10^{27}$ V

Page 92 duced as a mathematical tool, though, for the description of materials.) The strength of magnetism, used in any running electric motor, proves relativity right: there is a maximum speed in nature for all masses and charges. Both electric and magnetic fields carry energy and momentum. They are two faces of the same coin.

Curiosities and fun challenges about things electric and magnetic

> Alii vero et facta mirati et intellecta assecuti.*
> Augustine of Hippo

Before we study the motion of an electromagnetic field in detail, let's have some fun with electricity.

* *

Nowadays, having fun with sparks is straightforward. Tesla coils, named after Nikola Tesla** are the simplest devices that allow long sparks to be produced at home. Atten-

* 'Others however marvelled about the facts and understood their meaning.' Augustine, Sermon 98, 3. Augustine of Hippo (b. 354 Tagaste, d. 430 Hippo Regius) is an influential moral theologian. Despite this, he did not take care of his extramarital son, nor of his son's mother, because his own mother had forbidden him to do so.

** Никола Тесла (b. 1856 Smiljan, d. 1943 New York City), engineer and inventor. He invented and pro-

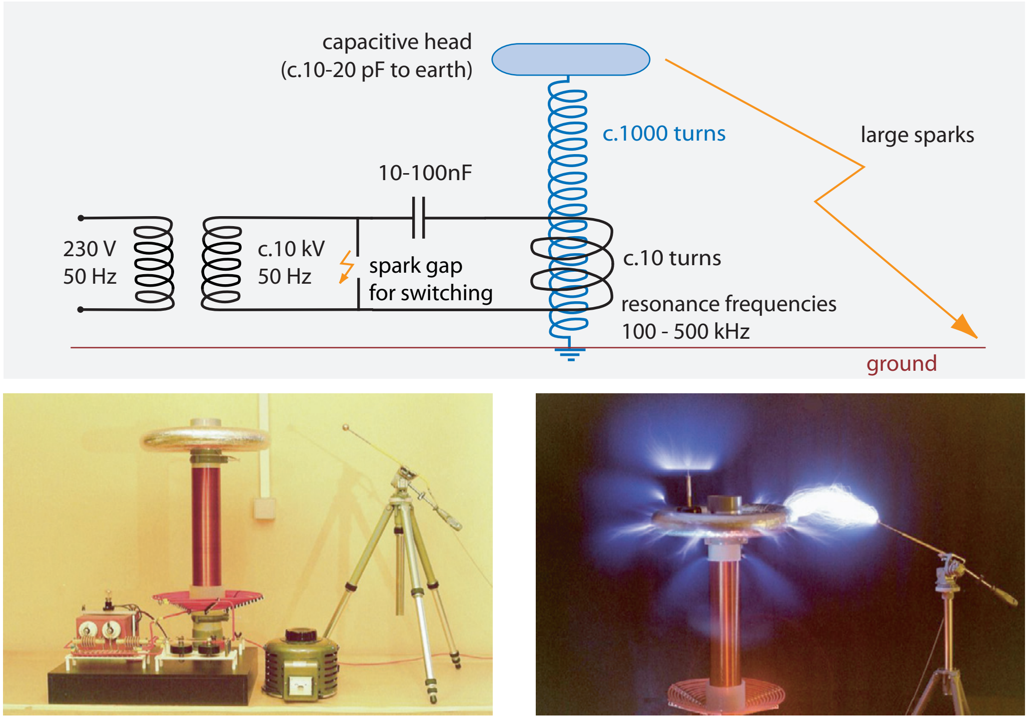


FIGURE 23 The schematics, the realization and the operation of a Tesla coil, including spark and corona discharges (photographs © Robert Billon).

tion: this is dangerous; that is the reason that such devices cannot be bought (almost) anywhere. The basic diagram and an example is shown in Figure 23. Tesla coils look like large metal mushrooms (to avoid unwanted discharges) and plans for their construction can be found on numerous websites or from numerous enthusiast's clubs, such as www.stefan-kluge.de.

* *

Challenge 35 s In 1722, George Graham discovered, by watching a compass needle, that the magnetic field of the Earth shows daily variations. Can you imagine why these variations occur?

* *

Challenge 36 d If even knocking on a wooden door is an electric effect, we should be able to detect fields when doing so. Can you devise an experiment to check this?

* *

Birds come to no harm when they sit on unprotected electricity lines. Nevertheless, one almost never observes any birds on tall, high voltage lines of 100 kV or more, which

moted the polyphase alternating current system, the alternating current electric motor, wireless communication, fluorescent lighting and many other applications of electricity. He is also one of the inventors of radio. The SI unit of the magnetic field is named after him. A flamboyant character, his ideas were sometimes unrealistic; for example he imagined that Tesla coils could be used for wireless power transmission.

Challenge 37 s transport power across longer distances. Why?

* *

How can you distinguish a magnet from an non-magnetized metal bar of the same size and material, using no external means? Challenge 38 s

* *

In the basement of a house there are three switches that control three light bulbs in the first floor. You are in the basement and are allowed to go to the first floor only once. How do you find out which switch controls which bulb? Challenge 39 s

* *

How do you wire up a light bulb to the mains and three switches so that the light can be switched on at any of the switches and off at any other switch? And for four switches? Nobody will take a physicist seriously who is able to write Maxwell's equations but cannot solve this little problem. Challenge 40 s

* *

The first appliances built to generate electric currents were large rubbing machines. Then, in 1799 Alessandro Volta (b. 1745 Como, d. 1827 Como) invented a new device to generate electricity and called it a *pile*; today its basic element is called a *(voltaic) cell*, a *primary cell** or, less correctly, a *battery*. (Correctly speaking, a battery is a collection of cells, as the one found in a car.) Voltaic cells are based on chemical processes; they provide much more current and are smaller and easier to handle than electrostatic machines. The invention of the battery changed the investigation of electricity so profoundly that Volta became world famous. At last, a simple and reliable source of electricity was available for use in experiments; unlike rubbing machines, cells and piles are compact, work in all weather conditions and make no noise.

An apple or a potato or a lemon with a piece of copper and one of zinc inserted is one of the simplest possible voltaic cells. It provides a voltage of about 1 V and can be used to run digital clocks or to produce clicks in headphones. Volta was also the discoverer of the charge 'law' $q = CU$ for capacitors (C being the capacity, and U the voltage) and the inventor of the high sensitivity capacitor electroscope. A modest man, nevertheless, the unit of electrical potential, or 'tension', as Volta used to call it, was deduced from his name. A 'battery' is a large number of voltaic cells; the term was taken from an earlier, almost purely military use.** A battery in a mobile phone is just an elaborated replacement for a number of apples or potatoes.

* *

Voltaic cells exist in all biological cells. For halobacteria, the internal voltaic cells are even essential to survival. Living in saltwater, internal voltaic cells help them to avoid death due to osmosis.

* A *secondary cell* is a rechargeable cell.

** A pile made of sets of a zinc plate, a sheet of blotting paper soaked with salt water and a copper coin is easily constructed at home and tested with a calculator or a digital watch. Challenge 41 e

FIGURE 24 A common playground effect (© Evan Keller).

∗ ∗

What happened in Figure 24? Why are most of such pictures taken in good weather and with blond children? Challenge 42 d

∗ ∗

A PC or a telephone can communicate without wires, by using radio waves. Why are these and other electrical appliances not able to obtain their *power* via radio waves, thus eliminating power cables? Challenge 43 s

∗ ∗

Magnetic storage looks far less mysterious if it is visualized. Figure 25 shows how simply with can be done. The method also allows taking films. What happens inside a metal when it is magnetized? The beautiful films at www.youtube.com/watch?v=HzxTqQ40wSU and www.youtube.com/watch?v=LFC6tbbMUaA, taken by Hendryk Richert of Matesy, show how the magnetization regions change when a magnet is approached to a piece of metal. Also these films have been made with a simple microscope, using as only help a polarizer and a layer of yttrium iron garnet deposited on glass.

∗ ∗

Also plants react to magnetic fields. In particular, different magnetic fields yield different growth patterns. The mechanisms, related to the cryptochrome system, are still a subject of research.

∗ ∗

Magnets can be used to accelerate steel balls. The most famous example is the *Gauss rifle* shown in Figure 26. If the leftmost ball is gently rolled towards the first magnet, the third ball is strongly kicked away. Then the process repeats: the speed increases even more for the fifth, the seventh and the ninth ball. The experiment never fails to surprise whoever

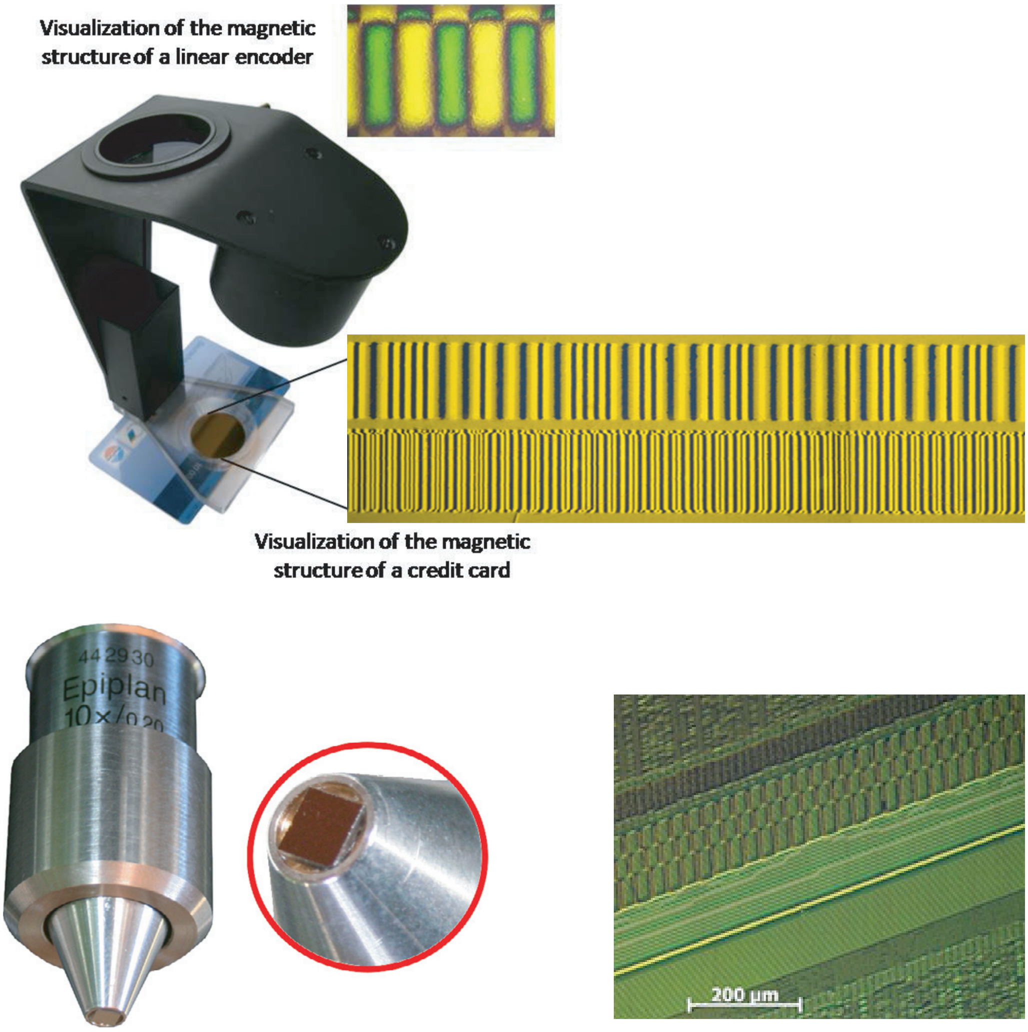


FIGURE 25 Top: how to see the information stored in the magnetic stripe on a credit card without any electronics, just using a lens, a polarizer and a magneto-optic layer; bottom: how to see the information on a hard disk in the same way, by adding a simple coated glass plate to a polarizing microscope (© Matesy).

Challenge 44 e sees it for the first time. Where does the momentum of the final ball come from?

* *

Objects that are not right–left symmetric are called *chiral*, from the Greek word for 'hand'. Can you make a mirror that does *not* switch chirality (i.e., does not 'switch left and right')? In two different ways? Challenge 45 s

* *

An adhesive tape roll is a dangerous device. Pulling the roll quickly leads to light emission (through triboluminescence) and to small sparks. It is suspected that several explosions

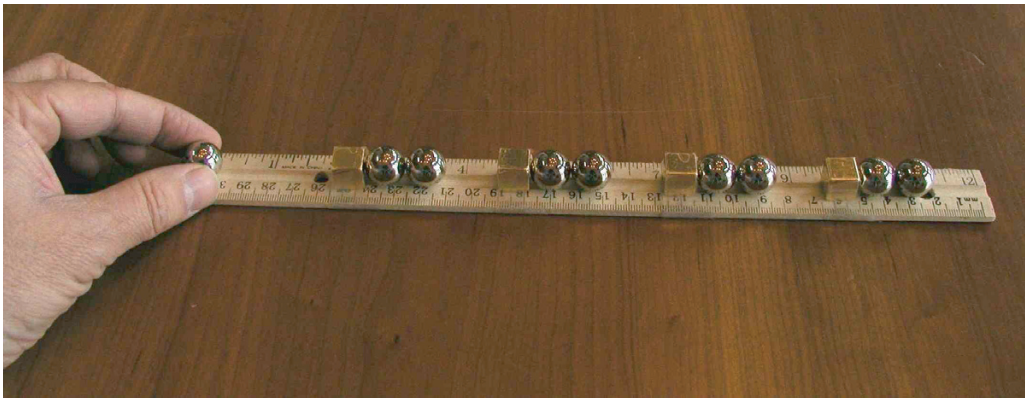


FIGURE 26 A Gauss rifle, made with a few steel balls and four magnets attached to a ruler with scotch tape (© Simon Quellen Field).

in mines were triggered when such a spark ignited a combustible gas mixture.

* *

Challenge 46 s

Take an envelope, wet it and seal it. After letting it dry for a day or more, open it in the dark. At the place where the two sides of paper are being separated from each other, the envelope glows with a blue colour. Why? Is it possible to speed up the test using a hair dryer?

* *

Challenge 47 e

A charge in an electric field feels a force. In other words, electric field produce a *potential energy* for charges. Since energy is conserved, electric potential energy can be transformed into kinetic energy or in thermal energy. What do these possibilities allow doing? What do they prevent from doing?

* *

Electromagnetism is full of surprises and offers many effects that can be reproduced at home. The internet is full of descriptions of how to construct Tesla coils to produce sparks, coil guns or rail guns to shoot objects, electrostatic machines to make your hair stand on end and much more. If you like experiments, just search for these terms. Some people earn their living by showing high voltage effects on stage, such as long discharges from their fingers or hair. A well-known example is Robert Krampf, also called 'Mr. Electricity', at thehappyscientist.com. Do not emulate these performers; it is rarely told that several of them have suffered dangerous accidents while doing so.

* *

Ref. 30

The moving discharges seen in so many displays, called *plasma globes*, are produced in a glass bowl filled with helium, neon or another inert gas at low pressure, typically 0.1 to 10 kPa, an applied voltage of 5 to 10 kV and usually a frequency of 30 to 40 kHz. At these conditions, the ion temperature of the discharges is room temperature, so that there is no danger; the electron temperature, which cannot be felt, is around 20 000 K. Approaching

FIGURE 27 A dangerous hobby, here demonstrated by Robert Krampf (© Wikimedia).

the hand to the sphere changes the electric potential and this also the shape of the discharges. If you approach a fluorescent tube to such a set-up, it will start glowing; and by moving your finger on the tube, you can 'magically' change the glow region. The internet is full of information on plasma globes.

* *

A high voltage can lead to current flow through air, because air becomes conductive in high electric fields. In such discharges, air molecules are put in motion. As a result, one can make objects that are attached to a pulsed high tension source lift up in the air, if one optimizes this air motion so that it points downwards everywhere. The high tension is thus effectively used to accelerate ionized air in one direction and, as a result, an object will move in the opposite direction, using the same principle as a rocket. An example is shown in Figure 29, using the power supply of a PC monitor. (Watch out: danger!) Numerous websites explain how to build these so-called lifters at home; in Figure 29, the bottle and the candle are used as high voltage insulator to keep one of the two thin high voltage wires (not visible in the photograph) high enough in the air, in order to avoid discharges to the environment or to interfere with the lifter's motion. Unfortunately, the majority of websites – not all – give incorrect or confused explanations of the phenomenon. These websites thus provide a good challenge for one to learn to distinguish fact from speculation. Challenge 48 e

* *

The electric effects produced by friction and by liquid flow are usually small. However, in the 1990s, a number oil tankers disappeared suddenly. The sailors had washed out the oil tanks by hosing sea water onto the tank walls. The spraying led to charging of the tank; a discharge then led to the oil fumes in the tank igniting. This led to an explosion and

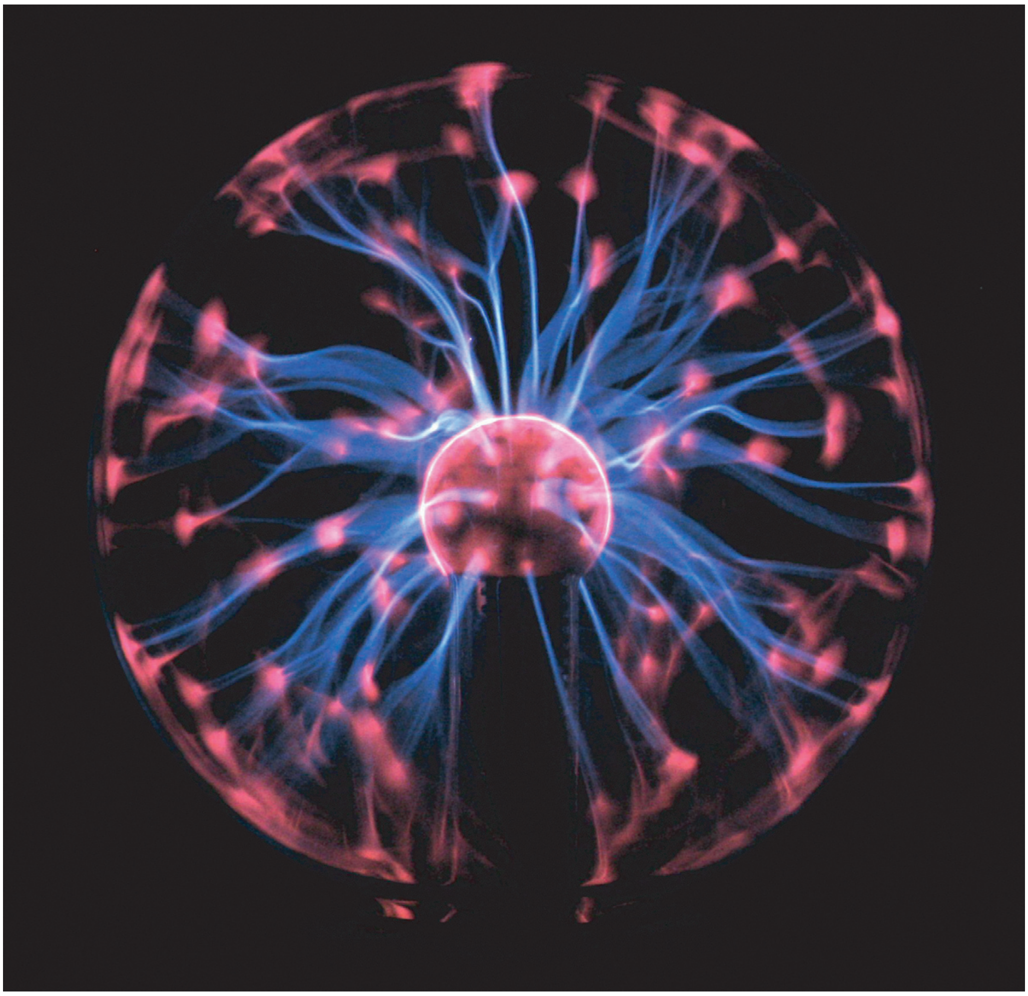


FIGURE 28 A low pressure glass sphere, or plasma globe, with a diameter of 30 cm and a built-in high voltage generator, showing its characteristic electric discharges. In a usual plasma globe, the discharges move around – slowly and irregularly. (© Philip Evans).

subsequently the tankers sank. Similar accidents also happen regularly when chemicals are moved from one tank to another.

* *

Rubbing a plastic spoon with a piece of wool charges it. Such a charged spoon can be used to extract pepper from a salt–pepper mixture by holding the spoon over the mixture. Why?

Challenge 49 s

* *

When charges move, they produce a magnetic field. In particular, when ions inside the Earth move due to heat convection, they produce the Earth's magnetic field. When the ions high up in the atmosphere are moved by solar wind, a geomagnetic storm appears;

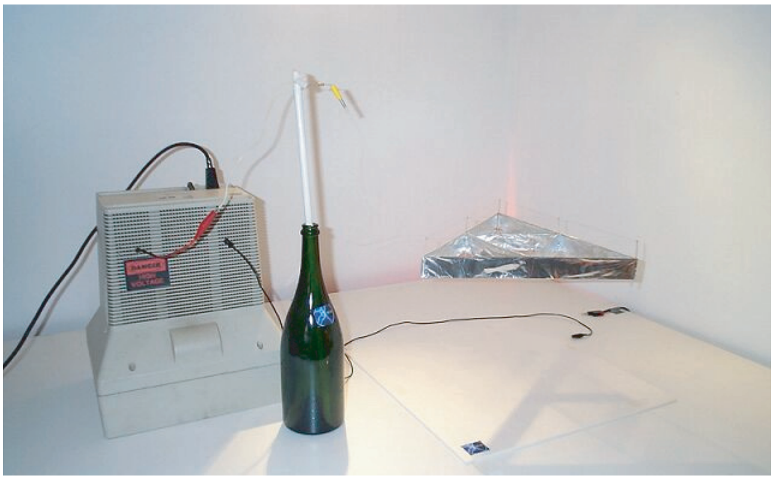


FIGURE 29 Lifting a light object – covered with aluminium foil – using high a tension discharge (© Jean-Louis Naudin at www.jlnlabs.org).

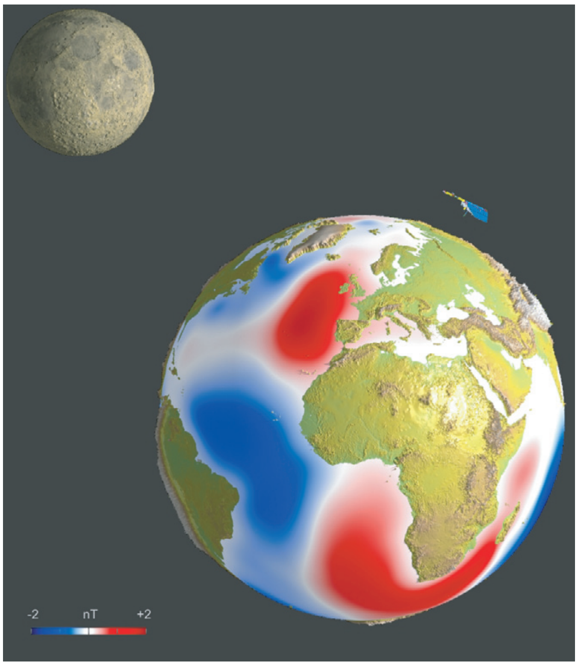


FIGURE 30 The magnetic field due to the tides (© Stefan Maus).

its field strength can be as high as that of the Earth itself. In 2003, an additional mechanism was discovered. When the tides move the water of the oceans, the ions in the salt water produce a tiny magnetic field; it can be measured by highly sensitive magnetometers in satellites orbiting the Earth. After two years of measurements from a small satellite it was possible to make a beautiful film of the oceanic flows. Figure 30 gives an impression. Ref. 31

* *

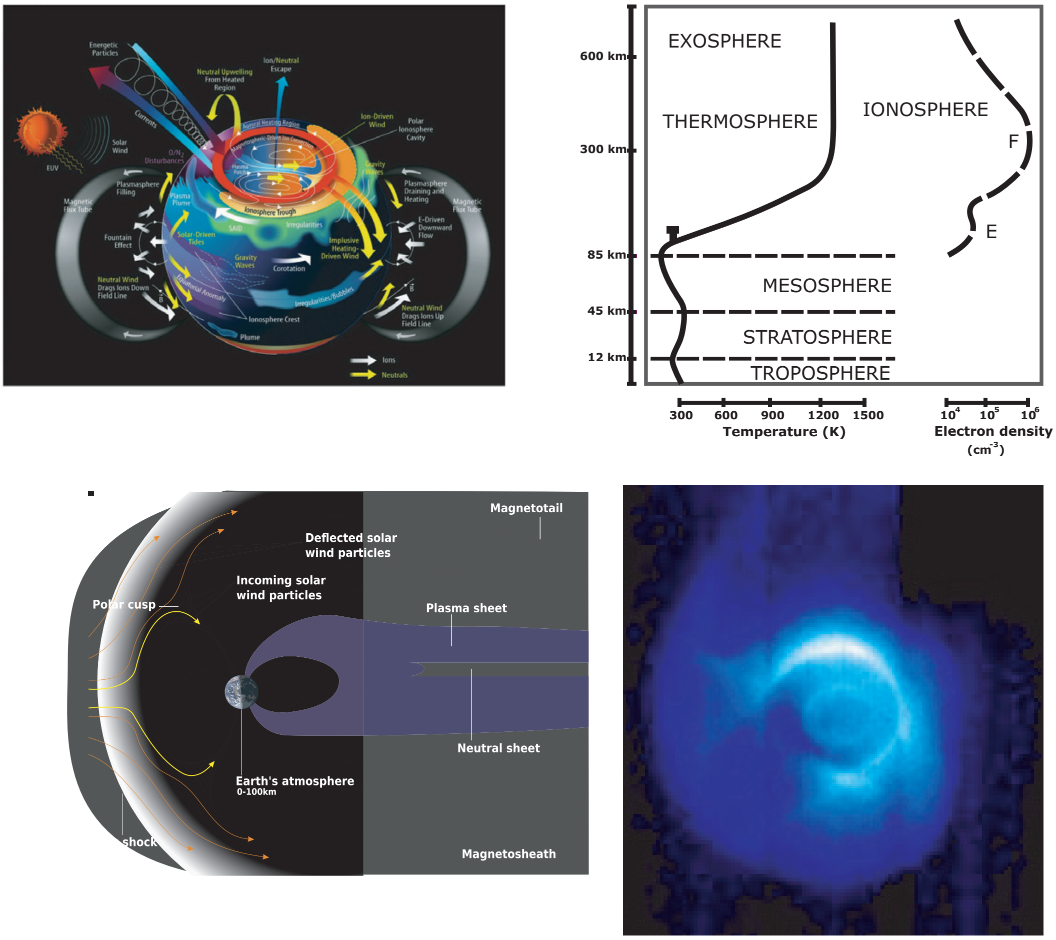


FIGURE 31 Top: the interaction of the solar wind and the Earth's magnetic field. Centre: the magnetic environment of the Earth. Bottom: the names of the layers around the Earth and a photograph of the cold plasma, or magnetosphere, surrounding the Earth, taken in the extreme ultraviolet, and showing both the ring at the basis of each aurora and a tail pointing towards the Sun (courtesy NASA).

The magnetic field of the Earth is clearly influenced by the Sun. Figure 31 shows the details of how the stream of charged particles from the Sun, the *solar wind*, influences the field lines and a several processes occurring in the higher atmosphere. The details of these fascinating processes are still a subject of research.

* *

The names electrode, electrolyte, ion, anode and cathode were suggested by William Whewell (b. 1794 Lancaster, d. 1866 Cambridge) on demand of Michael Faraday; Faraday had no formal education and asked his friend Whewell to form two Greek words for him. For anode and cathode, Whewell took words that literally mean 'upward street' and 'downward street'. Faraday then popularized these terms, like the other words mentioned above.

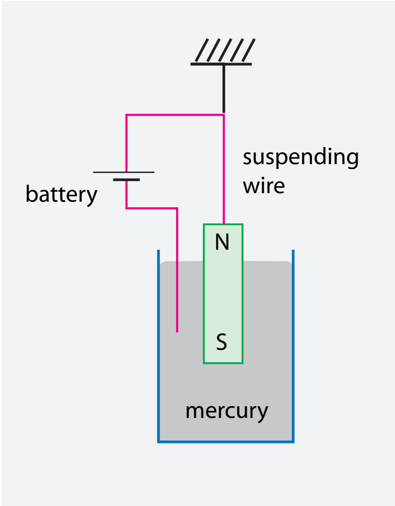


FIGURE 32 A unipolar motor.

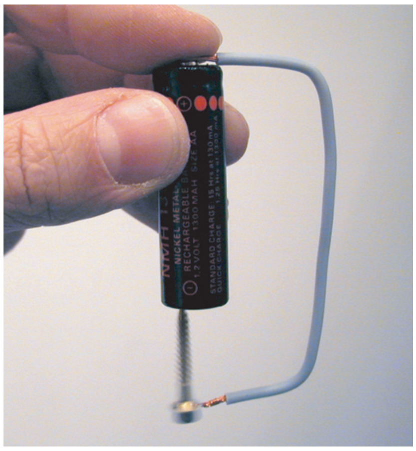


FIGURE 33 The simplest motor (© Stefan Kluge).

* *

Challenge 50 s
The shortest light pulse produced so far had a duration of 100 as. To how many wavelengths of green light would that correspond?

* *

How long can batteries last? At Oxford University, in Clarendon Hall, visitors can watch a battery-operated electric bell that is ringing since 1840. The two batteries, two Zamboni piles, produce a high voltage and low current, sufficient to keep the bell ringing. Several other similar devices, using Zamboni piles, have worked in Italy with the same batteries for over 100 years.

* *

Challenge 51 s
Why do we often see shadows of houses and shadows of trees, but never shadows of the electrical cables hanging over streets?

* *

Challenge 52 s
How would you measure the speed of the tip of a lightning bolt? What range of values do you expect?

* *

Ref. 32
One of the simplest possible electric motors was discovered by Faraday in 1831. A magnet suspended in mercury will start to turn around its axis if a current flows through it. (See Figure 32.) In addition, when the magnet is forced to turn, the device (often also called Barlow's wheel) also works as a current generator; people have even tried to generate domestic current with such a system! Can you explain how it works?
Challenge 53 s

The modern version of this motor makes use of a battery, a wire, a conductive samarium–cobalt magnet and a screw. The result is shown in Figure 33.

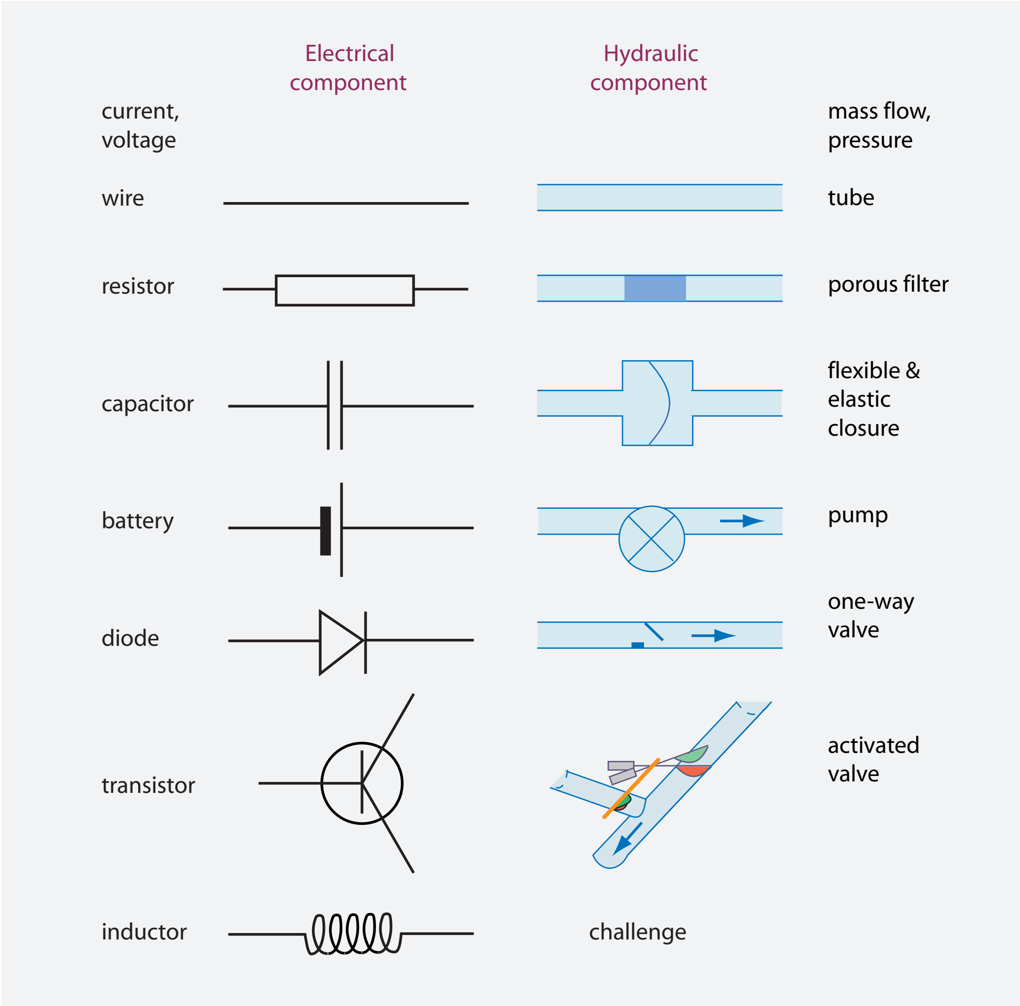


FIGURE 34 The correspondence of electronics and water flow.

* *

Ref. 33 The magnetic field of the Earth has a dipole strength of $7.8 \cdot 10^{22}$ A m^2. It shields us, together with the atmosphere, from lethal solar winds and cosmic radiation particles, by deflecting them to the poles. Today, a lack of magnetic field would lead to high radiation on sunny days; but in the past, its lack would have prevented the evolution of the human species. We owe our existence to the magnetic field of the Earth. At present, the magnetic field decreases by about 5 % per century. It seems that it might disappear temporarily in 1500 years; it is unclear whether this will lead to an increase of the cosmic radiation hitting the Earth's surface, or if the solar wind itself will take over the shielding effect.

* *

Comparing electricity with water is a good way of understanding electronics. Figure 34

shows a few examples that even a teenager can use. Can you fill in the correspondence
Challenge 54 s for the coil, and thus for a transformer?

The picture also includes the *transistor*. This device, as the hydraulic component shows, can be used to control a large current by using a small current. Therefore, transistors can be used as *switches* and as *amplifiers*. This is the reason that all electronic circuits, from radios to mobile phones and computers – make heavy use of transistors. A modern mobile phone or computer typically contains several million transistors, mostly assembled inside so-called *integrated circuits*. The design of these devices is a science on its own.

* *

There is even a way to push the previous analogy in another direction: it is possible to produce a mathematically consistent analogy between electric circuits and continuous fields. The required circuits are infinite grids or meshes in all directions of space, and are called *mimetic discretizations*. If you like to think in electric terms, you might enjoy pursuing this. Just search for the term on the internet.

* *

The ionosphere around the Earth has a resonant frequency of 7 Hz; for this reason any apparatus measuring low frequencies always gets a strong signal at this value. Can you
Challenge 55 s give an explanation of the frequency?

* *

The Kirlian effect, which allows one to make such intriguingly beautiful photographs, is not a property of objects, but a result of the applied time-varying electric field.

* *

At home, electricity is mostly used as alternating current. In other words, no electrons actually flow through cables; as the drift speed of electrons in copper wires is of the order
Page 232 of 1 μm/s, electrons just move back and forward by 20 nm. Nothing flows in or out of the cables! Why do the electricity companies require a real flow of money in return, instead
Challenge 56 e of being satisfied with a back and forth motion of money?

* *

Do electrons and protons have the same charge? Experiments show that the values are
Challenge 57 ny equal to within at least twenty digits. How would you check this?

* *

Charge values are velocity-independent, even near the speed of light. How would you
Challenge 58 ny confirm this?

* *

Magnets can be used, even by school children, to climb steel walls. Have a look at the www.physicslessons.com/TPNN.htm website.

* *

FIGURE 35 The floating bed problem: while the left model, with a length of around 40 cm and a floating height of a few centimetres, exists and has been admired by many, the scaled-up, real-size version on the right is impossible (© Janjaap Ruissenaars at www.UniverseArchitecture.com). The two images on the right are *not* photographs: they show a dream, not reality. Why?

Can magnets be used to make a floating bed? In 2006, a Dutch architect presented to the public a small model of a beautiful floating bed, shown on the left of Figure 35, kept floating in the air by permanent magnets. To prevent that the model bed falls over, it is fastened to the ground by four ropes. On his website, the architect also offers a real-size version of the same bed, for a price of over one million US dollars. However, the images of the scaled up bed – the only two images that exist – are not photographs, but computer graphics, as this dream bed is impossible. Why? Challenge 59 s

* *

Extremely high magnetic fields have strange effects. At fields of 10^{10} T, vacuum becomes effectively birefringent, photons can split and coalesce, and atoms get squeezed. Hydrogen atoms, for example, are estimated to get two hundred times narrower in one direction. Fortunately, these conditions exist only in specific neutron stars, called *magnetars*. Page 108

* *

Ohm's 'law', the observation that for almost all materials the current I is proportional to the voltage U, is

$$U \sim I \quad \text{or} \quad \frac{U}{I} = R = \text{const.} \tag{24}$$

and is due to a school teacher. Georg Simon Ohm (b. 1789 Erlangen, d. 1854 Munich), was a school teacher and physicist. He explored the validity of the proportionality in great depth and for many materials; in those days, such measurements were difficult to perform. Ohm discovered that the proportionality applies to most materials and to many

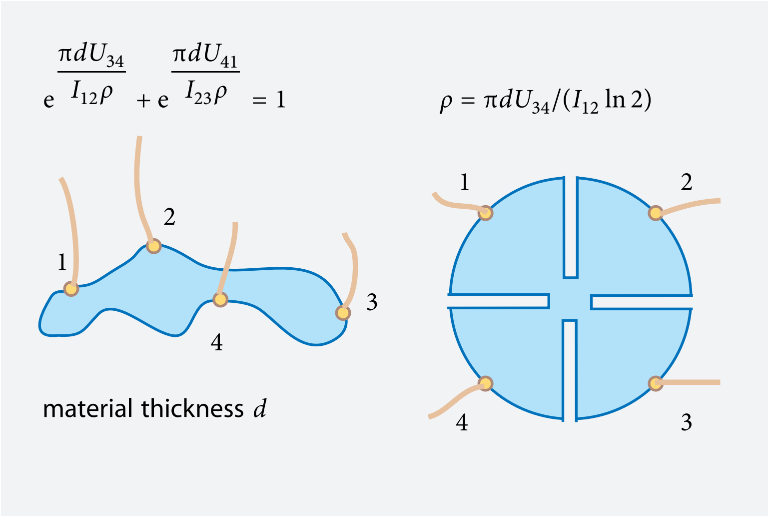


FIGURE 36 Can you deduce Van der Pauw's formula for the specific resistance ρ of homogeneous layers of any shape (left) or its special case for a symmetrical shape (right)?

current levels, as long as the temperature, the material density and the charge densities remain constant. The proportionality is thus not valid for situations with sparks or in semiconductors. But it is valid for most solid conductors, in particular for metals.

Ohm's efforts were recognized only late in his life, and he eventually was promoted to professor at the Technical University in Munich. Later the unit of *electrical resistance* R – this is the official name for the proportionality factor between voltage, which is measured in *volt*, and current, which measured in *ampere* – was named after him. One *ohm* is defined and written as 1 V/A=1 Ω.

Today, Ohm's relation is easy to measure. Recently, even the electrical resistance of single atoms has been measured: in the case of xenon it turned out to be about 10^5 Ω. It was also found that lead atoms are ten times more conductive than gold atoms. Can you imagine why?

Ref. 34
Challenge 60 ny

* *

Since many decades, Ohm's 'law' is taught in secondary school until every pupil in a class has lost his interest in the matter. For example, the electric power P transformed into heat in a resistor is given

$$P = UI = I^2 R = \frac{U^2}{R} . \qquad (25)$$

Vol. I, page 351
We mentioned this relation already earlier on; have a look. Now you know everything that needs to be known on the topic. Above all, the expression for electric power in a resistor describes electric heating, for example the heating in a modern kitchen stove or in a coffee machine.

* *

Ohm's 'law', so simple it seems, has many fascinating mathematical aspects. For example, in 1958, the Dutch physicist J.L. van der Pauw proved an astonishing formula and method

that allows measuring the specific resistance ρ of material layers of *any* shape. One only needs to attach four gold wires to the layer anywhere on its border. The specific resistance is then given by the expression shown in Figure 36. Can you imagine how the formula is deduced? (This is not an easy problem.) The formula reduced the workload in laboratories across the world by a significant amount; before the formula had been discovered, in every experiment, researchers also had to produce separate, dedicated samples that allowed measuring the specific resistance of the material they were investigating.

Challenge 61 d

* *

A good way to make money is to produce electricity and sell it. In 1964, a completely new method was invented by Fletcher Osterle. The method was presented to a larger public in a beautiful experiment in 2003. Larry Kostiuk and his group took a plate of glass, added a conducting layer on each side, and then etched a few hundred thousand tiny channels through the plate: 450 000 microchannels, each around 15 μm in diameter, in the 2 cm diameter plate. When water is made to flow through the channels, a current is generated. The contacts at the two conducting plates can be used like battery contacts and generated 1.5 μA of electric current.

Ref. 35

This simple device uses the effect that glass, like most insulators, is covered with a charged layer when it is immersed in a liquid. Can you imagine why a current is generated? Unfortunately, the efficiency of electricity generation is only about 1 %, making the method much less interesting than a simple blade wheel powering a dynamo.

Challenge 62 s

* *

For beautiful animations about magnetic and electric fields, see the website web.mit.edu/8.02t/www/802TEAL3D/visualizations.

* *

Electrostatics is sometimes counter-intuitive. Take an isolated, conducting sphere of radius R, and a point charge located outside the sphere, both with the same charge. Even though charges of equal sign repel each other, at small distances from the sphere, the point charge is attracted to the sphere. Why? At which distance d do they repel?

Challenge 63 s

* *

Gallium arsenide semiconductors can be patterned with so-called quantum dots and *point contacts*. These structures allow one to count single electrons. This is now routinely done in several laboratories around the world.

* *

Ref. 36

The charges on two capacitors in series are not generally equal, as naive theory states. For perfect, leak-free capacitors the voltage ratio is given by the inverse capacity ratio $V_1/V_2 = C_2/C_1$, due to the equality of the electric charges stored. This is easily deduced from Figure 37. However, in practice this is only correct for times between a few and a few dozen minutes. Why?

Challenge 64 s

* *

On certain high voltage cables leading across the landscape, small neon lamps, called

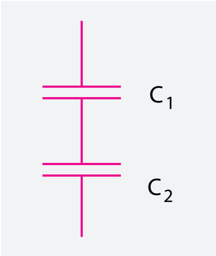


FIGURE 37 Capacitors in series.

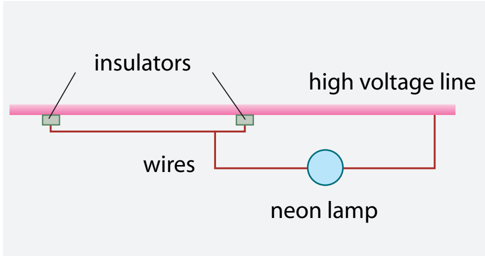


FIGURE 38 A neon lamp hanging from a high voltage line.

balisors, shine when the current flows, as shown in Figure 38. You can see them from the train when riding from Paris to Roissy airport. How do they work? Challenge 65 s

* *

During rain or fog, high-voltage lines often make noises; sometimes they even *sing*. What is going on? Challenge 66 s

* *

Electric polarizability is the property of matter responsible for the deviation of water flowing from a tap caused by a charged comb. Page 16 It is defined as the strength of electric dipole induced by an applied electric field. The definition simply translates the observation that many objects acquire a charge when an electric field is applied. Incidentally, how precisely combs get charged when rubbed, a phenomenon called *electrification*, is still one of the mysteries of modern science.

* *

A pure magnetic field cannot be transformed into a pure electric field by change of observation frame. The best that can be achieved is a state similar to an *equal mixture* of magnetic and electric fields. Can you provide an argument elucidating this relation? Challenge 67 s

* *

Calculating resistance of infinite grids is one of the most captivating problems in electricity, as shown in Figure 39. Can you find the solution? Challenge 68 ny

* *

To every limit value in nature there is a corresponding indeterminacy relation. This is also valid also for electricity and the lower charge limit. Indeed, there is an indeterminacy relation for capacitors, of the form

$$\Delta C \; \Delta U \geqslant e \qquad (26)$$

where e is the positron charge, C capacity and U potential difference. There is also an

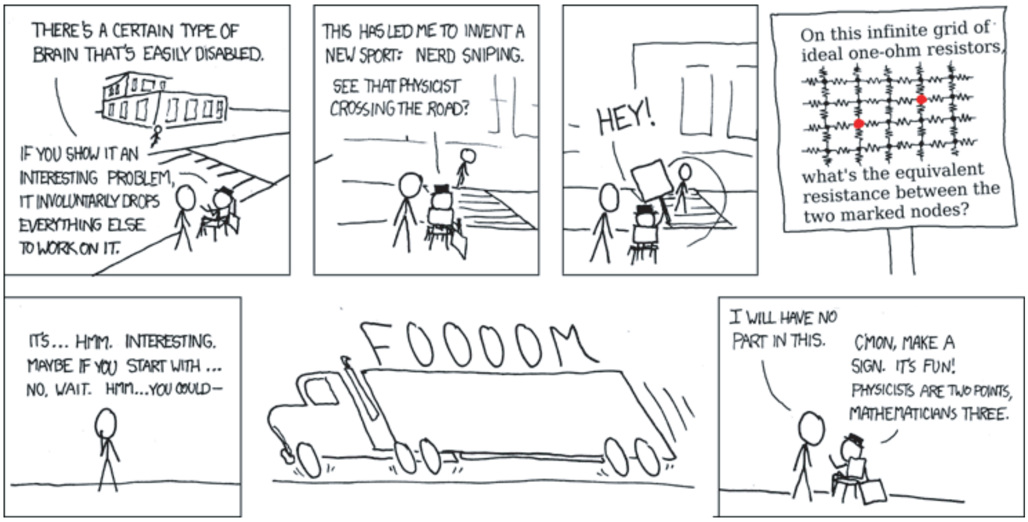


FIGURE 39 An electrical problem that is not easy (© Randall Munroe).

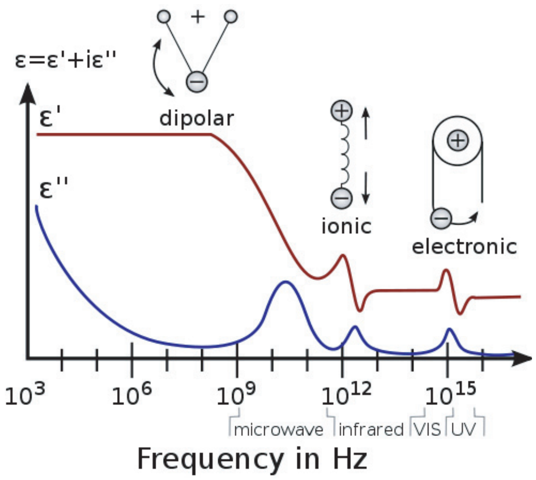


FIGURE 40 The change of the relative permittivity (real and imaginary) with frequency for an abstract material (mix), and the general processes responsible for the different domains (© Kenneth Mauritz).

indeterminacy relation between electric current I and time t

$$\Delta I \; \Delta t \geqslant e \; . \tag{27}$$

Ref. 37 Both these relations may be found in the literature.

* *

Electric properties of materials, in contrast to their magnetic properties, vary strongly with the frequency of the applied electric field. Figure 40 illustrates how the permittivity changes with frequency, and which microscopic processes are at the basis of the property at a specific frequency. The graph is only schematic: it shows features from different materials combined together. In nature, the real and imaginary parts of the permittivity

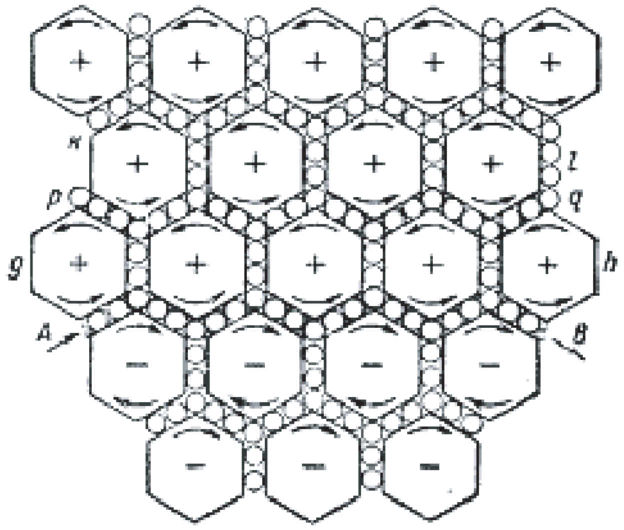


FIGURE 41 Maxwell's unsuccessful model of the vacuum.

are related by the so-called *Kramers-Kronig relations*, which are important for many material topics related to wave phenomena. The two curves in the graph do not follow them completely.

* *

Challenge 69 e If an axis rotates, one can attach a magnet to its end. With such a rotating magnet an extremely cheap tachymeter can be realized. How?

* *

Challenge 70 s In Maxwell's 1890 book on electrodynamics, he includes Figure 41 as a model of magnetic and electric fields of the vacuum. What is the biggest problem of this model of the vacuum?

* *

Ref. 38 For how long can silicon-based integrated circuits be made smaller and smaller? The opinions on this matter differ. Optimistic predictions, often called Moore's 'law', alternate with predictions that from 2011 onwards, the size reduction will be moderate due to the high cost of the required equipment. For example, the next generation of wafer steppers, the most expensive machines in the production of silicon chips, must work in the extreme ultraviolet – usually 13 nm – in order to achieve small transistor sizes. At this wavelength air is an absorber, and lenses have to be replaced by mirrors. It is unclear whether this will be technically and economically feasible. Future will tell.

* *

Challenge 71 s In the 1990s, microscope images showed, surprisingly, that the tusks of narwhals are full of nerve endings. Thus the tusk may be a sensory organ. However, the details and the exact use of the organ is not understood. How would you find out?

A summary: three basic facts about electricity

The experiments we have described so far show three basic results:

▷ Electric charges exert forces on other charges.
▷ Electric charges are conserved.
▷ Charges, like all matter, move slower than light.

From these three statements – the definition of charge, the conservation of charge, and the invariance of the speed of light – we can deduce every aspect of classical electrodynamics. Ref. 39 In particular, the Lagrangian of electrodynamics and Maxwell's field equations can be deduced from these three statements; they describe the way that charges *produce* any electric, magnetic or electromagnetic field. Also the Lorentz force can be deduced; it describes how the motion of massive charges and the motion of the electromagnetic field is related.

The proof of the connection between charge conservation and the field equations can Ref. 39 be given mathematically; we do not present it here, because the algebra is somewhat involved. The essential connection to remember is: all of electrodynamics follows from Ref. 40 the properties of charges that we have discovered so far.

CHAPTER 2

THE DESCRIPTION OF ELECTROMAGNETIC FIELD EVOLUTION

Electric and magnetic fields change: simply said, they move. How exactly does this happen? In the 1860s, James Clerk Maxwell* collected all experimental knowledge he could find, and deduced the precise description of electromagnetic field motion. Twenty years later, Heaviside and Hertz extracted the main points of Maxwell ideas, calling their summary *Maxwell's theory of the electromagnetic field*.

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The motion of the electromagnetic field is described by a set of evolution equations. In the relativistic description, the set consists of *two* equations, in the non-relativistic case of *four* equations. All observations of classical electrodynamics follow from these equations. In fact, if quantum effects are properly taken into account, *all* electromagnetic effects of nature are described.

The first field equation of electrodynamics

The first relativistic field equation of electrodynamics is the precise statement that electromagnetic fields *originate at charges*, and nowhere else. It can variously be written**

$$dF = j\mu_0 \quad \text{or}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0} \quad \text{and} \quad \nabla \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \boldsymbol{j} \; . \tag{28}$$

* James Clerk Maxwell (b. 1831 Edinburgh, d. 1879 Cambridge) is one of the most important and influential physicists. He founded electromagnetism by theoretically unifying electricity and magnetism, as described in this chapter. His work on thermodynamics forms the second pillar of his activity. In addition, he studied the theory of colours and developed the colour triangle; he was one of the first people to make a colour photograph. He is regarded by many as the greatest physicist ever. Both 'Clerk' and 'Maxwell' were his family names.

** There is a certain freedom in writing the equations, because different authors absorb different combinations of the constants c and μ_0 into the definitions of the quantities F, A and j. The one given here is the most common version. The equations can be generalized to cases where the charges are not surrounded by vacuum, but located inside matter. We will not explore these situations in our walk because, as we will discover later on, the seemingly special case of vacuum in fact describes all of nature.

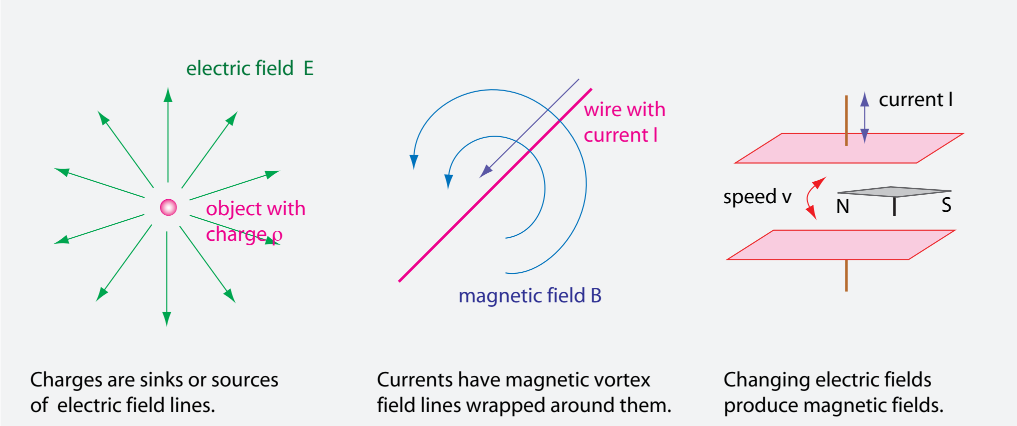

FIGURE 42 The first of Maxwell's field equations of electrodynamics illustrated in three drawings.

Each of these two equivalent ways* to write the first Maxwell equation makes a simple statement: *electrical charges carry the electromagnetic field*; they carry it along with them. For example, this first equation describes the attraction of dust by electrically charged objects and the working of electromagnets.

This first equation is equivalent to the three basic observations illustrated in Figure 42: Coulomb's relation, Ampère's relation, and the way changing electrical fields induce magnetic effects. More precisely, if we know where charges are and how they move, we can determine the electromagnetic field $\boldsymbol{F}$ they generate. *Static charges*, described by a density ρ, produce *electrostatic* fields, and moving charges, described by a 3-current density $\boldsymbol{j}$, produce a mix of electric and magnetic fields. *Stationary currents* produce *magnetostatic* fields.

Challenge 72 e

The first field equation also contains the *right hand rule* for magnetic fields around wires, through the vector product. And as already mentioned, the equation also states, most clearly in its last form, that changing electric fields induce magnetic fields. The effect is essential in the primary side of transformers. The small factor $1/c^2$ implies that the effect is small; therefore coils with *many* windings or *strong* electric currents are needed to produce or detect the effect.

The second field equation of electrodynamics

The second of Maxwell's field equations, illustrated in Figure 43, expresses the observation that in nature there are *no* magnetic charges, i.e., that magnetic fields have no sources. As a result, the equation also gives a precise description of how changing magnetic fields

* In component form, the first equation can be written

$$d_\mu \boldsymbol{F}^{\mu\nu} = j^\nu \mu_0 = (\rho c, \boldsymbol{j})\mu_0 = (\rho_0 \gamma c, \rho_0 \gamma \boldsymbol{v})\mu_0 \quad \text{or}$$

$$(\partial_t/c, \partial_x, \partial_y, \partial_z)\begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix} = \mu_0(\rho c, \boldsymbol{j})\ . \qquad (29)$$

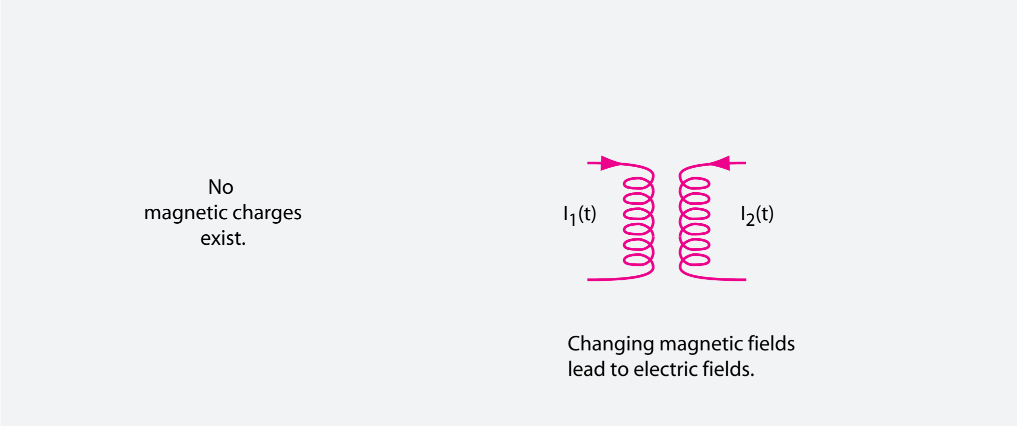

FIGURE 43 The second field equation of electrodynamics.

create electric fields, and vice versa. The second of Maxwell's equations for electrodynamics can variously be written

$$d\,{}^*\boldsymbol{F} = 0 \quad \text{with} \quad {}^*\boldsymbol{F}^{\rho\sigma} = \frac{1}{2}\varepsilon^{\rho\sigma\mu\nu}\boldsymbol{F}_{\mu\nu}$$

or

$$\nabla\cdot\boldsymbol{B} = 0 \quad \text{and} \quad \nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}\ . \qquad (30)$$

First of all, the second field equation* thus expresses the *lack of sources for the dual field tensor* ${}^*\boldsymbol{F}$. In other words, in nature there are no magnetic charges, i.e., no magnetic monopoles: there are no sources for magnetic fields. The equation thus states that cutting a magnet with a north and a south pole in any way always produces pieces with *two* poles, never a piece with a single pole.

Since there are no magnetic charges, magnetic field lines have *no beginning and no end*; not only the magnetic field lines induced by charges, no, *all* magnetic field lines have no beginning and no end. For example, field lines continue inside magnets. The lack of beginnings and ends is expressed mathematically by stating that the *magnetic flux*

* In component form, the second equation can be written

$$d_\mu {}^*\boldsymbol{F}^{\mu\nu} = 0 \quad \text{or}$$

$$(\partial_t/c, \partial_x, \partial_y, \partial_z)\begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z/c & -E_y/c \\ B_y & -E_z/c & 0 & E_x/c \\ B_z & E_y/c & -E_x/c & 0 \end{pmatrix} = (0,0,0,0) \quad \text{or}$$

$$\varepsilon^{\sigma\mu\nu\rho}\partial_\mu \boldsymbol{F}_{\nu\rho} = 0 \quad \text{or}$$

$$\partial_\mu \boldsymbol{F}_{\nu\rho} + \partial_\nu \boldsymbol{F}_{\rho\mu} + \partial_\rho \boldsymbol{F}_{\mu\nu} = 0\ . \qquad (31)$$

We note that the dual tensor ${}^*\boldsymbol{F}$ follows form the field tensor $\boldsymbol{F}$ by substituting E/c by B and B by $-E/c$. Page 88 This is the so-called *duality transformation*. More on this duality below.

through a *closed* surface S – such as a sphere or a cube – *always vanishes*: $\int_S \boldsymbol{B}\, \mathrm{d}\boldsymbol{A} = 0$. In other words, all field lines that enter a closed volume also leave it.*

Furthermore, the second Maxwell equation expresses that *changes in magnetic fields produce electric fields*. This effect is used in the secondary side of transformers and in dynamos. The cross product in the expression implies that an electric field generated in this way – also called an *electromotive field* – has no start and end points. The electromotive field lines thus can run in circles: in most practical cases they run along electric circuits. In short, an electric field can have vortices (like the magnetic field), but only when there is a changing magnetic field. The minus sign is essential to ensure energy conservation

Challenge 73 ny (why?) and has a special name: it is called *Lenz's rule*.

In practice, the second Maxwell equation is always needed together with the first. Can

Challenge 74 ny you see why?

The validity and the essence of Maxwell's field equations

We saw above that Lorentz' evolution equation

$$\begin{aligned} m\boldsymbol{b} &= q\boldsymbol{F}u \quad \text{or} \\ \mathrm{d}E/\mathrm{d}t &= q\boldsymbol{E}\boldsymbol{v} \quad \text{and} \quad \mathrm{d}\boldsymbol{p}/\mathrm{d}t = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \end{aligned} \tag{32}$$

describes how charges move given the motion of the fields. Together with Lorentz' evolution equation, the two Maxwell's evolution equations (28) and (30) describe *all* electromagnetic phenomena occurring on everyday scales, from mobile phones, car batteries, to personal computers, lasers, lightning, holograms and rainbows. In other words, this description of electromagnetic fields is complete for everyday life. Only quantum effects and the effects of curved space-time are not included.

Maxwell's equations seem very complex. But we should not forget that they contain only *four* basic ideas.

1. Electric charges follow Coulomb's rule.
2. Electric charges moves slower than light.
3. Electric charges are conserved.
4. Magnetic charges do not exist.

If we want to be simplistic, Maxwell's equations are just the relativistic formulation of Coulomb's rule. In fact, as we have seen before, Maxwell's equations follow from charge

Ref. 39 conservation alone.

We will not study many applications of the field equations. True, the range of applications is vast: modern medicine, transport, telecommunication, computers, most jobs and many pleasures depend on electricity. But we leave these topics aside and continue directly towards our aim to understand the connection between electromagnetic fields, everyday motion and the motion of light. In fact, the electromagnetic field has an impor-

* In contrast to what is often said and written in physics books, magnetic field lines are, in general, *not* closed

Ref. 41 lines; they are *not*, in general, loops or vortex lines. Closed magnetic field lines occur only for straight wires; they are not even loops for simple helical coils. In fact, in all usual, non-academic situations, magnetic field lines start and end at spatial infinity.

Magnetic field *lines* are a mathematical tool, they do not provide a completely useful description of the magnetic field. The magnetic field is best described by its *vector field*.

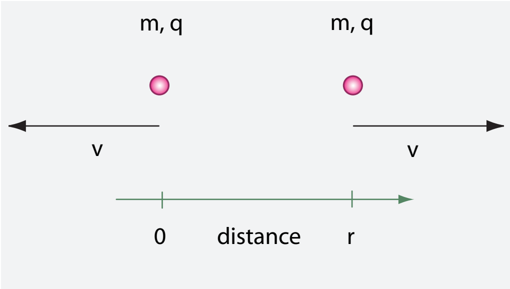

FIGURE 44 Charged particles after a collision.

tant property that we mentioned right at the start: the field itself can move. In particular, the field can carry energy, linear momentum and angular momentum.

Colliding charged particles

Electromagnetic fields move. A simple experiment clarifies the meaning of motion for fields: When two charged particles collide, their total momentum is *not* conserved. Let us check this.

Imagine two particles of identical mass and identical charge just after a collision, when they are moving away from one another. The situation is illustrated in Figure 44. Imagine also that the two masses are large, so that the acceleration due to their electrical repulsion is small. For an observer at the centre of gravity of the two, each particle feels an acceleration from the electric field of the other. This electric field E is given by the so-called

Challenge 75 ny *Heaviside formula*

$$E = \frac{q\,(1 - v^2/c^2)}{4\pi e_0 r^2} . \qquad (33)$$

In other words, the total system has a vanishing total momentum for this observer.

Take a second observer, moving with respect to the first with velocity v, so that the first charge will be at rest. Expression (33) leads to two *different* values for the electric

Ref. 42 fields, one at the position of each particle. In other words, the system of the two particles is not in inertial motion, as we would expect; the total momentum is not conserved for

Challenge 76 s this observer. The missing momentum is small, but where did it go?

This at first surprising effect has even been put in the form of a theorem by Van Dam

Ref. 43 and Wigner. They showed that, for a system of particles interacting at a distance, the total particle energy–momentum cannot remain constant in all inertial frames.

The total momentum of the system is conserved only because the electromagnetic field itself also carries some momentum. In short, momentum is conserved in the experiment, but some of it is carried by the field. The precise amount depends on the observer.

Two colliding charged particles thus show us that electromagnetic fields have momentum. If electromagnetic fields have momentum, they are able to *strike* objects and to be struck by them. As we will show below, light is also an electromagnetic field. Thus we should be able to move objects by shining light on to them. We should even be able to suspend particles in mid air by shining light on to them from below. Both predictions

Page 116 are correct, and some experiments will be presented shortly.

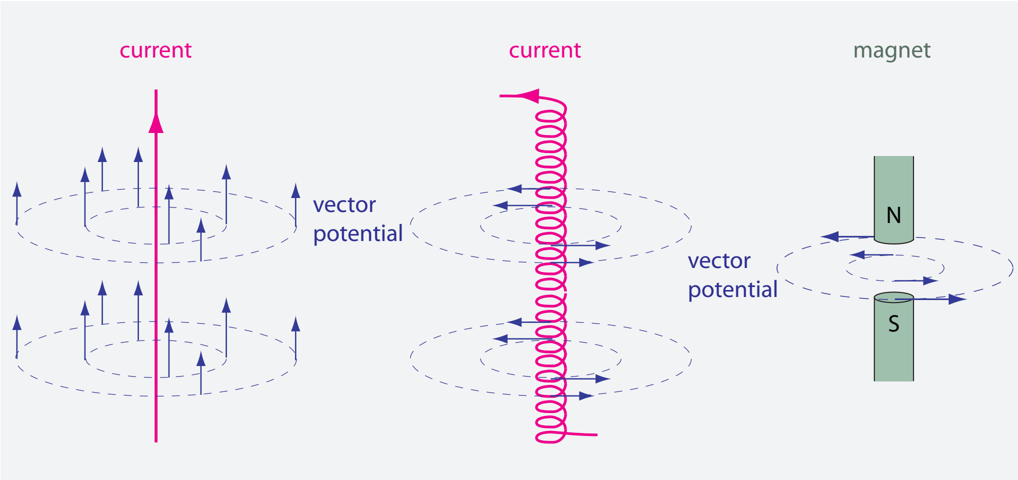

FIGURE 45 Vector potentials for selected situations.

We conclude that any sort of field leading to particle interactions must carry both energy and momentum, as the argument applies to all such cases. In particular, it applies to nuclear interactions. Indeed, in the quantum part of our adventure we will even find an additional result: all fields are themselves composed of particles. The energy and momentum of fields then become an obvious state of affairs. In short, it makes sense to say that electromagnetic fields move, because they carry energy and momentum.

The gauge field – the electromagnetic vector potential

The study of moving fields is called *field theory* and electrodynamics is the prime example. (The other classical example is fluid dynamics; moving electromagnetic fields and moving fluids are very similar mathematically.) Field theory is a beautiful topic; field lines, equipotential lines and vortex lines are some of the concepts introduced in this domain. They fascinate many.* However, in this mountain ascent we keep the discussion focused on motion.

We have seen that fields force us to extend our concept of motion. Motion is not only the change in state of objects and of space-time, but also the *change in state of fields*. We therefore need, also for fields, a complete and precise description of their state.

The observations using amber and magnets have shown us that *electromagnetic fields possess energy and momentum*. Fields can impart energy and momentum to particles. The experiments with motors have shown us that objects can add energy and momentum to fields. We therefore need to define a *state function* which allows us to define energy and momentum for electric and magnetic fields. And since electric and magnetic fields transport energy, their motion must follow the speed limit in nature.

Hertz and Heaviside defined the state function of fields in two standard steps. The first

Challenge 77 s * What is the relation, for static fields, between field lines and (equi-) potential surfaces? Can a field line cross a potential surface twice? For more details on topics such as these, see the *free* textbook by Bo Thidé, *Electromagnetic Field Theory*, on his www.plasma.uu.se/CED/Book website. And of course, in English, have Ref. 1, Ref. 24 a look at the texts by Schwinger and by Jackson.

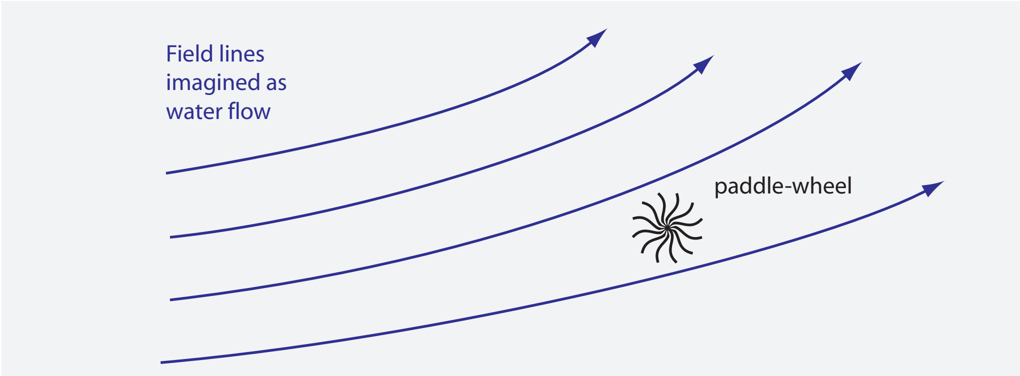

FIGURE 46 Visualizing the curl of a vector field. Imagine the field to be flowing air and check whether the small paddle-wheel rotates; if it does, the local curl is non-zero. The direction of the curl is the direction of the paddle-wheel axis that yields the highest rotation velocity.

step is the definition of the *(magnetic) vector potential*, which describes the momentum per charge that the field provides: Ref. 44

$$\boldsymbol{A} = \frac{\boldsymbol{p}}{q} \; . \tag{34}$$

When a charged particle moves through a magnetic potential $\boldsymbol{A}(\boldsymbol{x})$, its momentum changes by $q\Delta\boldsymbol{A}$; it changes by the difference between the potential values at the start and end points, multiplied by its charge. Owing to this definition, the vector potential has the property that

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} = \text{curl}\; \boldsymbol{A} \tag{35}$$

i.e., that the magnetic field is the *curl* of the magnetic potential. In most other languages the curl is called the *rotation* and abbreviated rot. To visualize what the curl or rotation is, imagine that the field vectors are the velocity vectors of flowing air. Now put a tiny paddle-wheel at a point, as shown in Figure 46. If it turns, the curl is non-zero. The rotation speed of the paddle-wheel is maximal for some direction of the axis; this maximal speed defines both the magnitude and the direction of the curl at the point. (The right-hand rule is implied.) For example, the curl for the velocities of a rotating solid body is everywhere 2ω, or twice the angular velocity. Challenge 78 ny

Ref. 45 The vector potential for a long straight current-carrying wire is parallel to the wire; it has the magnitude Challenge 79 ny

$$A(r) = -\frac{\mu_0 I}{4\pi} \ln \frac{r}{r_0} \; , \tag{36}$$

which depends on the radial distance r from the wire and an integration constant r_0. This expression for the vector potential, pictured in Figure 45, shows how the moving current produces a linear momentum in the (electro-) magnetic field around it. In the case of a solenoid, the vector potential ‘circulates’ around the solenoid. The magnitude obeys

$$A(\mathrm{r}) = -\frac{\Phi}{4\pi} \frac{1}{r} \; , \tag{37}$$

where Φ is the magnetic flux inside the solenoid. We see that, in general, the vector potential is *dragged along* by moving charges. The dragging effect decreases for larger distances. This fits well with the image of the vector potential as the momentum of the electromagnetic field.

This behaviour of the vector potential around charges is reminiscent of the way honey is dragged along by a spoon moving in it. In both cases, the dragging effect decreases with distance. However, the vector potential, unlike the honey, does *not* produce any friction that slows down charge motion. The vector potential thus behaves like a frictionless liquid.

Inside the solenoid, the magnetic field is constant and uniform. For such a field $\boldsymbol{B}$ we

Challenge 80 e

find the vector potential

$$\boldsymbol{A}(\mathrm{r}) = -\frac{1}{2}\boldsymbol{B} \times \boldsymbol{r} \, . \tag{38}$$

In this case, the magnetic potential thus increases with increasing distance from the origin.* In the centre of the solenoid, the potential vanishes. The analogy of the dragged honey gives exactly the same behaviour.

However, there is a catch. The magnetic potential is *not* defined uniquely. If $\boldsymbol{A}(\boldsymbol{x})$ is a vector potential, then the different vector potential

$$\boldsymbol{A}'(\boldsymbol{x}) = \boldsymbol{A}(\boldsymbol{x}) + \nabla \Lambda \, , \tag{39}$$

where $\Lambda(t, \boldsymbol{x})$ is some scalar function, is *also* a vector potential for the same situation. (The magnetic field $\boldsymbol{B}$ stays the same, though.) Worse, can you confirm that the corre-

Challenge 81 ny

sponding (absolute) momentum values also change? This unavoidable ambiguity, called *gauge invariance* or *gauge symmetry*, is a central property of the electromagnetic field. We will explore it in more detail below.

Not only the momentum, but also the energy of the electromagnetic field is defined ambiguously. Indeed, the second step in the specification of a state for the electromag-

Ref. 44

netic field is the definition of the *electric potential* as the energy U per charge:

$$\varphi = \frac{U}{q} \tag{40}$$

In other words, the potential $\varphi(\boldsymbol{x})$ at a point $\boldsymbol{x}$ is the energy needed to move a unit charge to the point $\boldsymbol{x}$ starting from a point where the potential vanishes. The potential energy is thus given by $q\varphi$. From this definition, the electric field $\boldsymbol{E}$ is simply the *change* of the potential with position corrected by the time dependence of momentum, i.e.,

$$\boldsymbol{E} = -\nabla\varphi - \frac{\partial}{\partial t}\boldsymbol{A} \, , \tag{41}$$

Obviously, there is a freedom in the choice of the definition of the potential. If $\varphi(\boldsymbol{x})$ is a

* This is only possible as long as the field is constant; since all fields drop again at large distances – because the energy of a field is always finite – also the vector potential drops at large distances.

possible potential, then

$$\varphi'(\boldsymbol{x}) = \varphi(\boldsymbol{x}) - \frac{\partial}{\partial t}\Lambda \tag{42}$$

is also a potential function for the same situation. This freedom is the generalization of the freedom to define energy up to a constant. Nevertheless, the electric field $\boldsymbol{E}$ remains the same for all potentials.

Ref. 44
Challenge 82 ny
To be convinced that the potentials really are the energy and momentum of the electromagnetic field, we note that for a moving charge we have

$$\begin{aligned} \frac{d}{\mathrm{d}t}\left(\frac{1}{2}mv^2 + q\varphi\right) &= \frac{\partial}{\partial t}q\ (\varphi - \boldsymbol{v}\boldsymbol{A}) \\ \frac{d}{\mathrm{d}t}\left(m\boldsymbol{v} + q\boldsymbol{A}\right) &= -\nabla q\left(\varphi - \boldsymbol{v}\boldsymbol{A}\right)\,, \end{aligned} \tag{43}$$

which show that the changes of generalized energy and momentum of a particle (on the left-hand side) are due to the change of the energy and momentum of the electromagnetic field (on the right-hand side).*

In relativistic 4-vector notation, the energy and the momentum of the field appear together in one quantity. The state function of the electromagnetic field becomes

$$A^\mu = (\varphi/c, \boldsymbol{A}) \tag{44}$$

and is called the *4-potential*. It is easy to see that the description of the field is complete, since we have

$$\boldsymbol{F} = d\,A \quad \text{or} \quad \boldsymbol{F}^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad (\text{and} \quad \boldsymbol{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)\,, \tag{45}$$

which means that the electromagnetic field F is completely specified by the 4-potential A.** But as just said, the 4-potential itself is *not* uniquely defined. Indeed, any other equivalent 4-potential A' is related to A by the *gauge transformation*

$$A'^{\mu} = A^\mu + \partial^\mu\Lambda \tag{46}$$

where $\Lambda = \Lambda(t, x)$ is any arbitrarily chosen scalar field. The new field A' leads to the *same* electromagnetic field, and to the same accelerations and evolutions. The 4-potential A is thus an *overdescription* of the physical situation as several *different* gauge choices correspond to the *same* physical situation.*** Therefore we have to check that all measurement results are independent of gauge transformations, i.e., that all observables are gauge invariant quantities. Such gauge invariant quantities are, as we just saw, the fields $\boldsymbol{F}$ and $^*\boldsymbol{F}$,

* This connection also shows why the expression $P^\mu - qA^\mu$ appears so regularly in formulae; indeed, it plays a central role in the quantum theory of a particle in the electromagnetic field.

** The connection between A_μ and A^μ, the same as for every other 4-vector, was mentioned earlier on; can
Challenge 83 e
you restate it?

*** Choosing a function Λ is often called *choosing a gauge*; the 4-potential A is also called the *gauge field*. These strange terms have historic reasons and are now common to all of physics.

and in general all classical quantities. We add that many theoretical physicists use the term 'electromagnetic field' loosely for both the quantities $F^{\mu\nu}$ and A^{μ}.

There is a simple image, due to Maxwell, to help overcoming the conceptual difficulties of the vector potential. It turns out that the closed line integral over A_μ is gauge invariant, because

Challenge 84 e

$$\oint A^{\mu} \mathrm{d}x_{\mu} = \oint (A^{\mu} + \partial^{\mu}\Lambda)\mathrm{d}x_{\mu} = \oint A'^{\mu} \mathrm{d}x_{\mu} \; . \tag{47}$$

In other words, if we picture the vector potential as a quantity allowing us to associate a number to a tiny ring at each point in space, we get a good, gauge invariant picture of the vector potential.*

Now that we have defined a state function that describes the energy and momentum of the electromagnetic field, let us look at what happens in more detail when electromagnetic fields move.

Energy and momenta of the electromagnetic field

All moving entities have energy, momentum and angular momentum. This also applies to the electromagnetic field. Indeed, the description so far allows us to write the *total* energy E_{nergy} of the electromagnetic field as

$$E_{\text{nergy}} = \frac{1}{4\pi} \int \frac{\varepsilon_0}{2} (\boldsymbol{E}^2 + c^2 \boldsymbol{B}^2) \; \mathrm{d}V \; . \tag{48}$$

Energy is thus quadratic in the fields.

For the total linear momentum $\boldsymbol{p}$ we obtain

$$\boldsymbol{p} = \frac{1}{4\pi} \int \varepsilon_0 \boldsymbol{E} \times \boldsymbol{B} \; \mathrm{d}V \; . \tag{49}$$

The expression inside the integral, is the *momentum density*. The related vector $\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{B}/\mu_0$, is called the *Poynting vector*** and describes the *energy flux*; it is a vector field and has the units $\mathrm{W/m^2}$. The Poynting vector is the momentum density divided by c^2; indeed, special relativity implies that the momentum and the energy flow for electromagnetic fields are related by a factor c^2. The Poynting vector thus describes the energy flowing per area per time, in other words, the power per area. As shown below, the Poynting vector is a part of the energy–momentum tensor.

Page 85

Can you produce a graph of the Poynting vector field for a cable carrying direct current? For a transformer?

Challenge 85 s

Ref. 47

For the total angular momentum we have

$$\boldsymbol{L} = \frac{\varepsilon_0}{4\pi} \int \boldsymbol{E} \times \boldsymbol{A} \; \mathrm{d}V = \frac{\varepsilon_0}{4\pi} \int \boldsymbol{r} \times (\boldsymbol{E} \times \boldsymbol{B}) \; \mathrm{d}V \; , \tag{50}$$

Ref. 46

* In the part of the text on quantum theory we will see that the exponent of this expression, namely $\exp(iq \oint A_\mu \mathrm{d}x^\mu)/\hbar$, usually called the *phase factor*, can indeed be directly observed in experiments.

** John Henry Poynting (b. 1852 Monton, d. 1914 Birmingham) introduced the concept in 1884.

where $\boldsymbol{A}$ is the magnetic vector potential.

In summary, the electromagnetic field has linear and angular momentum and energy. Nevertheless, for most everyday situations, the values are negligibly small, as you may want to check.

Challenge 86 e

The Lagrangian of electromagnetism

The motion of a charged particle and the related motion of the electromagnetic field can also be described using a Lagrangian, instead of using the three equations given above. It is not hard to see that the action S_{CED} for a particle in classical electrodynamics can be symbolically defined by*

Challenge 87 ny

$$S_{\mathrm{CED}} = -c^2 m \int \mathrm{d}\tau - \tfrac{1}{4\mu_0} \int \boldsymbol{F} \wedge * \boldsymbol{F} - \int j \wedge A \,, \tag{51}$$

which in index notation becomes

$$S_{\mathrm{CED}} = -mc \int_{-\infty}^{\infty} \sqrt{\eta_{\mu\nu} \frac{\mathrm{d}x_n^\mu(s)}{\mathrm{d}s} \frac{\mathrm{d}x_n^\nu(s)}{\mathrm{d}s}} \, \mathrm{d}s - \int_{\mathbf{M}} \left(\tfrac{1}{4\mu_0} \boldsymbol{F}_{\mu\nu} \boldsymbol{F}^{\mu\nu} + j_\mu A^\mu \right) \mathrm{d}^4 x \,, \tag{52}$$

or, in 3-vector notation

$$S_{\mathrm{CED}} = -c^2 m \int \mathrm{d}\tau + \int (qvA - q\varphi) \, \mathrm{d}t\mathrm{d}V + \int \left(\frac{\varepsilon_0}{2} E^2 - \frac{1}{2\mu_0} B^2 \right) \mathrm{d}t\mathrm{d}V \,. \tag{53}$$

The new part is the measure of the change – or action – due to the electromagnetic field. The pure field change is given by the term $\boldsymbol{F} \wedge * \boldsymbol{F}$, and the change due to interaction with matter is given by the term $j \wedge A$.

The least action principle, as usual, states that the change in a system is always as small as possible. The action S_{CED} leads to the evolution equations by requiring that the action be stationary under variations δ and δ' of the positions and of the fields which vanish at infinity. In other terms, the principle of least action requires that

$$\begin{aligned} \delta S = 0 \quad & \text{when} \quad x_\mu = x_\mu + \delta_\mu \quad \text{and} \quad A_\mu = A_\mu + \delta'_\mu \quad , \\ & \text{provided} \quad \delta x_\mu(\theta) \rightarrow 0 \quad \text{for} \quad |\theta| \rightarrow \infty \\ & \text{and} \quad \delta A_\mu(x_\nu) \rightarrow 0 \quad \text{for} \quad |x_\nu| \rightarrow \infty \,. \end{aligned} \tag{54}$$

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Challenge 88 ny

In the same way as in the case of mechanics, using the variational method for the two variables A and x, we recover the evolution equations for particle position and fields

$$b^\mu = \frac{q}{m} \boldsymbol{F}^\mu_\nu \, u^\nu \quad , \quad \partial_\mu \boldsymbol{F}^{\mu\nu} = j^\nu \mu_0 \quad , \quad \text{and} \quad \varepsilon^{\mu\nu\rho\sigma} \partial_\nu \boldsymbol{F}_{\rho\sigma} = 0 \,, \tag{55}$$

* The product described by the symbol ∧, 'wedge' or 'hat', and the duality operator ∗ have a precise mathematical meaning. The background, the concept of *(mathematical) form*, carries us too far from our walk.

Ref. 48

which we know already: they are the Lorentz relation and the two field equations. Obviously, they are equivalent to the variational principle based on S_{CED}. Both descriptions have to be completed by specifying *initial conditions* for the particles and the fields, as well as *boundary conditions* for the latter. We need the first and zeroth derivatives of the position of the particles, and the zeroth derivative for the electromagnetic field.

With the Lagrangian (51) all of classical electrodynamics can be described and understood. For the rest of our exploration of electrodynamics, we look at some specific topics from this vast field.

The energy–momentum tensor and its symmetries of motion

We know from classical mechanics that we get the definition of energy and momentum by using Noether's theorem. In particular, both the definition and the conservation of energy and momentum arise from the Lorentz symmetry of the Lagrangian. For example, we found that relativistic particles have an energy–momentum *vector*. At the point at which the particle is located, it describes its energy and momentum.

Since the electromagnetic field is not a localized entity, like a point particle, but an extended entity, a full description is more involved. In order to describe the energy–momentum of the electromagnetic field completely, we need to know the *flow* of energy and momentum at every point in space, separately *for each direction*. This makes a description with a *tensor* necessary, the so-called *energy–momentum tensor* $\boldsymbol{T}$ of the electromagnetic field.

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The electric field times a charge is the force on that charge, or equivalently, its momentum increase per time. The generalization for the full electromagnetic field $\boldsymbol{F}$, and for the full power–force (or 4-force) vector $\boldsymbol{K}$ is

$$F^{\mu\nu} j_\mu = K^\nu = \partial_\mu \boldsymbol{T}^{\mu\nu} \quad . \tag{56}$$

This short equation, which can also be derived from the Lagrangian, contains a lot of information. In particular, it expresses that every change in energy of the field is the sum of the energy radiated away (via the energy flow described by the Poynting vector $\boldsymbol{S}$) and of change in the kinetic energy of the charges. The equation also makes a similar statement on the momentum of the electromagnetic field.

The detailed parts of the energy–momentum tensor $\boldsymbol{T}$ are found to be

$$\begin{aligned} \boldsymbol{T}^{\mu\nu} &= \left(\begin{array}{c|c} \text{energy density} & \text{energy flow or momentum density} \\ \hline \text{energy flow or momentum density} & \text{momentum flow density} \end{array} \right) \\ &= \left(\begin{array}{c|c} u & \boldsymbol{S}/c = c\boldsymbol{p} \\ \hline c\boldsymbol{p} & T \end{array} \right) = \left(\begin{array}{c|c} (\varepsilon_0 E^2 + B^2/\mu_0)/2 & \varepsilon_0 c \boldsymbol{E} \times \boldsymbol{B} \\ \hline \varepsilon_0 c \cdot & -\varepsilon_0 E_i E_j - B_i B_j/\mu_0 \\ \boldsymbol{E} \times \boldsymbol{B} & 1/2 \delta_{ij} (\varepsilon_0 E^2 + B^2/\mu_0) \end{array} \right) \end{aligned} \tag{57}$$

where $\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{B}/\mu_0$ is the *Poynting vector* that describes the energy flow density of the electromagnetic field. The energy–momentum tensor $\boldsymbol{T}$ obeys a continuity relation: it

describes a conserved quantity.

We can sum up by stating that in nature, energy and momentum are conserved, if we take into account the momentum and energy of the electromagnetic field. And the energy–momentum tensor shows again that electrodynamics is a gauge invariant description: the energy and momentum values do not depend on gauge choices.

The energy–momentum tensor, like the Lagrangian, shows that electrodynamics is invariant under *motion inversion*. If all charges change direction of motion – a situation often confusingly called 'time inversion' – they move backwards along the same paths they took when moving forward. Every example of motion due to electric or magnetic causes can also take place backwards.

Challenge 89 e

On the other hand, everyday life shows many electric and magnetic effects which are not time invariant, such as the breaking of bodies or the burning of electric light bulbs. Can you explain how this fits together?

Challenge 90 s

We also note that charges and mass destroy a symmetry of the vacuum that we mentioned in special relativity: only the vacuum is invariant under conformal transformations. In particular, only the vacuum is invariant under the spatial inversion $r \rightarrow 1/r$. Any other physical system does not obey conformal symmetry.

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To sum up, electrodynamic motion, like all other examples of motion that we have encountered so far, is deterministic, slower than c, reversible and conserved. This is no big surprise. Nevertheless, two other symmetries of electromagnetism deserve special mention.

What is a mirror? Is nature parity-invariant?

We will study the strange properties of mirrors several times during our walk. We start with the simplest one first. Everybody can observe, by painting each of their hands in a different colour, that a mirror does *not* exchange right and left, as little as it exchanges up and down; however, a mirror does exchange right and left *handedness*. In fact, it does so by exchanging front and back.

Electrodynamics give a second answer: a mirror is a device that switches magnetic north and south poles. Can you confirm this with a diagram?

Challenge 91 s

But is it always possible to distinguish left from right? This seems easy: this text is quite different from a ɒɘɿoɿɿim version, as are many other objects in our surroundings. But take a simple landscape. Are you able to say which of the two pictures of Figure 47 is the original?

Astonishingly, it is actually impossible to distinguish an original picture of nature from its mirror image if it does not contain any human traces. In other words, everyday nature is somehow left–right symmetric. This observation is so common that all candidate exceptions have been extensively studied. Examples are the jaw movement of ruminating cows, the helical growth of plants, such as hops, the spiral direction of snail shells or the left turn taken by all bats when exiting their cave. The most famous example is the position of the heart. The mechanisms leading to this disposition are still being investigated. Recent research discovered that the oriented motion of the cilia on embryos, in the region called the *node*, determines the right–left asymmetry.

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Most human bodies have more muscles on the right side for right-handers, such as Albert Einstein and Pablo Picasso, and correspondingly on the left side for left-handers,

FIGURE 47 Which one is the original landscape? (NOAA).

such as Charlie Chaplin and Peter Ustinov. This asymmetry reflects an asymmetry of the human brain, called lateralization, which is essential to human nature.

Another asymmetry of the human body is the hair whirl on the back of the head; the majority of humans have only one, and in 80 % of the cases it is left turning. But many people have more than one. Can you name additional body asymmetries?

Challenge 92 s

The left–right symmetry of nature appears because everyday nature is described by gravitation and, as we will see, by electromagnetism. Both interactions share an important property: substituting all coordinates in their equations by the negative of their values leaves the equations unchanged. This means that for any solution of these equations, i.e., for any naturally occurring system, a mirror image is a possibility that can also occur naturally. Everyday nature thus cannot distinguish between right and left. Indeed, there are right *and* left handers, people with their heart on the left *and* others with their heart on the right side, etc.

To explore further this strange aspect of nature, try the following experiment: imagine you are exchanging radio messages with a Martian; are you able to explain to him what right and left are, so that when you meet, you are sure you are talking about the same thing?

Challenge 93 s

Actually, the *mirror symmetry* of everyday nature – also called its *parity invariance* – is so pervasive that most animals cannot distinguish left from right in a deeper sense. Most animals react to mirror stimuli with mirror responses. It is hard to teach them different ways to react, and it is possible almost only for mammals. The many experiments performed in this area gave the result that animals have symmetrical nervous systems, and possibly only humans show *lateralization*, i.e., a preferred hand and different uses for the left and the right parts of the brain.

Ref. 49

To sum up this digression, classical electrodynamics is left–right symmetric, or parity invariant. Can you show this using its Lagrangian?

Challenge 94 s

Why do metals provide good mirrors? Metals are strong absorbers of light. Any strong absorber has a metallic shine. This is true for metals, if they are thick enough, but also for dye or ink crystals. Any material that strongly absorbs a light wavelength also reflects it efficiently. The cause of the strong absorption of a metal are the electrons inside it; they can move almost freely and thus absorb most visible light frequencies; this leads to evanescent waves in the material and strong reflection. Strong reflection appears as soon as the absorption length is as low as about one wavelength. This is the reason that, for

example, strong coffee, strong tea and dense alkali vapour work as mirrors. (However, strong reflection is also possible without strong absorption, as the ubiquitous dielectric multilayers show.)

Page 86 Here is a puzzle: a concave mirror shows an inverted image; so does a plane mirror if it is partly folded along the horizontal. What happens if this mirror is rotated around Challenge 95 s the line of sight?

What is the difference between electric and magnetic fields?

Obviously, the standard answer is that electric fields have sources, and magnetic fields do not; as a result, magnetic fields are small relativistic effects of importance only when charge velocities are high or when electrical fields cancel out.

For situations involving matter, fields can indeed be distinguished with their sources. Up to the present day, no particle with a magnetic charge, called a *magnetic monopole*, has ever been found, even though its existence is possible in several speculative models Vol. V, page 262 of particle physics. If found, the action (51) would have to be modified by the addition of a fourth term, namely the magnetic current density. However, no such particle has yet been detected, despite intensive search efforts.

In empty space, when matter is not around, it is possible to take a completely different view. In empty space the electric and the magnetic fields can be seen as two faces of the same quantity, since a transformation such as

$$\begin{aligned} \boldsymbol{E} &\rightarrow c\,\boldsymbol{B} \\ \boldsymbol{B} &\rightarrow -\boldsymbol{E}/c \end{aligned} \tag{58}$$

called (electromagnetic) *duality* transformation, transforms each vacuum Maxwell equation into the other. The minus sign is necessary for this. (In fact, there are even more such Challenge 96 s transformations; can you spot them?) Alternatively, the duality transformation transforms $\boldsymbol{F}$ into ${}^{*}\boldsymbol{F}$. In other words, in empty space we *cannot* distinguish electric from magnetic fields. In particular, it is impossible to say, given a field line in vacuum, whether it is a magnetic or an electric field line.

Matter would be symmetric under duality only if magnetic charges, also called magnetic monopoles, could exist. In that case the transformation (58) could be extended to

$$c\rho_{\mathrm{e}} \rightarrow \rho_{\mathrm{m}} \quad , \quad \rho_{\mathrm{m}} \rightarrow -c\rho_{\mathrm{e}} \; . \tag{59}$$

For a long time, it was thought that duality can be used in the search for the final, unified Ref. 50 theory of physics. However, this hope has evaporated. The reason for this failure can be traced back to a small but ugly fact: the electromagnetic duality transformation changes the sign of the Lagrangian, and thus of the action. Therefore, electromagnetic duality is not a real symmetry of nature, and thus does not help to reach a deeper understanding of electromagnetism.

Duality, by the way, is a symmetry that works *only* in Minkowski space-time, i.e., in space-times of 3 + 1 dimensions. Mathematically, duality is closely related to the existence of quaternions, to the possibility of interpreting Lorentz boosts as rotations in 3 + 1 dimensions, and last, but not least, to the possibility of defining other smooth mathemat-

ical structures than the standard one on the space R^4. These mathematical connections are mysterious for the time being; they somehow point to the special role that *four* space-time dimensions play in nature. More details will become apparent in the last volume of our adventure.

Could electrodynamics be different?

Ref. 39 We saw that electrodynamics is based on three ideas: the conservation of charge, the speed limit for charges and Coulomb's inverse square relation. Could any of these be wrong or need modification?

Experiments imply that the only candidate for modification is Coulomb's relation. Indeed, any interaction, such as Coulomb's relation (4), which acts, for one given observer, between two particles independently of 3-velocity, must depend on 3-velocity for other inertial observers.* Such an interaction must also depend on the 4-velocity, to ensure the requirement from special relativity that the 4-acceleration must be 4-orthogonal to the 4-velocity. The simplest case of such an interaction is an interaction in which the acceleration is proportional to the 4-velocity. Together with the request that the interaction Ref. 51 leaves the rest mass constant, we then recover electrodynamics. Other interactions do not agree with experiment.

In fact, the requirements of gauge symmetry and of relativistic invariance also make it impossible to modify electrodynamics. In short, it does not seem possible to have a behaviour different from $1/r^2$ for a classical interaction.

Maybe a tiny deviation from Coulomb's relation is possible? An inverse square dependence implies a vanishing mass of light and light particles, the photons. Is the mass Ref. 52 really zero? The issue has been extensively studied. A massive photon would lead to a wavelength dependence of the speed of light in vacuum, to deviations from the inverse square 'law', to deviations from Ampère's 'law', to the existence of longitudinal electromagnetic waves and more. No evidence for these effects has ever been found. A summary of these studies shows that the photon mass is below 10^{-53} kg, or maybe 10^{-63} kg. Some arguments are not universally accepted, thus the limit varies somewhat from researcher to researcher.

A small non-vanishing mass for the photon would change electrodynamics somewhat. The inclusion of a tiny mass poses no special problems, and the corresponding Ref. 52 Lagrangian, the so-called *Proca Lagrangian*, has already been studied, just in case.

Strictly speaking, the photon mass cannot be said to vanish. In particular, a photon with a Compton wavelength of the radius of the visible universe cannot be distinguished from one with zero mass through any experiment. This gives a limit mass of 10^{-69} kg for the photon. Photons with such a small mass value would not invalidate electrodynamics as we know it. We note that at present, the experimental limits are still much larger. A surprise is still possible, in principle.

Interestingly, a non-zero mass of the photon would imply the lack of magnetic monopoles, as the symmetry between electric and magnetic fields would be broken. It is therefore important on the one hand to try to improve the experimental mass limit for

* This can be deduced from special relativity, from the reasoning of page 52 or from the formula in the footnote of page 80 in volume II.

photons, and on the other hand to explore whether the limit due to the universe's size has any implications for this issue. The question is still open.

In summary, it seems extremely difficult, if not impossible, to find modifications of electrodynamics that do not disagree with experiment. Electrodynamics is fixed once for all.

The brain: the toughest challenge for electrodynamics

Researchers working on classical electrodynamics still face a fascinating experimental and theoretical issue: understanding the process of thought. Researchers face two challenges in this domain. First, they must find ways to *model* the thought process. Second, the technology to *measure* the currents in the brain must be extended. In both domains, recent progress has been spectacular.

Important research has been carried out on many levels of thought modelling. For example, research using computer tomography, PET scans and MRI imaging has shown that the distinction between the *conscious* and the *unconscious* can be measured: it has a biological basis. Conscious and unconscious thoughts happen in different brain regions. Psychological processes, such as *repression* of unpleasant thoughts, can actually be observed in brain scans. Modellers of brain mechanisms are learning that various concepts of psychology are descriptions for actual physical processes. This research approach is still in its infancy, but very promising.

About the specific aspects of the working of the brain, such as learning, storage, recognition of shapes, location of sound sources or map formation, modern neurobiology and animal experimentation have allowed deducing models that make quantitative predictions. More on this will be told below.

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On the experimental side, research into magnetoencephalography devices is making rapid progress. The magnetic fields produced by brain currents are as low as 10 fT, which require sensors at liquid helium temperature and a good shielding of background noise. Improving the sensitivity and the spatial resolution of these systems is a central task. Also computer models and algorithms are making rapid progress.

The whole programme would be complete as soon as, in a distant future, a sensitive measuring apparatus could detect what is going on inside the brain and then could deduce or 'read' the thoughts of a person from these measurements. Thought reading might be the most complex of all challenges that science is facing. Clearly, such a feat will require involved and expensive machinery, so that there is no danger for a misuse of the technique. There are also good reasons to believe that full thought reading will never be possible in this way, due to the lack of localization of cognitive thought inside the brain and due to the variations in cognitive processing from one person to another. But the understanding and modelling of the brain will be a useful technology in numerous aspects of daily life, especially for the disabled.

On the path towards thought reading, the small progress that has been achieved so far is already fascinating. Wearing a cap full of electric contacts – a so-called *brain–computer interface* – and looking at a computer screen, it is now possible to type letters using the power of thought alone. Such a system is shown in Figure 48. The user controls the computer simply by *imagining* that he turns the arrow on the screen with his right hand. The brain currents created by the imagination process are read out and translated into

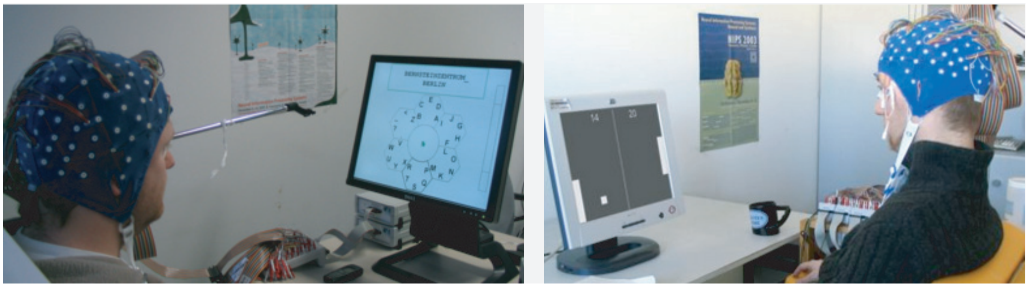


FIGURE 48 Typing a letter and playing video tennis using thought alone (© Fraunhofer FIRST).

Ref. 53 computer commands by an electronic device. The system, based on neural network algorithms, works after only 20 minutes of training with a particular person. In this way, the system allows people who are fully paralysed to communicate with others again. The system is so fast that it allows playing 'mental video tennis' on a computer screen.

Typing with thought alone is possible because the brain region responsible for the hand is near the skull, so that signals for hand rotation can be read out with sufficient spatial resolution by the electrodes on the cap. Researchers know that resolution limitations do not allow reading out the commands for single fingers in this way. For such high resolution tasks, electrodes still need to be *implanted* inside the relevant brain region. However, at present the functional lifetime for such electrodes is only a few months, so that the dream of controlling machines or even artificial limbs in this way is still distant.

Recent research with brain–computer interfaces suggests that in a not-too distant future a computer might be able to read out a secret number, such as a credit card PIN, Ref. 54 that a person is thinking about. The coming decades will surely yield more such research results.

Challenges and fun curiosities about electrodynamics

Not only animals, also plants can feel electric and magnetic fields. At least for magnetic fields, the sensors seem to use very similar mechanisms to those used by animals and bacteria.

* *

For everyday size – and larger – systems, electromagnetic motors are most effective. For microscopic sizes, electrostatic motors are more effective. They are used in sensors and small actuators. In contrast, large power systems use alternating current instead of direct current.

* *

If you calculate the Poynting vector for a charged magnet – or simpler, a point charge near a magnet – you get a surprising result: the electromagnetic energy flows in circles around the magnet. How is this possible? Where does this angular momentum come Challenge 97 s from?

Worse, any atom is an example of such a system – actually of two such systems. Why Ref. 55 is this effect not taken into account in calculations in quantum theory?

* *

Challenge 98 s

Perfectly spherical electromagnetic waves are impossible in nature. Can you show this using Maxwell's equation of electromagnetism, or even without them?

* *

Mirrors exist in many forms. An important mirror for radio waves is the ionosphere; especially during the night, when certain absorbing layers disappear, the ionosphere allows receiving radio stations from far away. When the weather is favourable, it is possible to receive radio stations sending from the antipodes. Another radio mirror is the Moon; with modern receivers it is possible to receive radio signals and, since a few years, even television signals reflected by the Moon.

* *

Challenge 99 s

In the past, textbooks often said that the Poynting vector, the electromagnetic energy flow, was not uniquely defined. Even Richard Feynman talks about this issue in his *Lectures on Physics*, in section 27-4. Can you show that there is no such ambiguity in the Poynting vector, and that those textbooks are all wrong?

* *

Ref. 56

No magnetic charges exist. More precisely, no particles with a single, non-zero magnetic charge exist. But we can introduce the mathematical quantity 'magnetic charge' nevertheless – it is usually called '*magnetic pole strength* – as long as we require that every object always has equal amounts of opposite magnetic charge values. With this condition, the magnetic charge is the divergence of the magnetization and obeys the magnetostatic Poisson equation, in a striking parallel to the electric case.

* *

Challenge 100 s

Any wall plug is a dipole driven by an alternating electric field. Why does a wall plug, delivering 230 V or 100 V at 50 Hz or 60 Hz, not radiate electromagnetic fields?

* *

Challenge 101 e

Why does a voltage transformer contain a ferromagnetic core?

* *

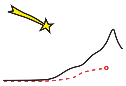
Challenge 102 s

Are there electromagnetic motors in biological systems?

Summary on electromagnetic field motion

In summary, the electromagnetic field carries energy, linear momentum and angular momentum. It is thus appropriate to say that the electromagnetic field *moves*. The motion of the electromagnetic field can be visualized as the motion of its electric and its magnetic field lines. The motion of the electromagnetic field is described by a least action principle. The motion conserves energy and momentum. The motion is continuous, relative, reversible and mirror-invariant.

We are directly lead to ask: what then is the nature of light?



Chapter 3

WHAT IS LIGHT?

Ref. 57

The nature of light has fascinated explorers of nature since at least the time of the ancient Greeks. The answer appeared in 1848, when Gustav Kirchhoff noted that the experimental values on both sides of the equation

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} . \tag{60}$$

agreed within measurement errors. This suggested the answer to the question asked two thousand years earlier:

▷ Light is an electromagnetic wave.

Ten years later, in 1858, Bernhard Riemann* proved mathematically that any electromagnetic wave must propagate with a speed c given by the above equation. Note that the right-hand side contains electric and magnetic quantities, and the left-hand side is an optical quantity. The expression of Kirchhoff and Riemann thus unifies electromagnetism and optics. The modern value for the speed of electromagnetic waves, usually called c from Latin *celeritas*, is

$$c = 299\,792\,458\,\mathrm{m/s} . \tag{61}$$

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The value for c is an integer number, because the meter is nowadays defined in such a way as to exactly achieve this number.

In 1865, Maxwell summarized all data on electricity and magnetism collected in the 2500 years in his equations. Almost nobody read his papers, because he wrote them using quaternions. The equations were then simplified independently by Heinrich Hertz and Oliver Heaviside. They deduced the original result of Riemann: in the case of empty space, the equations of the electromagnetic potentials can be written as

$$\Box \boldsymbol{A} = 0 \quad \text{or, equivalently} \quad \varepsilon_0 \mu_0 \frac{\partial^2 \varphi}{\partial t^2} + \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} = 0 . \tag{62}$$

* Bernhard Riemann (b. 1826 Breselenz, d. 1866 Selasca), important mathematician. A path-breaking mathematician, he also studied curved space, providing several of the mathematical and conceptual foundations of general relativity, but then died at an early age.

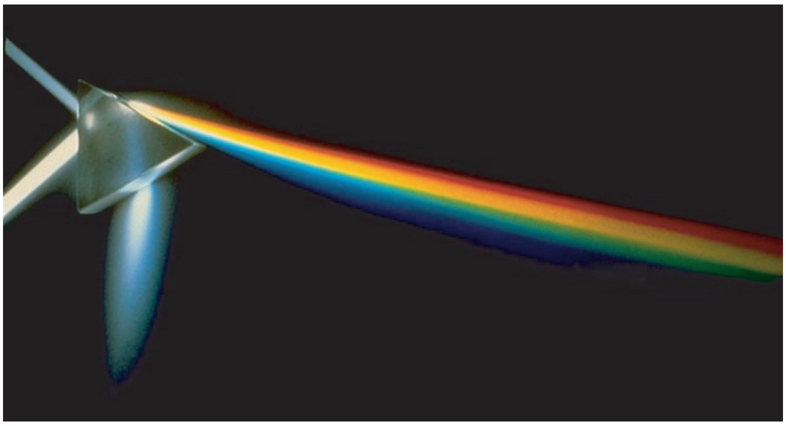

FIGURE 49 White light travelling through a glass prism (photograph by Susan Schwartzenberg, © Exploratorium www.exploratorium.edu).

Challenge 103 e This evolution equation is a *wave equation*, because it admits solutions of the type

$$\boldsymbol{A}(t, \boldsymbol{x}) = \boldsymbol{A}_0 \sin(\omega t - \boldsymbol{k}x + \delta) = \boldsymbol{A}_0 \sin(2\pi f t - 2\pi \boldsymbol{x}/\lambda + \delta) , \tag{63}$$

which are commonly called harmonic *plane electromagnetic waves*. We recall that a *wave* in physics is any propagating imbalance, and that a *harmonic* wave is a wave described by a sine curve. Vol. I, page 272

Such a harmonic plane electromagnetic wave satisfies equation (62) for any value of *amplitude* A_0, of *phase* δ, and of *angular frequency* ω, provided the angular frequency and the *wave vector* $\boldsymbol{k}$ satisfy the relation

$$\omega(\boldsymbol{k}) = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} k \quad \text{or} \quad \omega(\boldsymbol{k}) = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \sqrt{\boldsymbol{k}^2} . \tag{64}$$

The relation $\omega(\boldsymbol{k})$ between the angular frequency and the wave vector, the so-called *dispersion relation*, is the main property of any type of wave, be it a sound wave, a water wave, an electromagnetic wave, or any other kind.

The specific dispersion relation (64) is *linear* and implies a *phase velocity*, the velocity with which wave crests and troughs move, given by $\omega/k = 1/\sqrt{\varepsilon_0 \mu_0} = c$, thus reproducing the result by Kirchhoff and Riemann.

Experiments in empty space confirm that the phase velocity c is independent of the frequency of the wave. This phase velocity thus characterizes electromagnetic waves, and distinguishes them from all other types of waves in everyday life.

What are electromagnetic waves?

To get a clearer idea of electromagnetic waves, we explore their properties. The wave equation (62) for the electromagnetic field is *linear* in the field; this means that the sum of two allowed situations is itself an allowed situation. Mathematically speaking, any *superposition* of two solutions is also a solution. We therefore know that electromagnetic waves must show *interference*, as all linear waves do.

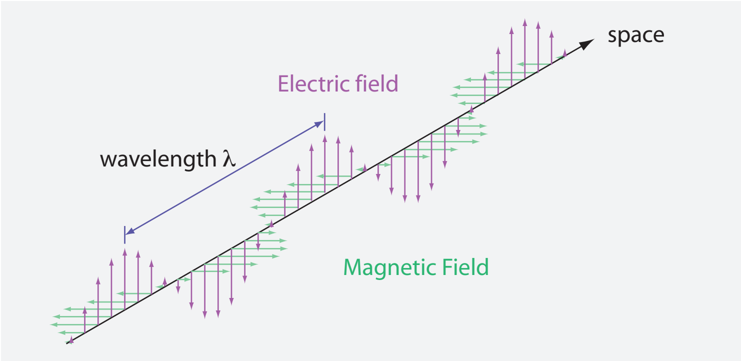

FIGURE 50 The general structure of a plane, monochromatic and linearly polarized electromagnetic wave at a specific instant of time.

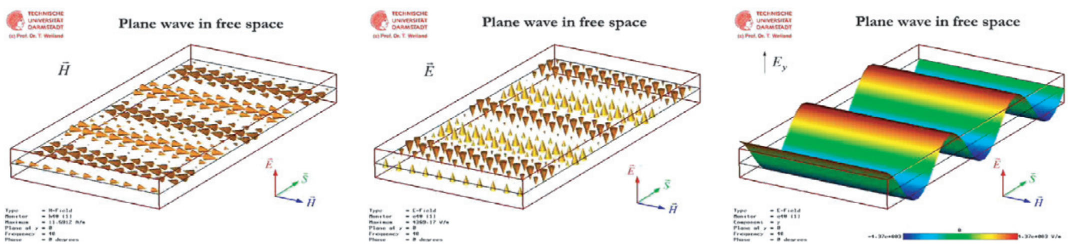

FIGURE 51 A plane, monochromatic and linearly polarized electromagnetic wave, showing the evolution of the electric field, the magnetic field, and again the electric field, in a further visualization (Mpg films © Thomas Weiland).

FIGURE 52 Heinrich Hertz (1857–1894).

Linearity also implies that two waves can cross each other without disturbing each other, and that electromagnetic waves can travel undisturbed across static electromagnetic fields.

Linearity also means that every electromagnetic wave can be described as a superposition of harmonic, or pure sine waves, each of which is described by expression (63). The simplest possible electromagnetic wave, the harmonic plane wave with linear polarization, is illustrated in Figure 50. Note that for this simplest type of waves, the electric and the magnetic field are *in phase*. (Can you prove this experimentally and by calculation?) The surfaces formed by all points of maximal field intensity are parallel planes, spaced by (half the) wavelength; these planes move along the direction of the propagation with the phase velocity.

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After Riemann and Maxwell predicted the existence of electromagnetic waves, in the

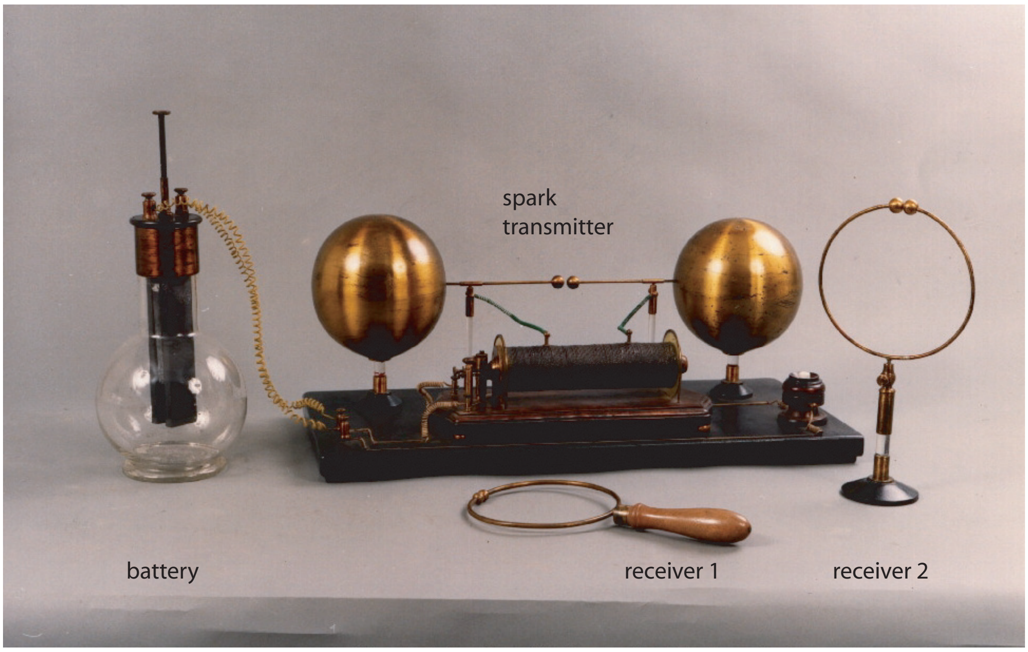

FIGURE 53 A reconstruction of one of the first transmitters and receivers of electromagnetic waves by Heinrich Hertz (© Fondazione Guglielmo Marconi).

years between 1885 and 1889, Heinrich Hertz* discovered and studied them. He fabricated a very simple transmitter and receiver for 2 GHz waves, shown in Figure 53. Such waves are still used today: cordless telephones and the last generation of mobile phones work at this frequency – though the transmitters and the receivers look somewhat differently nowadays. Such waves are now also called *radio waves*, since physicists tend to call all moving force fields *radiation*, recycling somewhat incorrectly a Greek term that originally meant 'light emission.'

Today Hertz's experiment can be repeated in a much simpler way. As shown in Figure 54, a budget of a few euro is sufficient to remotely switch on a light emitting diode with a gas lighter. (After each activation, the coherer has to be gently tapped, in order to get ready for the next activation.) Attaching longer wires as antennas and ground allows this set-up to achieve transmission distances up to 30 m.

Hertz also measured the *speed* of the waves he produced. In fact, you can also measure the speed at home, with a chocolate bar and a (older) kitchen microwave oven. A microwave oven emits radio waves at 2.5 GHz – not far from Hertz's value. Inside the oven, these waves form standing waves. Just put the chocolate bar (or a piece of cheese) in the oven and switch the power off as soon as melting begins. You will notice that the bar melts at regularly spaced spots. These spots are half a wavelength apart. From the measured wavelength value and the frequency, the speed of light and radio waves simply

* Heinrich Rudolf Hertz (b. 1857 Hamburg, d. 1894 Bonn), important theoretical and experimental physicist. The unit of frequency is named after him. Despite his early death, Hertz was a central figure in the development of electromagnetism, in the explanation of Maxwell's theory and in the unfolding of radio communication technology. More about him on page 219 in volume I.

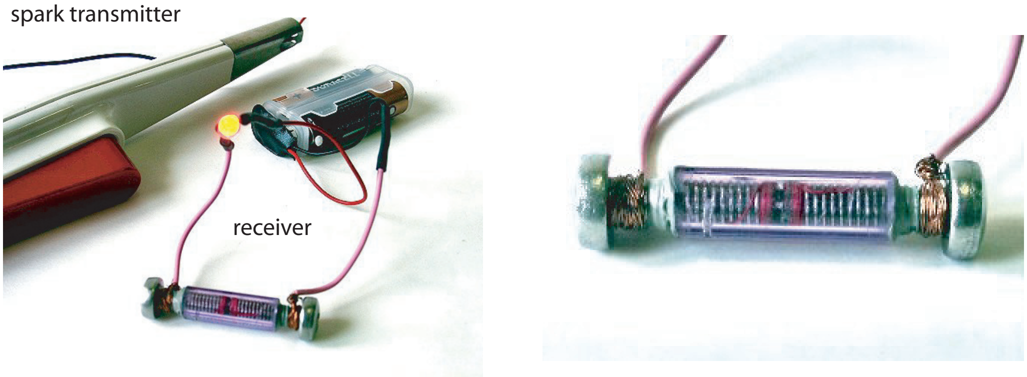

FIGURE 54 The simplest radio transmitter possible, a gas lighter and a wire, together with the simplest radio receiver possible, built from a battery pack, a light emitting diode, and a simple *coherer* made from a ball pen housing, two screws and some metal powder (© Guido Pegna).

follows as the product of the two.

If you are not convinced, you can measure the speed directly, by telephoning a friend on another continent, if you can make sure of using a satellite line (choose a low cost provider). There is about half a second additional delay between the end of a sentence and the answer of the friend, compared with normal conversation. In this half second, the signal goes up to the geostationary satellite, down again and returns the same way. This half second gives a speed of $c \approx 4 \cdot 36\,000\,\text{km}/0.5\,\text{s} \approx 3 \cdot 10^5\,\text{km/s}$, which is close to the precise value. Radio amateurs who reflect their signals from the Moon can perform a similar experiment and achieve higher precision.

In summary, electromagnetic waves exist and move with the speed of light.

Light as a wave

The electromagnetic wave equation has more interesting stories to tell. Above all, the wave equation confirmed earlier predictions that *light* itself is an electromagnetic wave, albeit with a much higher frequency and much shorter wavelength than radio waves. We check this in two steps: we first show that light is a wave and then show that it is electromagnetic.

The first to suggest that light is a (kind of) *wave* was, around the year 1678, the important physicist Christiaan Huygens.* You can confirm that light is a wave with your own fingers. Simply place your hand one or two centimetres in front of your eye, look towards the sky through the gap between the middle and the index finger and let the two fingers almost touch. You will see a number of dark lines crossing the gap. These lines are the interference pattern formed by the light behind the slit created by the fingers. Figure 55

* Christiaan Huygens (b. 1629 's Gravenhage, d. 1695 Hofwyck) was one of the main physicists and mathematicians of his time. Huygens clarified the concepts of mechanics; he also was one of the first to show that light is a wave. He wrote influential books on probability theory, clock mechanisms, optics and astronomy. Among other achievements, Huygens showed that the Orion Nebula consists of stars, discovered Titan, the moon of Saturn, and showed that the rings of Saturn consist of rock. (This is in contrast to Saturn itself, whose density is lower than that of water.)

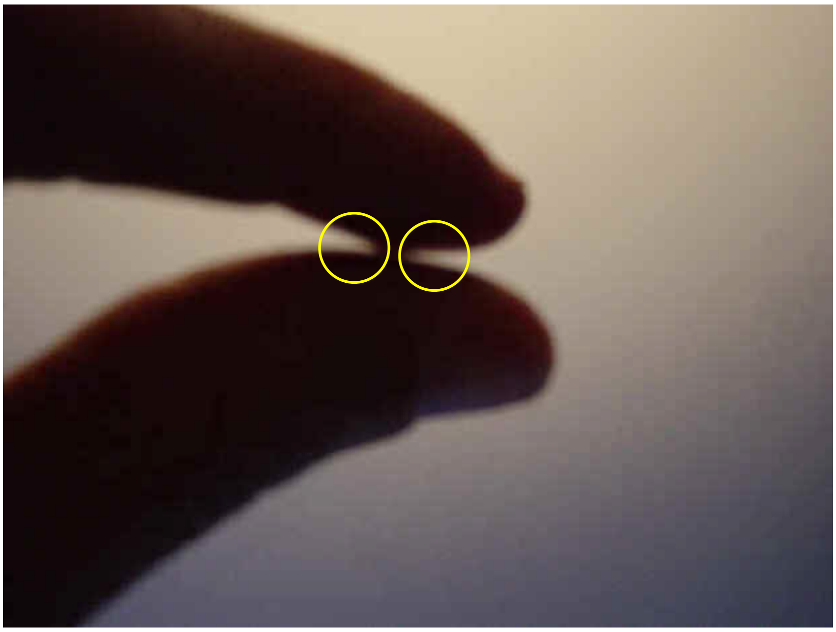

FIGURE 55 Diffraction lines can be seen between the fingers, if one looks carefully enough. (© Chuck Bueter)

FIGURE 56 The primary and secondary rainbow, and the supernumerary bows below the primary bow (© Antonio Martos and Wolfgang Hinz).

shows an example. *Interference* is the name given to the effect and the amplitude patterns that appear when several waves superpose.* The interference patterns depend on the spacing between the fingers. This experiment therefore allows you to estimate the Challenge 105 s wavelength of light, and thus, if you know its speed, its frequency. Can you do this?

Historically, another effect was central in convincing everybody that light was a wave: supernumerary rainbows, the additional bows below the main or primary rainbow. If we look carefully at a rainbow, below the main red–yellow–green–blue–violet bow, we Ref. 58 observe weaker, additional green, blue and violet bows. Depending on the intensity of the rainbow, several of these supernumerary rainbows can be observed. They are due to interference of light triggered by the water droplets, as Thomas Young showed around Page 126 1803.** Indeed, the repetition distance of the supernumerary bows depends on the radius

Challenge 104 s * Where does the energy go in an interference pattern?

** Thomas Young (b. 1773 Milverton, d. 1829 London), read the bible at two, spoke Latin at four; a doctor of

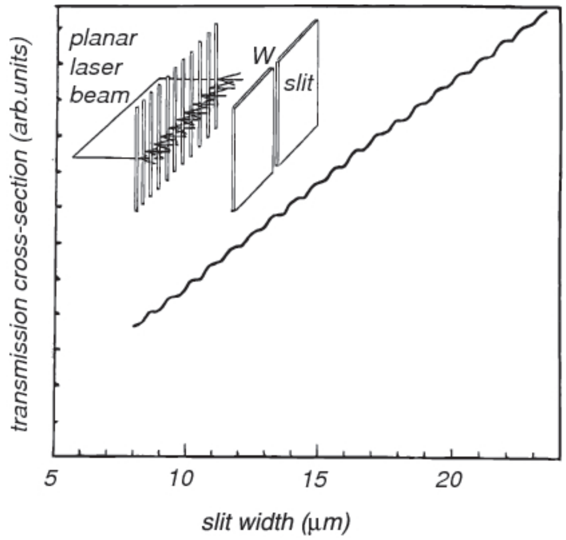

FIGURE 57 The light power transmitted through a slit as function of its width (© Nature).

Ref. 59 and shape distribution of the average water droplets that form them. (Details about the Page 121 normal rainbows are given below.) Both supernumerary rainbows and Thomas Young where essential to convince people that light is a wave. It seems that in those times scientists either did not trust their own eyes or fingers, or did not have any.

There are many other ways in which the wave character of light can be made apparent. Maybe the most beautiful is an experiment carried out by a team of Dutch physicists in Ref. 60 1990. They simply measured the light transmitted through a *slit* in a metal plate. It turns out that the transmitted intensity depends on the width of the slit. Their surprising result is shown in Figure 57. Can you explain the origin of the unexpected intensity steps in the curve? Challenge 106 ny

Interference of light is a common effect. It is commonly seen when lasers are used. A few examples are shown in Figure 58. Both white light interference and laser interference are used for measurements; nowadays, a whole industry makes use of interference effects.

Given an interference pattern like the green one in Figure 58, you may wish to calculate the distance between the lines, given the slit distance s, the colour and the distance d to the screen. (This experiment was used to determine the wavelength of the light for the first time.) Challenge 107 s

Another proof that light is a wave is the discovery of light polarization. We will explore it shortly. Numerous other experiments on the creation, detection and measure-

medicine, he became a professor of physics. He introduced the concept of *interference* into optics, explaining Newtonian rings and supernumerary rainbows; he was the first person to determine light's *wavelength*, a concept that he also introduced, and its dependence on colour. He was the first to deduce the three-colour vision explanation of the eye and, after reading of the discovery of polarization, explained light as a transverse wave. In short, Young discovered most of what people learn at secondary school about light. He was a universal talent: he also worked on the deciphering of hieroglyphs, studied languages and introduced the term 'Indo-European', explored ship building and many engineering problems. Young collaborated with Fraunhofer and Fresnel. In Britain his ideas on light were not accepted, since Newton's followers crushed all opposing views. Towards the end of his life, his results were finally made known to the physics community by Fresnel and Helmholtz.

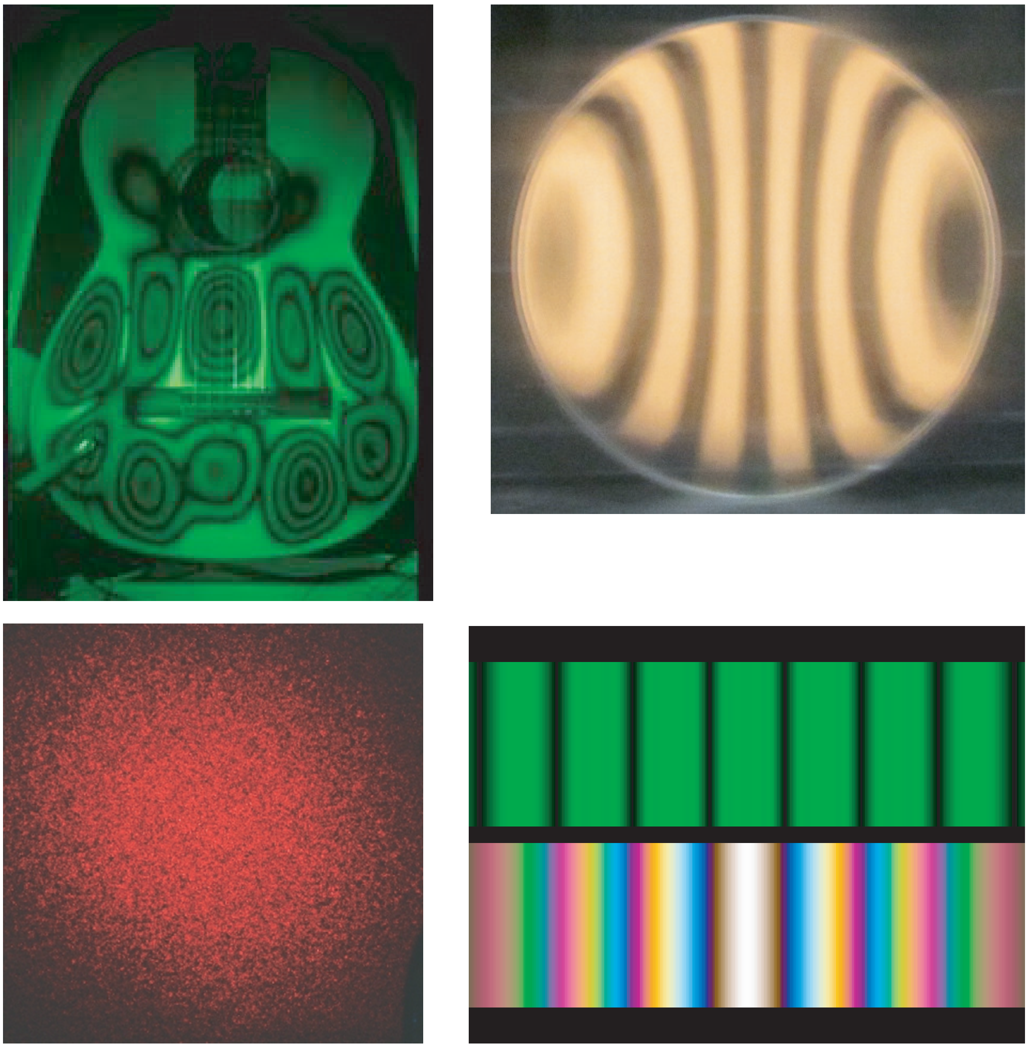

FIGURE 58 Some interference patterns: the interference that a playing guitar produces in laser holography that show how the body vibrates, the interference produced by a good parabolic telescope mirror of 27 cm diameter, a speckle laser pattern on a rough surface and the diffraction pattern produced by two parallel narrow slits illuminated with green light and with white light respectively (© Bernard Richardson, Cardiff University, Mel Bartels, Epzcaw and Dietrich Zawischa).

ment of light waves were performed between the seventeenth and the twentieth century. For example, in 1800, William Herschel discovered *infrared light* using a prism and a thermometer. (Can you guess how?) In 1801, Johann Wilhelm Ritter (b. 1776 Samitz, d. 1810 Munich) a more than colourful figure of natural Romanticism, discovered *ultraviolet light* using silver chloride, AgCl, and again a prism. Modern cameras can image infrared light, as shown beautifully in Figure 59.

Challenge 108 s

At the end of the twentieth century a beautiful confirmation of the oscillations in light waves became possible. Using quite sophisticated experiments, researchers measured the oscillation frequency of light *directly*. They actually managed to count how often light

Ref. 61

FIGURE 59 The same rainbow in the visible and in the infrared, showing how infrared comes before red (© Stefan Zeiger).

wave oscillate in a second! The frequency value, between 375 and 750 THz, is exactly as predicted. The frequency value is so high that its detection was impossible for a long time. But with these modern experiments the dispersion relation of light, $\omega = ck$, has finally been confirmed in all its details, and to extremely high precision. Ref. 62

The result of all these experiments is: light waves, like all other waves, can be distinguished by their wavelength or frequency values. The most important categories are listed in Table 14. Page 104 For visible light, the wavelength lies between 0.4 μm, corresponding to violet, and 0.8 μm, corresponding to red. The wavelength of a visible harmonic light wave determines its *colour*.

Light and other electromagnetic waves

The experiments mentioned so far showed that electromagnetic waves exist, that they move with the same speed as light, and that light is a wave. To confirm that light waves are indeed *electromagnetic* is more difficult. The most convincing proof would be to repeat Hertz's experiments for light. Page 97 In Hertz's experiment, shown in Figure 53, the receiver is a simple open metal circle; when the wave – more precisely, its magnetic field – arrives, a spark is generated and the wave is thus detected.

In an almost incredible feat of miniaturization, in 2009, the research group of Kobus Ref. 63 Kuipers managed to make metal rings much smaller than a micrometre, and repeat Hertz's experiment for light. An impression of their experiment is given in Figure 60. They could clearly discern the maxima and minima of waves, as well as their polarization. They thus showed that light is an electromagnetic wave in exactly the same way as Hertz did for radio waves.

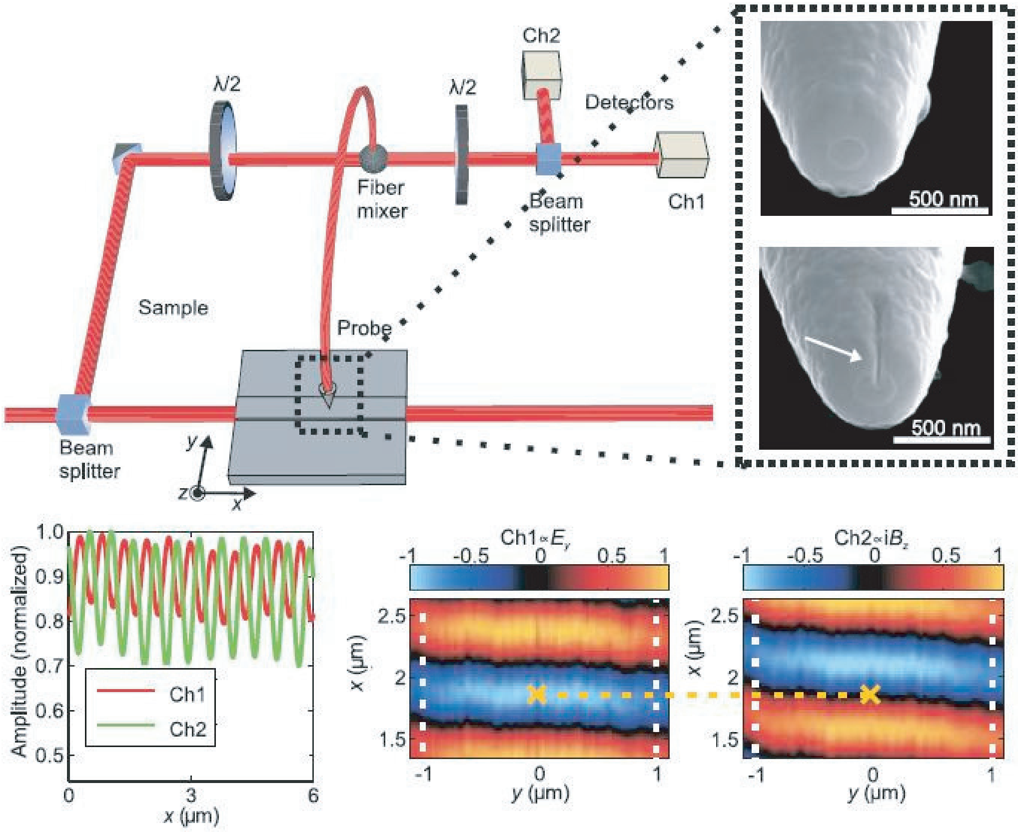

FIGURE 60 An experiment measuring the electric and magnetic field of light. Top left: the general set-up; top right: the antenna, indicated by an arrow; bottom: the measurement data (© Kobus Kuipers)

Of course, people in the 19th century had less technology at their disposal and were not easily convinced. They had to look for other ways to show that light is electromagnetic in nature. Now, since the evolution equations of the electrodynamic field are linear, additional electric or magnetic fields alone do not influence the motion of light. On the other hand, we know that electromagnetic waves are emitted only by accelerated charges, and that all light is emitted from matter. It thus follows that matter is full of electromagnetic fields and accelerated electric charges. This in turn implies that the influence of matter on light can be understood from its internal electromagnetic fields and, in particular, that subjecting matter to an *external* electromagnetic field should change the light it emits, the way matter interacts with light, or generally, the material properties as a whole.

Challenge 109 e

Searching for effects of electricity and magnetism on matter has been a main effort of physicists for over a hundred years. For example, electric fields influence the light transmission of oil, an effect discovered by John Kerr in 1875.* Also the discovery that certain gases change colour when subject to a field yielded several Nobel Prizes for physics. With time, many more influences on light-related properties by matter subjected to fields were

* John Kerr (b. 1824 Ardrossan, d. 1907 Glasgow), was mathematician and physicist, as well as friend and collaborator of William Thomson.

found. An extensive list is given below, in the table on page 216. It turns out that apart from a few exceptions the effects can *all* be described by the electromagnetic Lagrangian (51), or equivalently, by Maxwell's equations (55). In summary, classical electrodynamics indeed unifies the description of electricity, magnetism and optics; all phenomena in these fields, from the rainbow to radio and from lightning to electric motors, are found to be different aspects of the evolution of the electromagnetic field.

Page 84

After two centuries of research, it became clear that light and radio waves form only a small section of the full *spectrum of electromagnetic waves*, which contains the waves from the smallest possible to the largest possible wavelengths. The full spectrum is given in the following table.

TABLE 14 The electromagnetic spectrum.

Frequency	Wavelength	Name	Main properties	Appearance	Use
$3 \cdot 10^{-18}$ Hz	10^{26} m	**Lower frequency limit**		see the section on cosmology	
< 10 Hz	> 30 Mm	**Quasistatic fields**		intergalactic, galactic, stellar and planetary fields, brain, electrical fish	power transmission, accelerating and deflecting cosmic radiation
		Radio waves		electronic devices	
10 Hz–50 kHz	30 Mm–6 km	ELW	go round the globe, penetrate into water, penetrate metal	nerve cells, electromechanical devices	power transmission, communication through metal walls, communication with submarines www.vlf.it
50–500 kHz	6 km–0.6 km	LW	follow Earth's curvature, felt by nerves ('bad weather nerves')	emitted by thunderstorms	radio communications, telegraphy, inductive heating
500–1500 kHz	600 m–200 m	MW	reflected by night sky		radio
1.5–30 MHz	200 m–10 m	SW	circle world if reflected by the ionosphere, destroy hot air balloons	emitted by stars	radio transmissions, radio amateurs, spying
15–150 MHz	20 m–2 m	VHF	allow battery operated transmitters	emitted by Jupiter	remote controls, closed networks, tv, radio amateurs, radio navigation, military, police, taxi

Frequency	Wavelength	Name	Main properties	Appearance	Use
150 – 1500 MHz	2 m–0.2 m	UHF	*idem*, line of sight propagation		radio, walkie-talkies, tv, mobile phones, internet via cable, satellite communication, bicycle speedometers
		Microwaves			
1.5 – 15 GHz	20 cm–2 cm	SHF	*idem*, absorbed by water	night sky, emitted by hydrogen atoms	radio astronomy, used for cooking (2.45 GHz), telecommunications, radar
15 – 150 GHz	20 mm–2 mm	EHF	*idem*, absorbed by water		
		Infrared	allows night vision	emitted by every warm object	satellite photography of Earth, astronomy
0.3 – 100 THz	1000 –3 μm	IRC or far infrared		sunlight, living beings	seeing through clothes, envelopes and teeth
100 – 210 THz	3 μm–1.4 μm	IRB or medium infrared		sunlight	used for optical fibre communications for telephone and cable television
210 – 384 THz	1400–780 nm	IRA or near infrared	penetrates for several cm into human skin	sunlight, radiation from hot bodies	healing of wounds, rheumatism, sport physiotherapy, hidden illumination
375 – 750 THz	800–400 nm	**Light**	not (much) absorbed by air, detected by the eye (up to over 900 nm at sufficient power)	heat ('hot light'), lasers & chemical reactions e.g. phosphor oxidation, fireflies ('cold light')	definition of straightness, enhancing photosynthesis in agriculture, photodynamic therapy, hyperbilirubinaemia treatment
384 – 484 THz	780–620 nm	Red	penetrate flesh	blood	alarm signal, used for breast imaging Ref. 64
	700 nm	Laboratory primary red		filtered tungsten lamp	colour reference for printing, painting, illumination and displays
484 – 511 THz	620–587 nm	Orange		various fruit	attracts birds and insects

Frequency	Wavelength	Name	Main properties	Appearance	Use
511 – 525 THz	587–571 nm	Yellow		majority of flowers	*idem*; best background for reading black text
525 – 614 THz	571–488 nm	Green	maximum eye sensitivity	algae and plants	highest luminous efficiency response ('felt brightness') per light energy for the human eye
	546.1 nm	Laboratory primary green		mercury lamp	colour reference
614 – 692 THz	488–433 nm	Blue		sky, gems, water	
	435.8 nm	Laboratory primary blue		mercury lamp	colour reference
692 – 789 THz	433–380 nm	Indigo, violet		flowers, gems	
		Ultraviolet			
789 – 952 THz	380–315 nm	UVA	penetrate 1 mm into skin, darken it, produce vitamin D, suppress immune system, cause skin cancer, destroy eye lens	emitted by Sun, stars and flames	seen by certain birds, integrated circuit fabrication
0.95 – 1.07 PHz	315–280 nm	UVB	*idem*, destroy DNA, cause skin cancer	*idem*	*idem*
1.07 – 3.0 PHz	280–100 nm	UVC	form oxygen radicals from air, kill bacteria, penetrate 10 μm into skin	emitted by Sun, stars and welding arcs	disinfection, water purification, waste disposal, integrated circuit fabrication
3 –24 PHz	100–13 nm	EUV			sky maps, silicon lithography
		X-rays	penetrate materials	emitted by stars, plasmas and black holes	imaging human tissue
24 – 240 PHz	13–1.3 nm	Soft X-rays	*idem*	synchrotron radiation	*idem*
> 240 PHz or > 1 keV	< 1.2 nm	Hard X-rays	*idem*	emitted when fast electrons hit matter	crystallography, structure determination

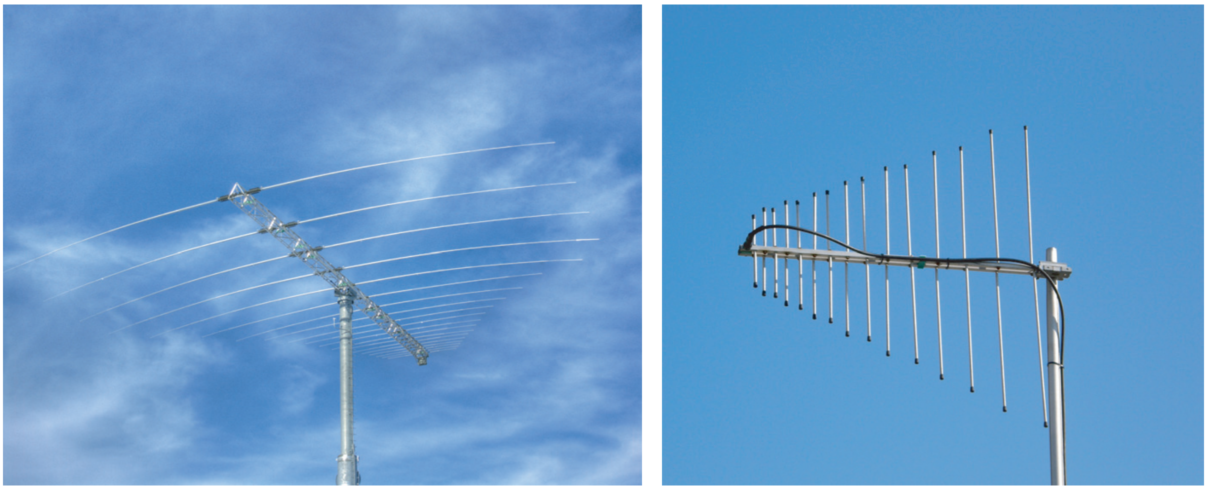

FIGURE 61 Antennas for horizontally and vertically polarized electromagnetic waves (© Martin Abegglen, K. Krallis).

Frequency	Wavelength	Name	Main properties	Appearance	Use
> 12 EHz or > 50 keV	< 24 pm	**γ-rays**	*idem*	radioactivity, cosmic rays	chemical analysis, disinfection, astronomy
$2 \cdot 10^{43}$ Hz	$\approx 10^{-35}$ m	**Planck limit**		see last volume of this series	

Polarization of electromagnetic waves

The electric field in light or in an electromagnetic wave looks like the amplitude of a water Page 96 wave, generalized to three dimensions, as shown in Figure 50 and Figure 51. The same is valid for magnetic fields, and the two fields are perpendicular to each other.

One question about light and all other electromagnetic waves arises: In which spatial direction does the oscillation occur? The answer is hidden in the parameter $\boldsymbol{A}_0$ in expression (63), but shown in Figure 50 and Figure 51. Generally speaking, the fields in electromagnetic waves oscillate in directions *perpendicular* to their motion. Therefore, we follow:

▷ Even for identical frequency and phase, waves can still differ: they can have different *polarization* directions.

For example, the polarization of radio transmitters determines whether radio antennas of receivers have to be kept horizontal or vertical, as shown in Figure 61. For all electromagnetic waves, the polarization is defined, by convention, by the orientation of the *electric* field vector, because practically all effects of electromagnetic waves are due to the electric field.

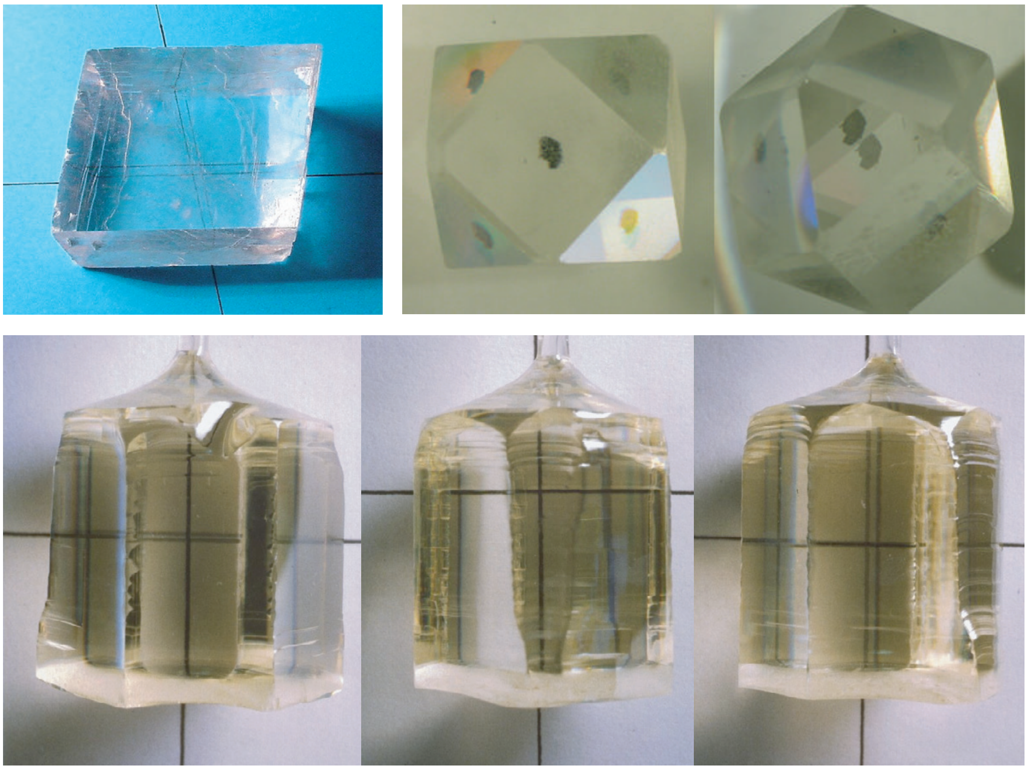

FIGURE 62 Birefringence in crystals: *calcite* lying on crossed lines (top left, crystal size around 4 cm), *rutile* lying on an ink spot, photographed along the optical axis (middle) and at an angle to it (top right, crystal size around 1 cm), and an octagonal sodium vanadate crystal doped with manganese, showing *three* different behaviours (bottom, crystal diameter 1.9 cm) (© Roger Weller/Cochise College, Brad Amos, Martin Pietralla).

Polarization is easily achieved also for light, e.g., by shining it through a stretched plastic film, called a polarizer, or by using glass, water or some special stones. Polarization was discovered in 1808, by Louis Malus (b. 1775 Paris, d. 1812 Paris). Malus discovered polarization when he looked at the strange double images produced by calcite, a transparent crystal found in many minerals. Figure 62 shows two examples. Calcite ($CaCO_3$) splits light beams into two – it is *birefringent* – and polarizes them differently. That is the reason that calcite – or feldspar, ($KAlSi_3O_8$), which shows the same effect – is part of every crystal collection. If you ever get hold of a piece of transparent calcite, do look through it at something written on paper, and rotate the crystal around the vertical. (Can you show that *trirefringence*, if defined as the appearance of three images, cannot exist?)

Challenge 110 ny

Challenge 111 d

When Malus discovered the polarization of light, he did not know yet that light was electromagnetic. But he definitively established the wave nature of light.

The light from the sky – not that from the Sun – is partially polarized. The polarization occurs when the light is scattered by the molecules in the air. The polarization is perpendicular to the direction towards the Sun, as illustrated in Figure 63. The shape is easy to remember with the following connection: *A rainbow is polarized everywhere in tangential direction.* Photographers know that when the Sun is rising or setting, the sky

Ref. 65

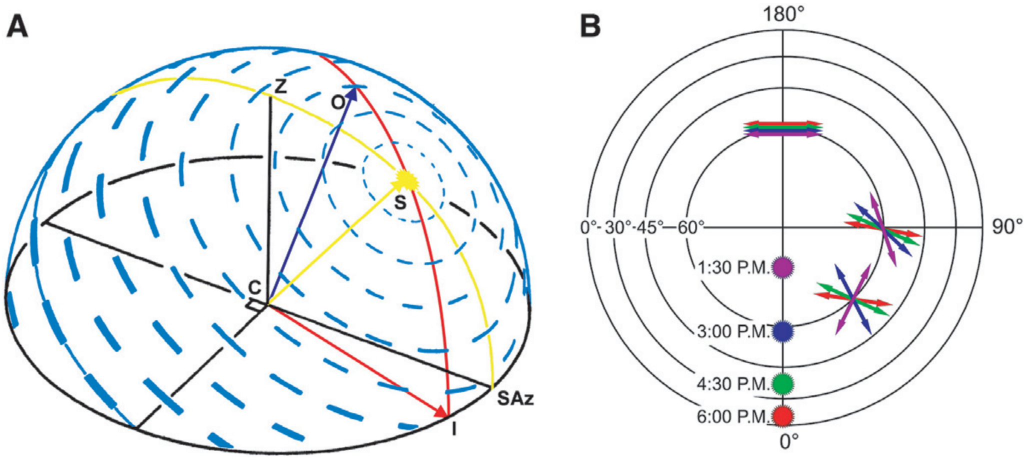

FIGURE 63 Left: the polarization of daylight in the clear sky as a solar elevation of 53°. The orientation and the thickness of the blue bars illustrate the orientation and degree of polarization of the electric field as seen by an observer in the center C of the sphere. The orientation is always perpendicular to a great circle (red) that is defined by connecting a given observation point in the sky O with the position of the Sun S. SAz indicates the solar azimuth of the Sun. Right: the zenithal projection of solar elevation and electric field orientation for different light colours at four times of August 1, at 23.4° N, 5.2° E. Circles represent elevation and the straight lines represent azimuth. The circular polarization pattern of the sky is used by photographers to modify sky photographs and by insects and birds to navigate. (© Keram Pfeiffer/Elsevier).

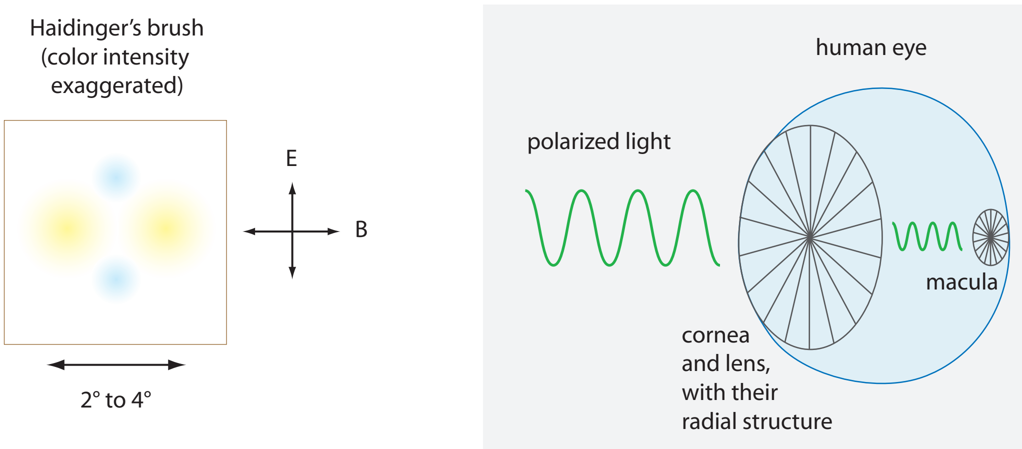

FIGURE 64 Haidinger's brush and its origin in the human eye.

is mainly polarized in north-south direction. This fact can make a lake or a digital watch look black when observed in the evening in northern or southern direction – at a certain observation angles. Also the sunlight below water is partially polarized.

Many insects, spiders, certain birds and certain shrimps can detect polarization with their eyes. Honey bees and many other insects use polarization to deduce the position of Ref. 66 the Sun, even when it is hidden behind clouds, and use the effect for navigation. Some beetles of the genus *Scarabeus* even use the polarization of moonlight for navigation,

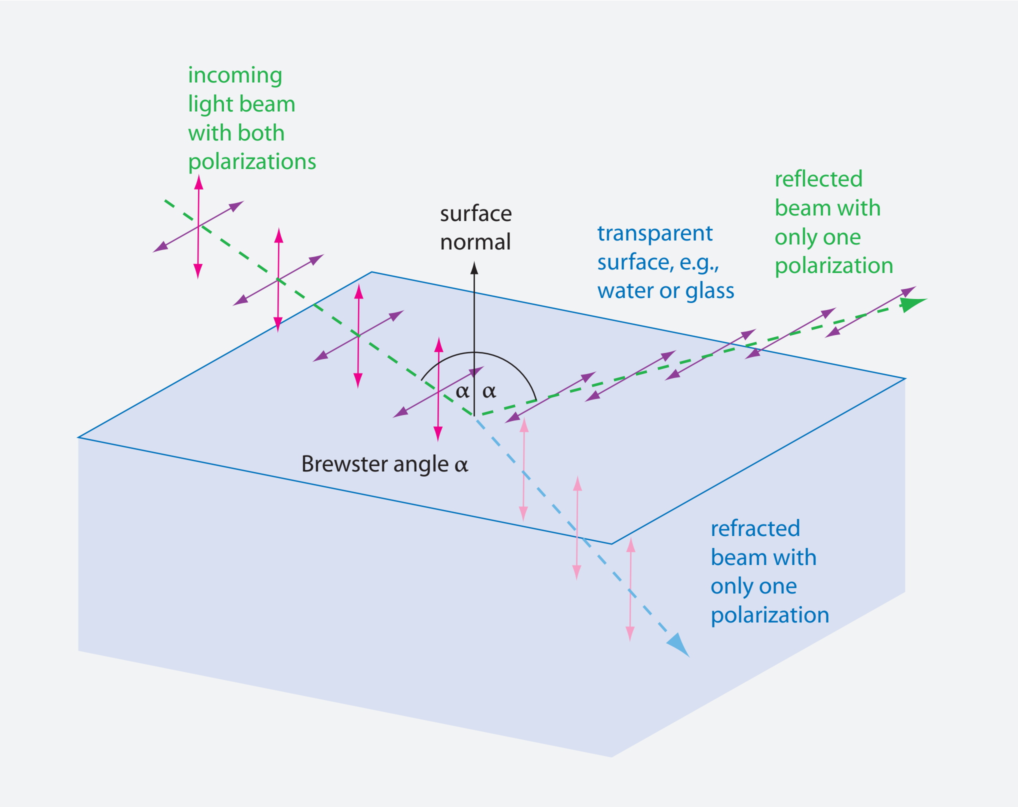

FIGURE 65 For every transparent material, at the so-called *Brewster angle*, only the horizontally polarized light is reflected; the vertically polarized light is then fully refracted. The Brewster angle is a material-dependent quantity. The value for water is, for most wavelengths, 53° and for glass 56(1)°, measured from the line that is normal to the surface.

and many insects use polarization of sunlight to distinguish water surfaces from mirages. (Can you find out how?) Challenge 112 s

In 1844, the mineralogist Wilhelm Haidinger (b. 1795 Vienna, d. 1871 Dornbach) discovered that there is a way to observe the polarization of light with the unaided human eye. (Ref. 67) The best way to observe the effect is by looking at a distance of about an arms's length on a white LCD screen and slowly tilt your head. You will note an *extremely faint* yellow or yellow-blue pattern, about two fingers wide, that is superimposed on the white background. This pattern is called *polarization brush* or *Haidinger's brush*. A rough illustration is given in Figure 64. The weak effect disappears after a few seconds if the head stops rotating along the line of sight. Haidinger's brush is due to the birefringence of the cornea and the lens of the human eye, together with the morphology of the macula lutea inside the eye. (Ref. 68) The cornea acts as a radially oriented, colour-dependent polarizer, whereas the yellow spot acts as a radially oriented analyser. In short, the human eye is indeed able to see the directions in which the electric and magnetic field of light are oscillating.

Haidinger's brush, being yellow, is also visible in the blue sky, provided that the air is clear. (Indeed, it is easily drowned out by multiple scattering, and therefore provides

a test of atmospheric transparency.) In the sky, Haidinger's brush is barely the size of a thumbnail at arm's length. (The angular size is the angular size of the macula.) The yellow arm of the cross points to the Sun, if you look about 90° away from it, high in the sky. To see it really clearly, hold a polarizer (or polarizing sunglasses) upwards and look through it, and then rotate it about the line of sight.

When polarized light is directed to a transparent medium, the ratio between the reflected and the transmitted light intensity depends on the polarization. The transmitted intensity can be zero or near zero for certain critical combinations of angles and polarizations. When the engineers at the Mercedes Benz car company forgot this, it cost the company millions of Euros. Behind the windshield, one of their car models had a sensor that detects whether it is day or night. The photodiode sensor worked well, except when the weather was extremely good, with a blue sky and no clouds; in that case, the sensor gave "night" as output. The mystery was solved when people recognized that the geometry was near the Brewster angle, that in such weather, the light from the sky is polarized and had a low amount of infrared light, at which the – wrongly chosen – photodiode was most sensitive. As a result, tens of thousands of cars had to be repaired.

Note that all possible polarizations of light form a continuous set. However, a general plane wave can be seen as the superposition of two orthogonal, linearly polarized waves with different amplitudes and different phases. Mathematically, all linearly polarized electromagnetic waves with the same frequency and direction for a two-dimensional vector space.

Light can also be *unpolarized*. Unpolarized light is a mixture of light of various polarizations. Light from the Sun and from other hot sources is typically unpolarized, due to the Brownian motion of the emitting sources. *Partially* polarized light is a mixture of polarized and unpolarized light.

In summary, for a wave in three-dimensional space, there are two basic types of polarization. One often classifies them into horizontal and vertical polarization, or, with other terms, into parallel and perpendicular polarization. A generally polarized wave is a superposition of these two basis states. These are the so-called *linear* polarization states.

Interestingly, a generally polarized plane wave can also be seen as the superposition of right and left *circularly polarized waves*. An illustration of a circularly polarized wave is given in Figure 66. In nature, circular polarization is extremely rare. Firefly larvae emit circularly polarized light. The light reflected by many species of scarab beetles is circularly polarized, as is the case for various stomatopod crustaceans, such as the mantis shrimp. Ref. 69 The latter – and probably the former – are also able to detect circularly polarized light.

The range of electromagnetic radiation

Electromagnetic waves of lower frequency, or radio waves, are commonly used to transmit mobile phone signals as well as television, radio and satellite programs. Like light, radio waves are due to moving electrons. In everyday life, light is (usually) generated by electrons accelerated inside atoms or molecules. Radio waves, which have lower frequency and thus larger wavelength, are more easily generated by electrons that are accelerated in metals roughly of the size of the wavelength; such pieces of metal are called *antennas*.

Radio waves emitted by a hand-held device can carry signals round the Earth. In other

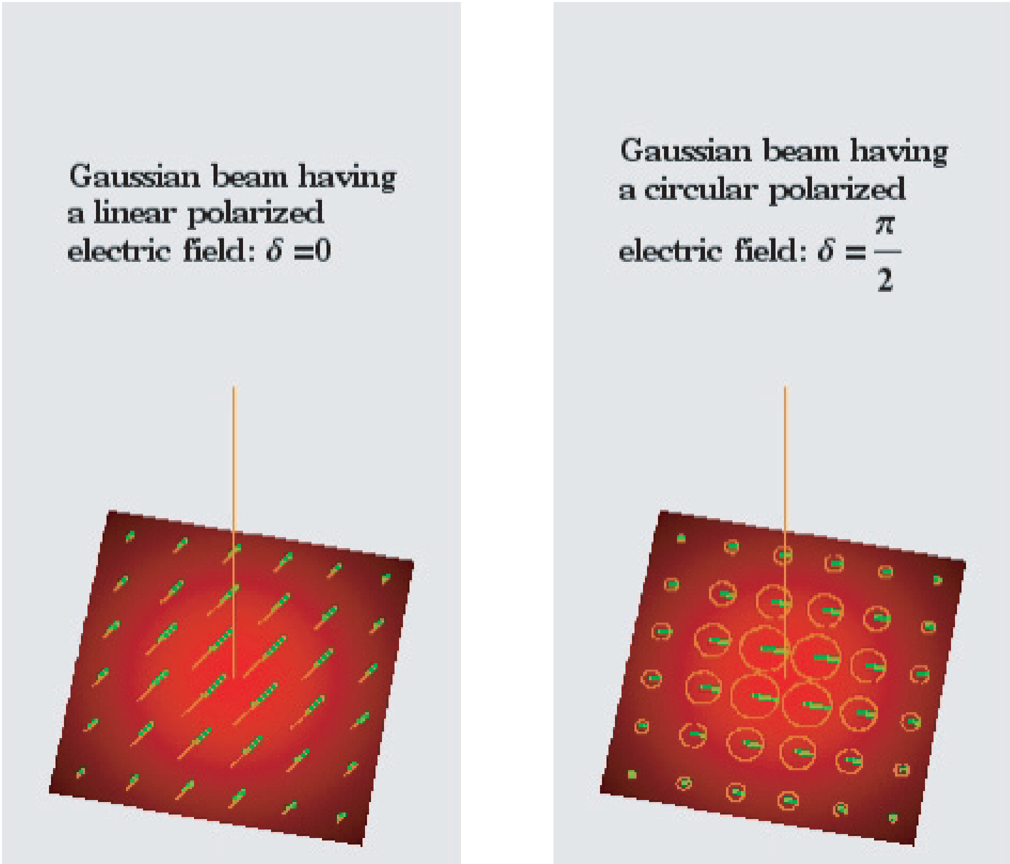

FIGURE 66 Left: the electric field of a Gaussian, linearly polarized electromagnetic wave (a beam); right: a Gaussian, circularly polarized beam (QuickTime film © José Antonio Díaz Navas).

words, radio waves have a large range. How is this possible? After all, a static electric field is usually unmeasureable after a distance of a dozen meters. It turns out that the field strength of radio waves decreases as $1/r$, where r is the distance from the source. The field strength thus decreases much more slowly than for static fields, which decrease as
Ref. 70 $1/r^2$. Why is this the case?

The slow $1/r$ dependence of radio waves can be understood qualitatively from the drawing shown in Figure 67. It shows the electric field around a charged particle that undergoes the simplest possible accelerated motion: a bounce on a wall. In fact, the last, lower diagram is sufficient to show that the transverse field, given by the *kink* in the
Challenge 113 d electric field lines, decreases as $1/r$. Can you deduce the dependence?

If we perform the construction of the field lines for a charge that undergoes *repeated* bounces, we get field lines with regularly spaced kinks that move away from the source. For a charge undergoing *harmonic* motion, we get the field lines shown in Figure 68. The figure thus shows the mechanism of the simplest antenna (or light source) one can imagine.

The magnitude of the transverse electric field can also be used to deduce the relation

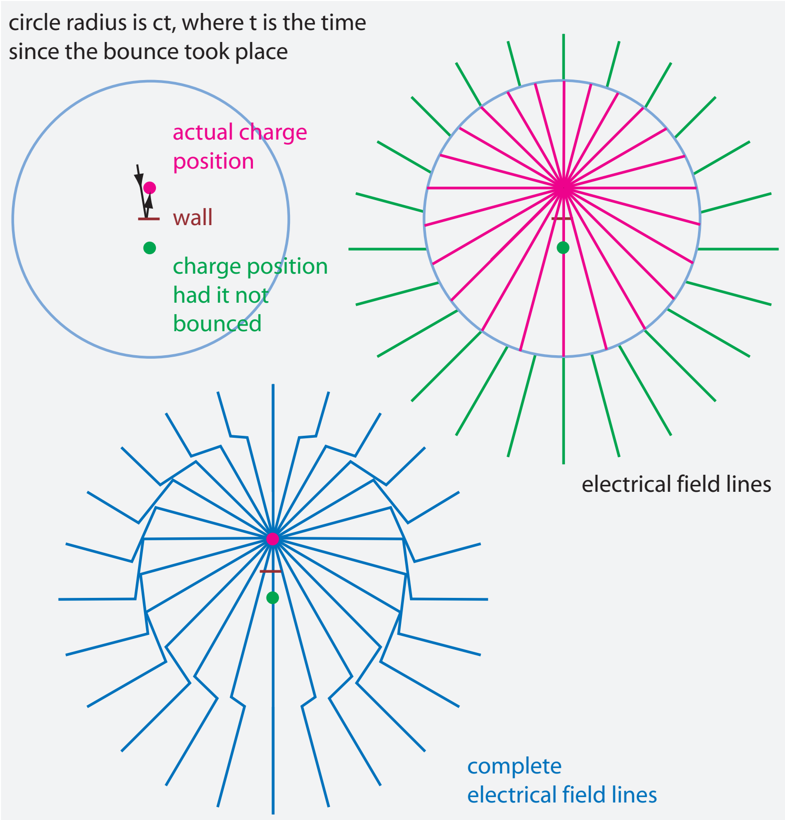

FIGURE 67 Constructing, in three steps, the electrical field around a charged particle bouncing from a wall.

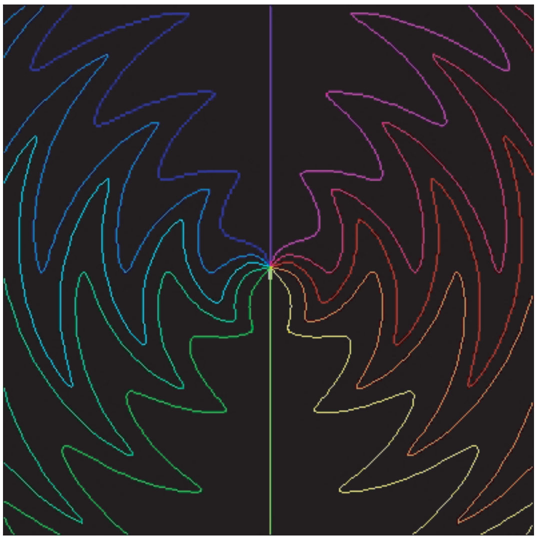

FIGURE 68 The electrical field around a particle oscillating in vertical direction (QuickTime film © Daniel Schroeder).

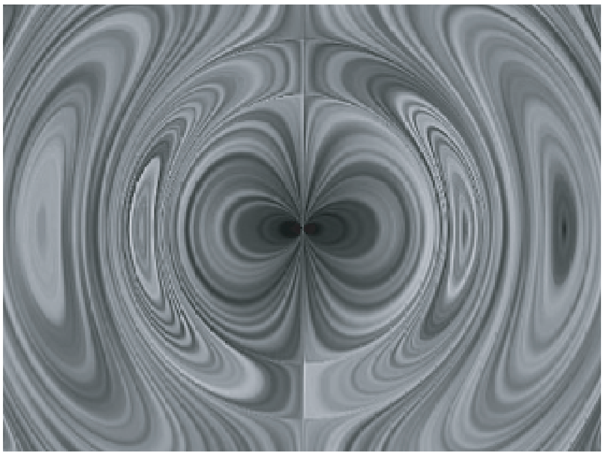

FIGURE 69 The electrical field around an oscillating *dipole* (QuickTime film © Daniel Weiskopf).

between the acceleration a of a charge q and the radiated electromagnetic power P. First, the transverse electric field (calculated in the last challenge) has to be squared, to give the local electric energy density. Then it has to be doubled, to include magnetic energy. Finally, we have to integrate over all angles; this gives a factor of 2/3. In total we get

$$P = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3} . \tag{65}$$

The total radiated power P thus depends on the square of the acceleration and on the square of the charge that is being accelerated. This is the so-called *Larmor formula*. It shows why radio transmitters need power supplies and allows deducing how large they need to be. Note that Figure 67 and Figure 68 and also show that transmitter antennas have a *preferred* direction of power emission.

Usually, the source of electromagnetic radiation is described more accurately as an oscillating dipole. A visualization of the electric field in this case is given in Figure 69. At large distances, a wave section can be approximated as a plane wave.

In all cases, we find that the intensity of radio waves decrease slowly with distance and that radio communication is possible.

The slowness of progress in physics – and relativity

Gustav Kirchhoff's and Bernhard Riemann's expression from the 1850s for the speed of light and all other electromagnetic waves

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{66}$$

is so strange that we should be intrigued whenever we see it. Something essential is missing. The expression states that the speed c is *independent* of the proper motion of the

observer measuring the electromagnetic field and *independent* of the speed of the emitting source. In other words, the speed of light is predicted to be independent of the lamp speed and independent of the observer speed. This is indeed confirmed by all experiments, as explained in the volume on relativity.

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In addition, expression (66) implies that no observer can outrun light. In other words, light does *not* behave like a stream of bullets: the speed of bullet depends on the speed of the gun and of the target. A target can always outrun a bullet, if it moves rapidly enough. The speed of light is a *limit speed*.

Experiments confirm that also the speed of radio waves, of X-rays and of γ-rays is independent of the transmitter and the receiver. Experiments confirm that these speeds have the same value as the speed of light. All this is contained in expression (66).

In short, the speed c is *invariant* and is the *limit energy speed* in nature. Incredibly, *nobody* explored the consequences of this invariance until Lorentz and others started doing so in the 1890s, triggering Einstein until he settled the issues in 1905. The theory of relativity remained undiscovered for two generations! As in so many other cases, the progress of physics was much slower than necessary.

The invariance of the speed of light c is the essential point that distinguishes special relativity from Galilean physics. Since every electromagnetic device – such as every electric motor – makes use of expression (66), every electromagnetic device is a working proof of special relativity.

How does the world look when riding on a light beam?

At the end of the nineteenth century, the teenager Albert Einstein read a book series by Aaron Bernstein discussing the speed of light. The book asked what would happen if an observer moved at the same speed as light. Einstein thought much about the issue, and in particular, asked himself what kind of electromagnetic field he would observe in that case. Einstein later explained that this Gedanken experiment convinced him already at that young age that *nothing* could travel at the speed of light, since the field observed would have a property not found in nature. Can you find out which one he meant?

Ref. 71

Challenge 114 s

Riding on a light beam situation would have strange consequences:

- You would have no mirror image, like a vampire.
- Light would not be oscillating, but would be a static field.
- Nothing would move, like in the tale of sleeping beauty.

But also at speeds *near* the velocity of light observations would be interesting. You would:

- see a lot of light coming towards you and almost no light from the sides or from behind; the sky would be blue/white in the front and red/black behind;
- observe that everything around happens very very slowly;
- experience the smallest dust particle as a deadly bullet.

Can you think of more strange consequences? It is rather reassuring that our planet moves rather slowly through its environment, when compared to the speed of light.

Challenge 115 s

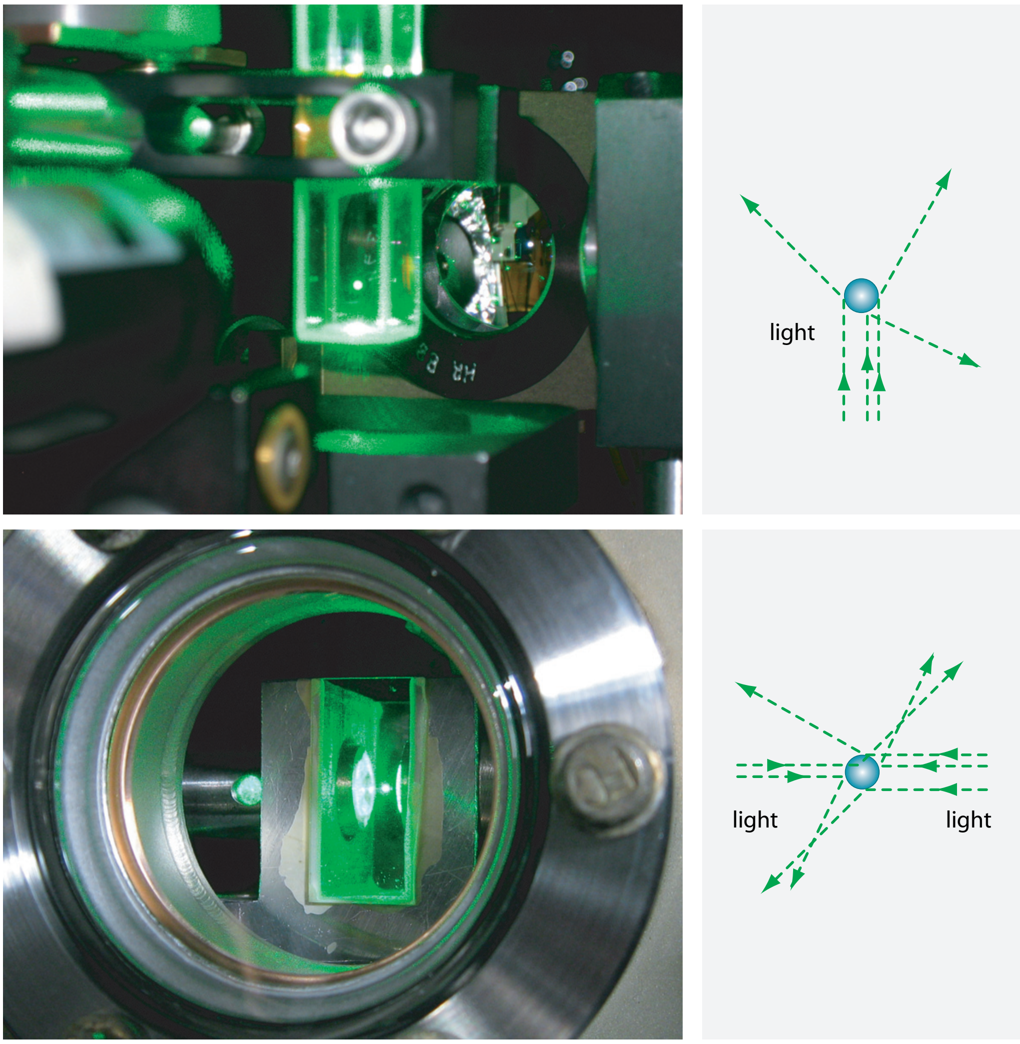

FIGURE 70 Levitating a small glass bead with a laser from below and with two opposed horizontal laser beams (© Mark Raizen, Tongcang Li).

Can we touch light?

Ref. 72

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If a little glass bead is put on top of a powerful laser, the bead remains suspended in mid-air, as shown in Figure 70.* This example of optical levitation proves that light has momentum. Therefore, contrary to what we said in the beginning of our mountain ascent, images *can* be touched! In fact, the ease with which objects can be pushed even has a special name. For planets and planetoids, it is called the *albedo*, and for general objects it is called the *reflectivity*, abbreviated as *r*.

* The heaviest object that has been levitated with a laser had a mass of 20 g; the laser used was the size of a building, and the method also made use of a few additional effects, such as internal shock waves, to keep the object in the air.

FIGURE 71 The tail of comet McNaught, photographed in Australia in 2007 (© Flagstaffotos).

Like each type of electromagnetic field, and like every kind of wave, light carries energy; the energy flow $\boldsymbol{T}$ per surface and time is

Challenge 116 e

$$\boldsymbol{T} = \frac{1}{\mu_0}\boldsymbol{E} \times \boldsymbol{B} \quad \text{giving an average} \quad \langle T \rangle = \frac{1}{2\mu_0} E_{max} B_{max} \; . \qquad (67)$$

Obviously, light also has a momentum P. It is related to the energy E by

$$P = \frac{E}{c} \; . \qquad (68)$$

Challenge 117 e

As a result, the pressure p exerted by light on a body is given by

$$p = \frac{T}{c}(1 + r) \qquad (69)$$

where for black bodies we have a reflectivity $r = 0$ and for mirrors $r = 1$; other bodies have values in between. What is your guess for the amount of pressure due to sunlight on a black surface of one square metre? Is this the reason that we feel more pressure during the day than during the night?

Challenge 118 s

If lasers are not available, rather delicate equipment is needed to detect the momentum or the radiation pressure of light. Already in 1619, Johannes Kepler had suggested in *De cometis* that the tails of comets exist only because the light of the Sun hits the small

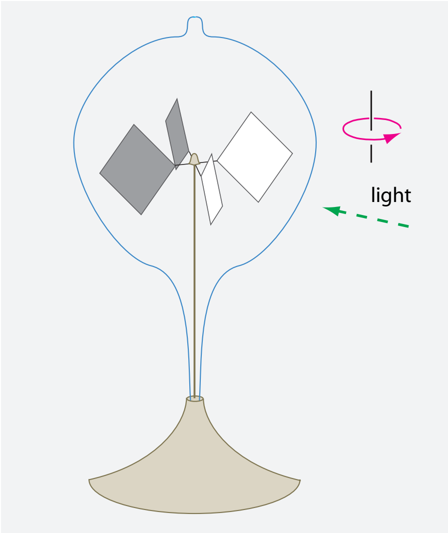

FIGURE 72 A commercial light mill turns *against* the light (Wikimedia).

dust particles that detach from it. For this reason, the tail always points *away* from the Sun, as you might want to check at the next opportunity. Today, we know that Kepler was right; but proving the hypothesis is not easy.

Challenge 119 e

In order to detect the radiation pressure of light, in 1873, William Crookes* invented the *light mill radiometer*. The light mill consists of four thin plates, black on one side and shiny on the other, that are mounted on a vertical axis, as shown in Figure 72. However, when Crookes finished building it – it was similar to those sold in shops today – he found, like everybody else, that it turned in the wrong direction, namely with the shiny side towards the light! (Why is it wrong?) You can check it by yourself by shining a laser pointer on to it. The behaviour has been a puzzle for quite some time. Explaining it involves the tiny amount of gas left over in the glass bulb and takes us too far from the path of our adventure. It was only in 1901, with the advent of much better pumps, that the Russian physicist Pyotr Lebedew managed to create a sufficiently good vacuum to allow him to measure the light pressure with such an improved, true radiometer. Lebedew also confirmed the predicted value of the light pressure and proved the correctness of Kepler's hypothesis about comet tails. Today it is even possible to build tiny propellers that start to turn when light shines on to them, in exactly the same way that the wind turns windmills.

Challenge 120 s

Ref. 73

Ref. 74

Ref. 75

But light cannot only touch and be touched, it can also *grab*. In the 1980s, Arthur Ashkin and his research group developed actual *optical tweezers* that allow one to grab, suspend and move small transparent spheres of 1 to 20 μm diameter using laser beams. It is possible to do this through a microscope, so that one can also observe at the same time what is happening. This technique is now routinely used in biological research around the world, and has been used, for example, to measure the force of single muscle fibres,

Ref. 76

* William Crookes (b. 1832 London, d. 1919 London), chemist and physicist, president of the Royal Society, discoverer of thallium, and believer in spiritualism.

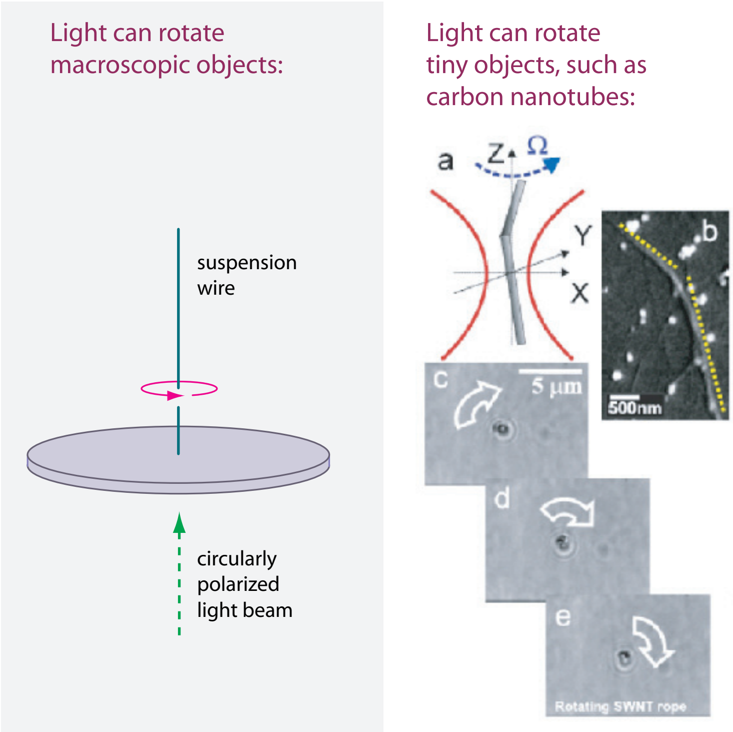

FIGURE 73 Light can rotate objects (© A.C. Ferrari)

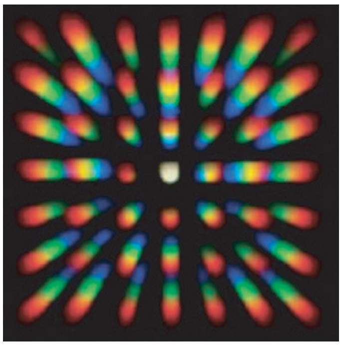

FIGURE 74 Umbrellas decompose white light: look at a small lamp through a black umbrella at night (© Wikimedia).

by chemically attaching their ends to glass or Teflon spheres and then pulling them apart with such optical tweezers.

But that is not all. In the last decade of the twentieth century, several groups even managed to *rotate* objects, thus realizing actual *optical spanners*. They are able to rotate particles at will in one direction or the other, by changing the optical properties of the laser beam used to trap the particle.

Ref. 76

In fact, it does not take much to deduce that if light has linear momentum, circularly polarized light also has *angular* momentum. In fact, for such a wave the angular momen-

tum $\boldsymbol{L}$ is given by

$$\boldsymbol{L} = \frac{E_{\text{nergy}}}{\omega} \; . \tag{70}$$

Challenge 121 e Equivalently, the angular momentum of a wave is $\lambda/2\pi$ times its linear momentum. For Ref. 77 light, this result was already confirmed in the early twentieth century: a light beam can Challenge 122 ny Ref. 78 put certain materials (which ones?) into rotation; in liquids, this is now standard practice. Two examples are shown in Figure 73. Of course, the whole thing works even better with a laser beam. In the 1960s, a beautiful demonstration was performed with microwaves. A circularly polarized microwave beam from a maser – the microwave equivalent of a laser – can put a metal piece absorbing it into rotation. Indeed, for a beam with cylindrical symmetry, depending on the sense of rotation, the angular momentum is either parallel or antiparallel to the direction of propagation. All these experiments confirm that light also carries angular momentum, an effect which will play an important role in the quantum part of our mountain ascent.

We note that not for all waves angular momentum is energy per angular frequency. This is only the case for waves made of what in quantum theory will be called spin 1 particles. For example, for gravity waves the angular momentum is *twice* this value, and they are therefore expected to be made of spin 2 particles.

What does this mean for the comet tails mentioned above? The issue was settled def- Ref. 79 initely in 1986. A satellite was shot up to an altitude of 110 000 km and made to release a cloud of barium. The cloud was visible from the Earth, and it soon developed a tail that was visible from Earth: that was the first artificial comet. It turns out that comet tails shapes are partly due to hitting photons, partly due to the solar wind, and partly to magnetic fields.

In summary, light can touch, light can rotate, and light can be touched. Obviously, if light can rotate bodies, it can also *be* itself rotated. Could you imagine how this can be Challenge 123 s achieved?

War, light and lies

From the tiny effects of equation (69) for light pressure we deduce that light is not an efficient tool for hitting objects. On the other hand, light is able to heat up objects, as we can feel in the sun or when the skin is touched by a laser beam of about 100 mW or more. For the same reason even cheap laser pointers are dangerous to the eye.

In the 1980s, and again in 2001, a group of people who had read too many science fiction novels managed to persuade the military – who also indulge in this habit – that lasers could be used to shoot down missiles, and that a lot of tax money should be spent on developing such lasers. Using the definition of the Poynting vector and a hitting time of about 0.1 s, are you able to estimate the weight and size of the battery necessary for Challenge 124 ny such a device to work? What would happen in cloudy or rainy weather?

Other people tried to persuade NASA to study the possibility of propelling a rocket using emitted light instead of ejected gas. Are you able to estimate that this is not feasible?

Challenge 125 e

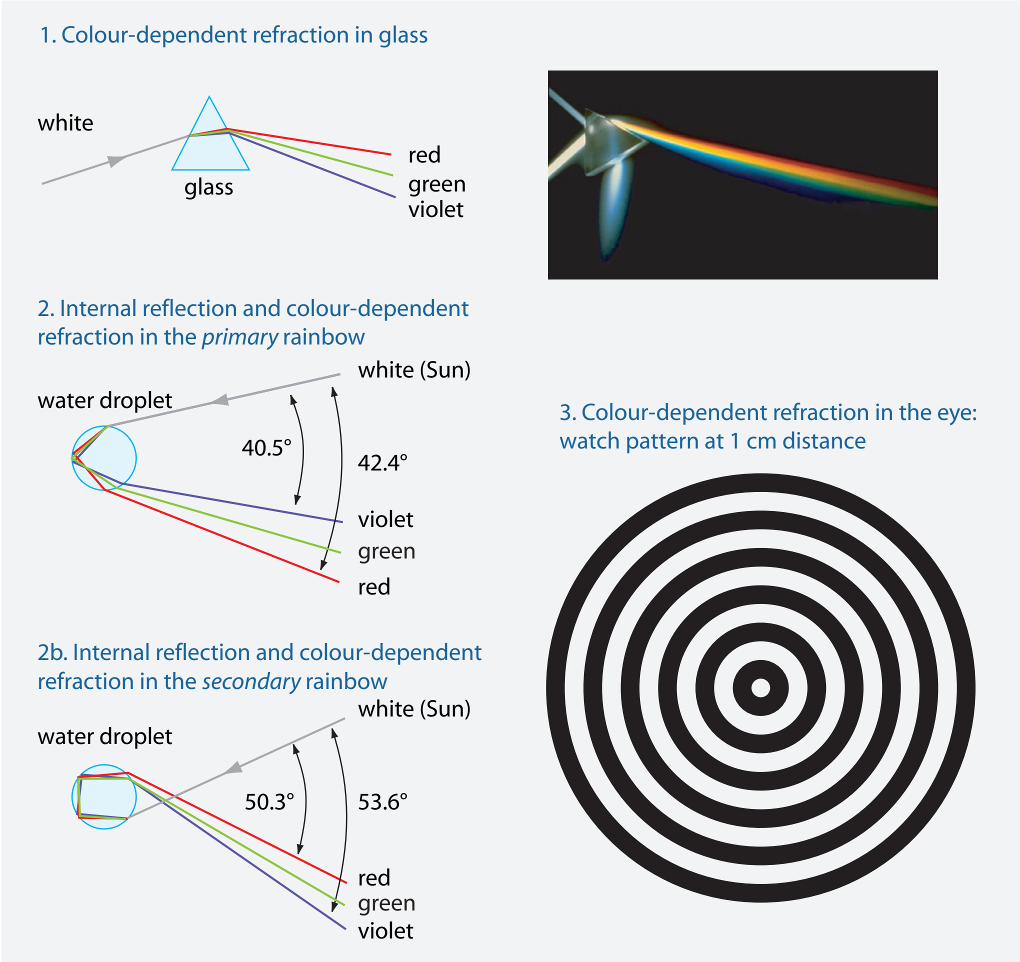

FIGURE 75 Three proofs that *white* light is a mixture of colours (with exaggerated angle differences): prism decomposition, rainbow formation and the coloured borders seen on a circular black and white pattern (photograph by Susan Schwartzenberg, © Exploratorium www.exploratorium.edu).

What is colour?

We saw that radio waves of certain frequencies are visible. Within that range, different frequencies correspond to different colours. (Are you able to convince a friend about this?) Challenge 126 s But the story does not finish here. Numerous colours can be produced either by a single wavelength, i.e., by *monochromatic* light, or by a *mixture* of several different colours. For example, standard yellow can be, if it is pure, an electromagnetic beam of 575 nm wavelength or it can be a mixture of standard green of 546.1 nm and standard red of 700 nm. The eye cannot distinguish between the two cases; only spectrometers can. In everyday life, all colours turn out to be mixed, with the exceptions of those of yellow street lamps, of laser beams and of laboratory spectra. You can check this for yourself, using an umbrella or a compact disc: they decompose light mixtures, but they do not decompose pure colours, such as those from a laser pointer or an LED display. Challenge 127 e

In particular, *white* light is a mixture of a continuous range of colours with a specific

intensity per wavelength. If you want to check that white light is a mixture of colours without any light source, simply hold the lower right-hand side of Figure 75 so close to your eye that you cannot focus the stripes any more. The unsharp borders of the white stripes have either a pink or a green shade. These colours are due to the imperfections of the human eye, its so-called *chromatic aberrations*. Aberrations have the consequence that not all light frequencies follow the same path through the lens of the eye, and therefore they hit the retina at different spots. This is the same effect that occurs in prisms or in water drops showing a rainbow.

The left-hand side of Figure 75 explains how rainbows form. Above all, the internal reflection inside the water droplets in the sky is responsible for throwing back the light coming from the Sun, whereas the wavelength-dependent refraction at the air–water surface is responsible for the different paths of each colour. The first two persons to verify this explanation were Theodoricus Teutonicus de Vriberg (*c.* 1240 to *c.* 1318), in the years from 1304 to 1310 and, at the same time, the Persian mathematician Kamal al-Din al-Farisi. (Ref. 80) To check the explanation, they did something smart and simple that anybody can repeat at home. (Challenge 128 e) They built an enlarged water droplet by filling a thin spherical (or cylindrical) glass container with water; then they shone a beam of white light through it. Theodoricus and al-Farisi found exactly what is shown in Figure 75. With this experiment, each of them was able to reproduce the opening angle of the main or *primary* rainbow, its colour sequence, as well as the existence of a *secondary* rainbow, its observed angle and its inverted colour sequence.* All these rainbows are found in Figure 56. (Page 99) Theodoricus's beautiful experiment is sometimes called the most important contribution of natural science in the Middle Ages.

By the way, the shape of the rainbow tells something about the shape of the water droplets. Can you deduce the connection? (Challenge 130 s)

Incidentally, the explanation of the rainbow given in Figure 75 is not complete. It assumes that the light ray hits the water droplet at a specific spot on its surface. If the light ray hits the droplet at other spots, the rainbows appear at other angles; however, all those rainbows wash out. Only the visible rainbow remains, because its deflection angles are extremal. The primary rainbow is, in fact, the coloured edge of a white disc. And indeed, the region above the primary bow is always darker than the region below it.

Incidentally, at sunset the atmosphere itself also acts as a prism, or more precisely, as a cylindrical lens affected by spherochromatism. Therefore, especially at sunset, the Sun is split into different images, one for each colour, which are slightly shifted with respect to each other; the total shift is about 1 % of the diameter. As a result, the rim of the evening Sun is coloured. (Ref. 83) If the weather is favourable, if the air is clear up to and beyond the horizon, and if the correct temperature profile is present in the atmosphere, a colour-dependent mirage will appear: for about a second it will be possible to see, after or near the red, orange and yellow images of the setting Sun, the green–blue image, sometimes even detached. (Ref. 82) This is the famous *green flash* described by Jules Verne in his novel *Le*

(Challenge 129 s) * Can you guess where the ternary and quaternary rainbows are to be seen? There are rare reported sightings of them; only two or three photographs exist world-wide. The hunt to observe the fifth-order rainbow is still open. (Ref. 81) (In the laboratory, bows around droplets up to the thirteenth order have been observed.) For more details, see the beautiful website at www.atoptics.co.uk. There are several formulae for the angles of the various orders of rainbows; they follow from straightforward geometric considerations, but are too involved to be given here.

FIGURE 76 A green flash above the setting Sun and one above the Moon, showing also the colour change of the Moon rim (© Andrew Young and Laurent Laveder/PixHeaven.net).

FIGURE 77 Milk and water simulate the evening sky (© Antonio Martos).

Rayon-vert. The green flash is often seen on tropical beaches, for example in Hawaii, and from the decks of ships in warm waters. Ref. 83, Ref. 84

Even pure air splits white light. However, this effect is not due to dispersion, but to scattering. Wavelength-dependent scattering, mainly *Rayleigh scattering*, is the reason that the sky and distant mountains look blue and that the Sun looks red at sunset and sunrise. (The sky looks black even during the day from the Moon.) You can repeat this effect by looking through water at a black surface or at a lamp. Adding a few drops of milk to the water makes the lamp yellow and then red, and makes the black surface blue (like the sky seen from the Earth as compared to the sky seen from the Moon) as shown in Figure 77. More milk increases the effect. For the same reason, sunsets are especially red after volcanic eruptions.

In the evening, however, the sky is blue for another, far less known reason: at the time around sunset, the sky is blue mainly because of the ozone layer. Ozone is a blue gas. Without ozone, the sky would be yellowish during sunsets. Ref. 85

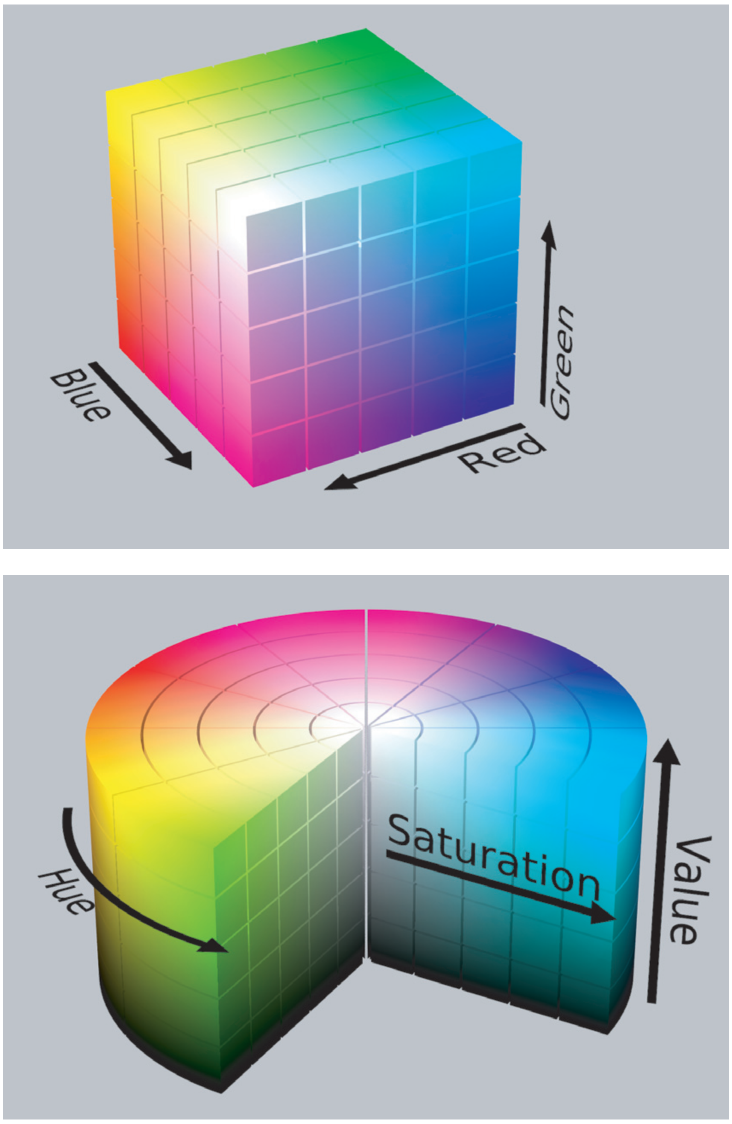

FIGURE 78 Two of the many ways to illustrate the set of all possible human colours: (top) as mixtures of red, green and blue values that increase along the three coordinate axes, and (bottom) using hue, saturation and brightness value coordinates (© SharkD).

In summary, light is, in general, a mixture of wavelengths. As a result, light wavelength or frequency are *not* sufficient to describe colour. Colour experts call *hue* that aspect of colour that matches most closely the change with wavelength. But every colour has two additional characteristics. For example, any given colour can be bright or dark; *brightness* is a second, independent property of colour. A third independent property of colour is its *saturation*; it expresses how strongly a colour differs from white. A strongly saturated colour is the opposite of a pale, or weakly saturated colour.

Human colour space is three-dimensional. Humans are thrichromatic. Figure 78 illustrates the point. Every colour we see is described by *three* independent parameters, because the human eye has three types of cones, thus three types of colour-sensitive cells. This is the reason that any colour selection scheme, for example on a computer, has –

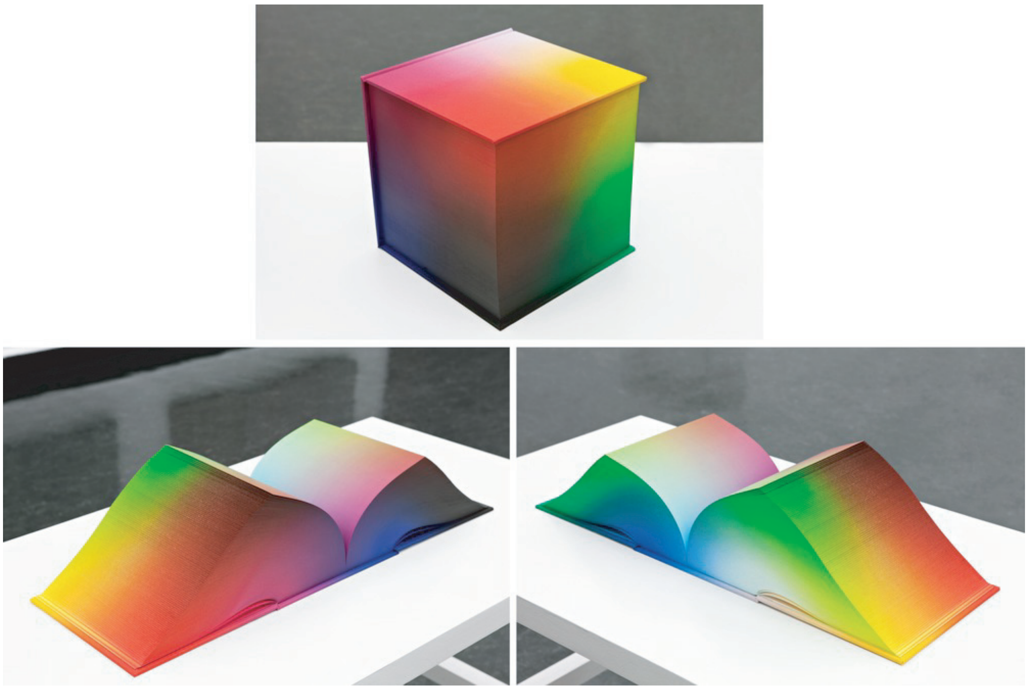

FIGURE 79 A unique colour book that illustrates, on each page and on all its outside surfaces, the three-dimensional colour space of humans (© Tauba Auerbach).

at least – three parameters that can be varied. A modern artist, Tauba Auerbach, even produced a beautiful book version of the colour space, shown in Figure 79. The number three is also the reason that every display has at least three different types of pixels. These three parameters do not need to be hue, saturation and brightness value. They can also be taken to be the intensities of red, green and blue. Many other colour properties can be used to describe colour, such as lightness, chroma, purity, luma and others. Also descriptions with four and more parameters – which then are not independent from each other – are used, especially in the printing industry.

Ref. 86

Many birds, reptiles, fish and various insects have four-dimensional colour spaces that include the ultraviolet; butterflies and pigeons have five-dimensional colour spaces, and other bird species have even higher-dimensional colour spaces. Mantis shrimps possibly have the most complex eyes in the animal kingdom, with up to twelve-dimensional colour spaces. (One species of mantis shrimps, *Gonodyctylus smithii*, can also detect circular and linear light polarization in complete detail.) In contrast to humans and apes, most mammals have only two-dimensional colour spaces. Also colour-blind persons can have lower-dimensional colour spaces. In other terms, the number of dimensions of the perceived colour space is not a property of light, nor a property of nature, but a specific property of our human eyes. *Colours in nature and colours perceived by humans differ.* The is no colour space in nature.

Colours in nature and colours in human perception differ in an additional way, discovered by linguists. In human language, colours have a natural *order*. All people of the world, whether they come from the sea, the desert or the mountains, order colours in

FIGURE 80 Exceptionally many supernumerary rainbows (© Denis Betsch).

the following sequence: 1. black and white, 2. red, 3. green and yellow, 4. blue, 5. brown, 6. mauve, pink, orange, grey and sometimes a twelfth term that differs from language to language. (Colours that refer to objects, such as aubergine or sepia, or colours that are not generally applicable, such as blond, are excluded in this discussion.) The precise discovery is the following: if a particular language has a word for any of these colours, then it also has a word for all the preceding ones. The result also implies that people use these basic colour classes even if their language does *not* have a word for each of them. These strong statements have been confirmed for over 100 languages. Ref. 87

Fun with rainbows

The width of the usual, primary rainbow is 2.25°, for the secondary rainbow it is about twice that value (which is one reason why it is less bright). The width is larger than the dispersion angle difference given in Figure 75 because the angular size of the sun, about 0.5°, has (roughly) to be added on top of the angle difference.

The finite size of droplets leads, via interference, to the supernumemrary rainbows, as mentioned above. If the droplets are small and all of the same size, the number of supernumerary rainbows increases, as Figure 80 shows strikingly. Page 99

If the droplets are extremely fine, the rainbow becomes white; it is then called a *fogbow*. Such bows are also often seen from aeroplanes. If the droplets are not round, for example due to strong wind, one can get a so-called *irregular* or *twinned rainbow*. An example is Ref. 59

FIGURE 81 Five rare types of rainbows: a fogbow (top left), an irregular, split rainbow in a windy situation due to non-spherical rain drops (top right, shown with increased colour saturation), a six-fold rainbow (middle left), a red rainbow at sunset (middle right), and a moonbow, created by the Moon, not by the Sun, and brightened digitally (© Michel Tournay, Eva Seidenfaden, Terje Nordvik, Zhu XiaoJin and Laurent Laveder).

shown in Figure 81.

Light from the rainbow is tangentially polarized. You can check that easily with polarizing sunglasses. During the internal reflection in the water droplets, as the reflection angle is very near to the angle at which total reflection sets in, light gets polarized. (Why does this lead to polarization?) More on polarization will be told in the next section.

Challenge 131 e

Challenge 132 ny

If the air is full of ice crystals instead of droplets, the situation changes again. One can then get additional images of the sun in the direction of the sun. They are called *parhelia*,

FIGURE 82 A composite photograph showing the parhelia, the light pillars, the halo and the upper tangent arc formed by ice crystals in the air, if all are oriented in the same direction (© Phil Appleton).

FIGURE 83 A rare circumzenithal arc formed by hexagonal ice crystals in upper regions of the atmosphere (© Paul Gitto).

sometimes also or sundogs. This happens most clearly with no wind, if the crystals are all oriented in the same direction. In that case one can take photographs such as the one shown in Figure 82.

Rare bows and other astonishing atmospheric effects are best explored on the website providing the 'optical picture of the day' at www.atoptics.co.uk/opod.htm. There one can find third- and fourth-order rainbows, fogbows that include supernumerary bows, lunar fogbows, rainbows whose secondary bow has supernumeraries, irregular rainbows, moonbows, circumzenithal arcs, Sun's halos, Sun's pillars, green flashes, and much more.

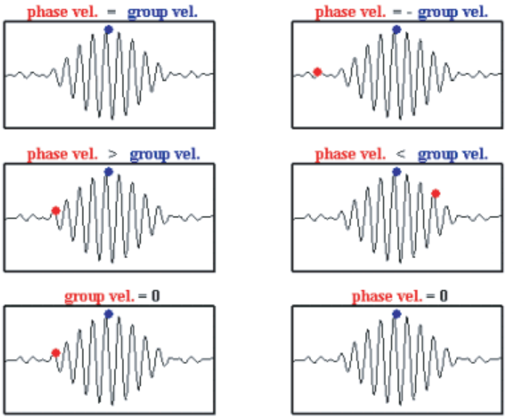

FIGURE 84 A visualisation of group velocity (blue) and phase velocity (red) for different types of waves (QuickTime film © ISVR, University of Southampton).

The website presents the beauty of light in nature – and all effects are also explained in detail.

What is the speed of light? What is signal speed?

Physics talks about motion. Talking is the exchange of sound; and sound is an example of a signal.

▷ A *(physical) signal* is the transport of information using the transport of energy.

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There are no signals without a motion of energy. Indeed, there is no way to store information without storing energy. To any signal we can thus ascribe a propagation speed. We call it the *signal speed*. The highest possible signal speed is also the maximal velocity of the general influences, or, to use sloppy language, the maximal velocity with which effects spread causes.

If the signal is carried by matter, such as by the written text in a letter, the signal velocity is the velocity of the material carrier. Experiments show that this speed is limited by the speed of light.

For a wave carrier, such as water waves, sound, light or radio waves, the situation is less evident. What is the speed of a wave? The first answer that comes to mind is the speed with which wave crests of a sine wave move. This already introduced *phase velocity* is given by the ratio between the frequency and the wavelength of a monochromatic wave, i.e., by

$$v_{ph} = \frac{\omega}{k} . \tag{71}$$

For example, the phase velocity determines interference phenomena. Light in a vacuum has the same phase velocity $v_{ph} = c$ for all frequencies. Are you able to imagine an experiment to test this to high precision?

Challenge 133 s

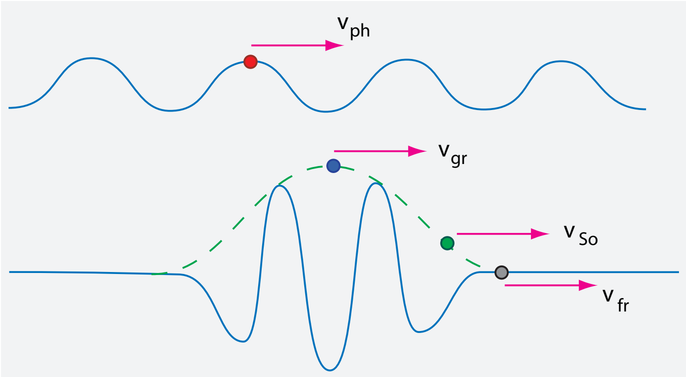

FIGURE 85 The definition of the important velocities in wave phenomena, including Sommerfeld's front velocity and the forerunner velocity.

On the other hand, there are cases where the phase velocity is *greater* than c, most notably when light travels through an absorbing substance, and when at the same time the frequency is near to an absorption maximum. In these cases however, experiments show that the phase velocity is *not* the signal velocity. For such situations, a better approximation to the signal speed is the *group velocity*, i.e., the velocity at which a group maximum will travel. This velocity is given by

Ref. 88

$$v_{gr} = \left.\frac{d\omega}{dk}\right|_{k_0} , \tag{72}$$

where k_0 is the central wavelength of the wave packet. We observe that $\omega = c(k)k = 2\pi v_{ph}/\lambda$ implies the relation

$$v_{gr} = \left.\frac{d\omega}{dk}\right|_{k_0} = v_{ph} - \lambda \frac{dv_{ph}}{d\lambda} . \tag{73}$$

This means that the sign of the last term determines whether the group velocity is larger or smaller than the phase velocity. For a travelling group, as shown by the dashed line in Figure 85, this means that new maxima appear either at the end or at the front of the group. Experiments show that this is only the case for light passing *through matter*; for light *in vacuum*, the group velocity has the same value $v_{gr} = c$ for all values of the wave vector k.

You should be warned that many publications are still propagating the incorrect statement that the group velocity *in a material* is never greater than c, the speed of light in vacuum. Actually, the group velocity in a material can be zero, infinite or even negative; this happens when the light pulse is very narrow, i.e., when it includes a wide range of frequencies, or again when the frequency is near an absorption transition. In many (but not all) cases the group is found to widen substantially or even to split, making it difficult to define precisely the group maximum and thus its velocity. Many experiments have confirmed these predictions. For example, the group velocity in certain materials has been measured to be *ten times* that of light. The refractive index then is smaller than

Challenge 134 ny

Ref. 89

1. However, in all these cases the group velocity is *not* the same as the signal speed.*

What then is the best velocity describing signal propagation? Arnold Sommerfeld** almost solved the main problem in the beginning of the twentieth century. He defined the signal velocity as the velocity v_{So} of the *front slope* of the pulse, as shown in Figure 85. The definition cannot be summarized in a formula, but it does have the property that it describes signal propagation for almost all experiments, in particular those in which the group and phase velocity are larger than the speed of light. When studying its properties, it was found that for no material is Sommerfeld's signal velocity greater than the speed of light in vacuum.

Ref. 88

Sometimes it is conceptually easier to describe signal propagation with the help of the energy velocity. As previously mentioned, every signal transports energy. The *energy velocity* v_{en} is defined as the ratio between the energy flow density $\boldsymbol{S}$, i.e., the Poynting vector, and the energy density W, both taken in the direction of propagation. For electromagnetic fields – the only ones fast enough to be interesting for eventual superluminal signals – this ratio is

$$\boldsymbol{v}_{en} = \frac{\langle \boldsymbol{P} \rangle}{\langle W \rangle} . \tag{74}$$

However, as in the case of the front velocity, in the case of the energy velocity we have to specify the underlying averaging procedure, denoted by $\langle\rangle$, i.e., whether we mean the energy transported by the main pulse or by the front of it. In vacuum, neither speed is ever greater than the speed of light.*** (In general, the velocity of energy in matter has a value slightly different from Sommerfeld's signal velocity.)

Ref. 88

In recent years, the progress in light detector technology, allowing one to detect even the tiniest energies, has forced scientists to take the fastest of all these energy velocities to describe signal velocity. Using detectors with the highest possible sensitivity we can use as signal the first point of the wave train whose amplitude is different from zero, i.e., the first tiny amount of energy arriving. This point's velocity, conceptually similar to Sommerfeld's signal velocity, is commonly called the *front velocity* or, to distinguish it even more clearly from Sommerfeld's case, the *forerunner velocity*. It is simply given by

Challenge 135 s

$$v_{fr} = \lim_{\omega\to\infty} \frac{\omega}{k} . \tag{75}$$

* In quantum mechanics, SchrödingerSchrödinger, Erwin proved that the velocity of an electron is given by the group velocity of its wave function. Therefore the same discussion reappeared in quantum theory, as we will find out in the next volume of our mountain ascent.

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** Arnold Sommerfeld (b. 1868 Königsberg, d. 1951 Munich) was a central figure in the spread of special and general relativity, of quantum theory, and of their applications. A professor in Munich, an excellent teacher and text book writer, he worked on atomic theory, on the theory of metals and on electrodynamics, and was the first to understand the importance and the mystery around 'Sommerfeld's famous fine structure constant.'

*** Signals not only carry energy, they also carry negative entropy ('information'). The entropy of a transmitter increases during transmission. The receiver decreases in entropy (but less than the increase at the transmitter, of course).

Ref. 91

Note that the negative group velocity implies energy transport against the propagation velocity of light. This is possible only in *energy loaded* materials.

Ref. 92

The forerunner velocity is *never* greater than the speed of light in a vacuum, even in materials. In fact it is precisely c because, for extremely high frequencies, the ratio ω/k is independent of the material, and vacuum properties take over.

▷ The forerunner velocity is the true signal velocity or the *true velocity of light*.

Using the forerunner speed, all discussions on light speed become clear and unambiguous.

To end this section, here are two challenges for you. Which of all the velocities of light
Challenge 136 s is measured in experiments determining the velocity of light, e.g. when light is sent to the Moon and reflected back? And now a more difficult one: why is the signal speed of
Challenge 137 s light inside matter less than the speed in vacuum, as all experiments show?

Signals and predictions

When one person reads a text over the phone to a neighbour who listens to it and maybe repeats it, we speak of communication. For any third person, the speed of communication is always less than the speed of light. But if the neighbour already knows the text, he can recite it without having heard the readers' voice. To the third observer such a situation appears to imply motion that is faster than light. Prediction can thus *mimic* communication and, in particular, it can mimic faster-than-light (superluminal) communication. Such a situation was demonstrated most spectacularly in 1994 by Günter
Ref. 90 Nimtz, who seemingly transported music – all music is predictable for short time scales – through a 'faster-than-light' system. To distinguish between the two situations, we note that in the case of prediction, no transport of energy takes place, in contrast to the case of communication. In other words, the definition of a signal as a transporter of information is not as useful and clear-cut as the definition of a signal as a *transporter of energy*. In the above-mentioned experiment, no energy was transported faster than light. The same distinction between prediction on the one hand and signal or energy propagation on the other will be used later to clarify some famous experiments in quantum mechanics.

> “If the rate at which physics papers are being published continues to increase, physics journals will soon be filling library shelves faster than the speed of light. This does not violate relativity since no useful information is being transmitted.”
>
> David Mermin

Aether good-bye

Gamma rays, X-rays, light and radio waves are moving electromagnetic waves. All exist in empty space. What is oscillating when light travels? Maxwell himself called the oscillating 'medium' the *aether*. The properties of the oscillating medium that are measured in experiments are listed in Table 15. The strange numerical values are due to the definition
Page 327 of the units henry and farad.

Ref. 93 The last item of Table 15 is the most important: despite intensive efforts, nobody has been able to detect any *motion* of the so-called aether. In particular, there is no motion

TABLE 15 Experimental properties of *flat*, classical vacuum, thus neglecting all quantum effects and all effects of general relativity.

Physical property	Experimental value
Permeability	μ_0 =1.3 μH/m
Permittivity	ε_0 =8.9 pF/m
Wave impedance/resistance	Z_0 = 376.7 Ω
Conformal invariance	applies
Spatial dimensionality	3
Topology	R^3
Friction on moving bodies	none
Components	none
Mass and energy content	none
Motion	none

of the aether relative to the vacuum. In other words, even though the aether supposedly oscillates, it does not move. Together with the other data, all these results can be summed up in one sentence: there is no way to distinguish the aether from the vacuum. Challenge 138 e

Sometimes one hears that certain experiments or even the theory of relativity show that the aether does not exist. There is a lot of truth in this statement; in fact, experiments show something even more important: *the aether is indistinguishable from the vacuum*. This statement is true in all cases. For example, we found out in the section on general relativity that a curved vacuum *can* move; but the aether still remains indistinguishable from it.* Also quantum field theory confirms the identity of aether and vacuum.

What then is oscillating in the case of electromagnetic waves? We now have a simple answer to this old question: the vacuum. The vacuum is the carrier, or carrier medium, of electromagnetic waves. The flat, Lorentz-invariant vacuum carries waves, even though it cannot move and it does not provide a favourite coordinate system. Flat vacuum is thus something special,** and it is also acceptable to avoid the terms 'carrier' or 'medium' altogether. In some bizarre clubs it is even compulsory to do so. However, this avoidance is impossible in general relativity, as we have seen, and is equally impossible in quantum field theory, as we will find out.***

In short, experiments in the domain of special relativity have abolished the aether: it is a superfluous concept; the physical vacuum has many of the properties that were once ascribed to the aether. From now on, we will drop the concept of aether from our Ref. 94

Ref. 94 * Historically, the term 'aether' has been used as an expression for several different ideas, depending on the author. First of all it was used for the idea that a vacuum is not empty, but *full*; secondly, that this fullness can be described by *mechanical models*, such as gears, little spheres, vortices, etc.; thirdly, it was imagined that the aether is a *substance*, similar to matter. All these ideas are put to rest by relativity. Nevertheless, these issues will reappear in the last part of our mountain ascent, when the description of the vacuum itself is explored.

** We will find a way to explain the properties of vacuum at the end of our adventure.

*** In 2013, the German Physical Society published an official expert opinion stating that "electromagnetic waves do not need vacuum as carrier." The society also wants all physics teachers to tell this false statement to their pupils. Physicists are still laughing.

vocabulary. On the other hand, we have not yet finished the study of the vacuum; vacuum will keep us busy for the rest of our walk, starting with the following part of adventure, on
Challenge 139 d quantum physics. In fact, quantum physics shows that all experimental values in Table 15 require amendments.

Challenges and fun curiosities about light, polarization and the geometric phase

Since light is a wave, something must happen if it is directed to a hole less than its wave-
Challenge 140 s length in diameter. What exactly happens?

* *

On a sunny day at moderate latitudes on the Earth, sunlight has a power density of $1\,\mathrm{kW/m^2}$. What is the corresponding energy density and what are the average electric
Challenge 141 s and magnetic fields?

* *

Challenge 142 s Spectrally pure light is often called 'monochromatic'. Why is this a misnomer?

* *

Electrodynamics shows that light beams always push; they never pull. Can you confirm
Challenge 143 e that 'tractor beams' are impossible in nature?

* *

It is well known that the glowing material in light bulbs is tungsten wire in an inert gas. This was the result of a series of experiments that began with the grandmother of all lamps, namely the cucumber. The older generation knows that a pickled cucumber, when attached to the 230 V of the mains, glows with a bright green light. (Be careful; the experiment is dirty and dangerous.)

* *

Light beams have an effective temperature and entropy. Though not often discussed nowadays, the thermodynamics of light has been explored in great detail by Max von Laue (b. 1879 Koblenz, d. 1960 Berlin) in the years between 1900 and 1906. Von Laue
Ref. 95 showed that usual light propagation in empty space is a reversible process and that the entropy of a beam indeed remains constant in this case. When light is diffracted, scattered or reflected diffusively, the effective temperature decreases and the entropy increases. The most interesting case is interference, where entropy usually increases, but sometimes decreases.

* *

The wave impedance of the vacuum of $376.7\,\Omega$ has practical consequences. If an electromagnetic wave impinges on a large, thin, resistive film along the normal direction, the numerical value of the film resistance determines what happens. If the film resistance is much *larger* than $376.7\,\Omega$ per square, the film is essentially transparent, and the wave will be *transmitted*. If the film resistance is much lower than $376.7\,\Omega$ per square, the film

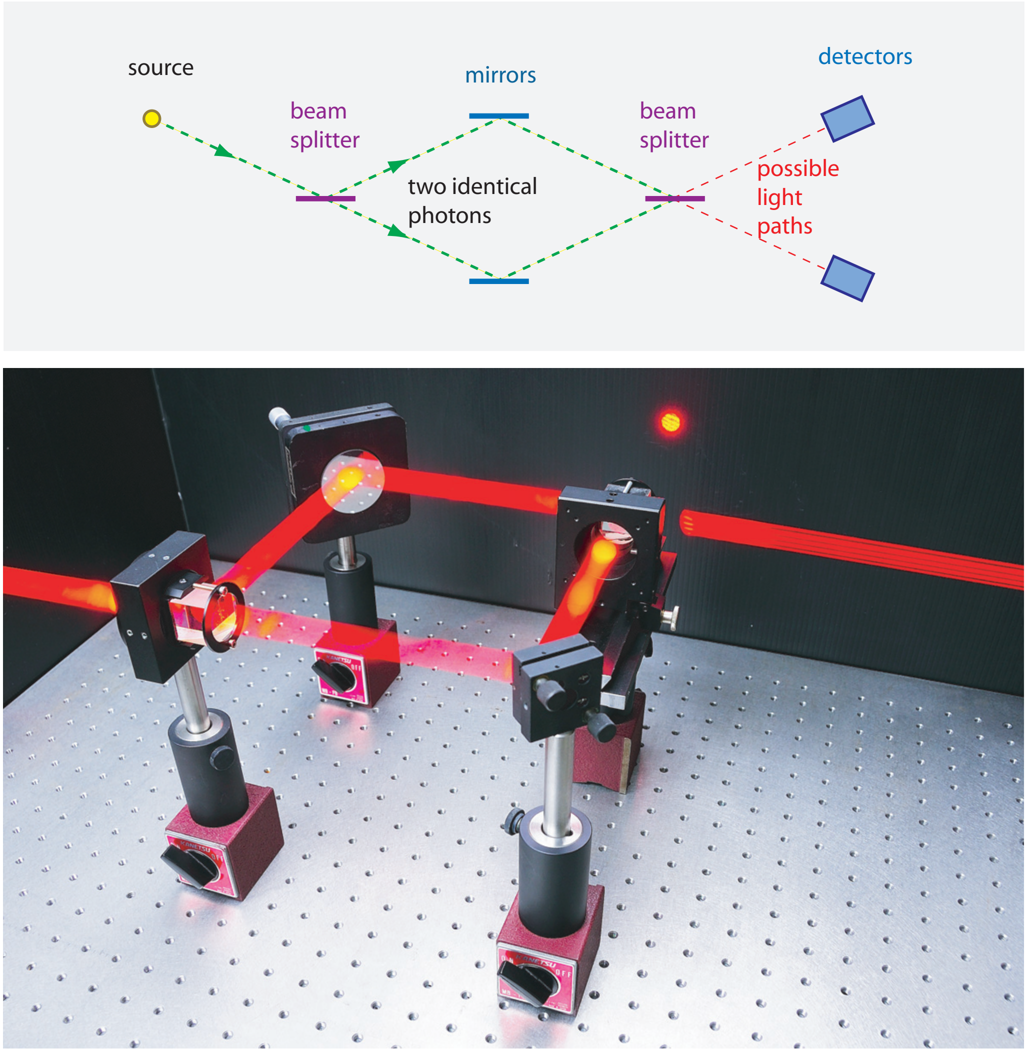

FIGURE 86 A conventional two-dimensional (Mach-Zehnder) interferometer, with sides of equal lengths, and its outputs A and B. Light exits in direction A, the direction of constructive interference (photo © Félix Dieu and Gaël Osowiecki).

is essentially a short circuit for the wave, and the wave will be *reflected*. Finally, if the film resistance is *comparable* to 376.7 Ω per square, the film is impedance-matched and the wave will be *absorbed*.

* *

If the light emitted by the headlights of cars were polarized from the bottom left to the upper right (as seen from the car's driver) one could vastly improve the quality of driving at night: one could add a polarizer to the wind shield oriented in the same direction. As a result, a driver would see the reflection of his own light, but the light from cars coming towards him would be considerably dampened. Why is this not done in modern cars? Challenge 144 s

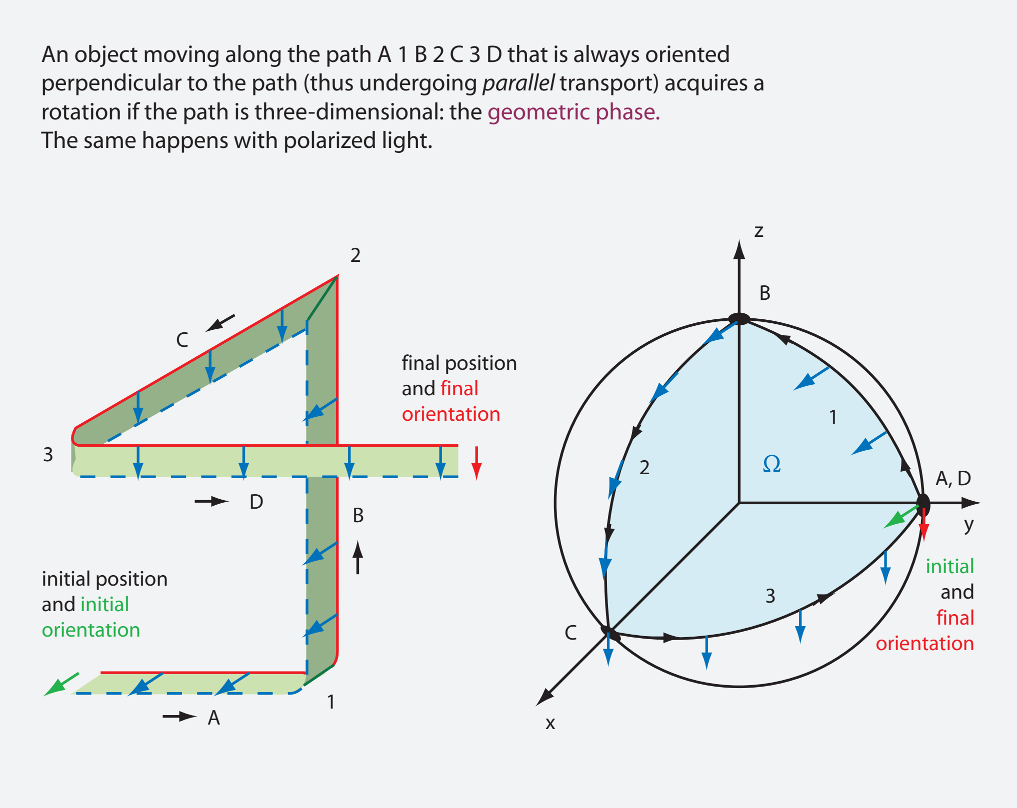

FIGURE 87 Left: a three-dimensional path traced by a pointed object that behaves like the polarization of light. The bends 1, 2 and 3 could be induced by mirrors. Right: the rotation angle of the polarization is given by the solid angle Ω, the geometric phase, enclosed by the path.

* *

Could light have a tiny mass, and move with a speed just below the maximal speed possible in nature? The question has been studied extensively. If light had mass, Maxwell's equations would have to be modified, the speed of light would depend on the frequency and on the source and detector speed, and longitudinal electromagnetic radiation would exist. Despite a promise for eternal fame, no such effect has been observed. Ref. 96

* *

A beam of light can be polarized. The direction of polarization can be changed by sending the light through materials that are birefringent, such as liquid crystals, calcite or stressed polymers. But polarization can also be changed with the help of mirrors. To achieve such a polarization change, the path of light has to be genuinely three-dimensional; the path must not lie in a plane.

To understand the rotation of polarization with mirrors, the best tool is the so-called *geometric phase*. The geometric phase is an angle that occurs in three-dimensional paths of any polarized wave. The geometric phase is a general phenomenon that appears both for light wave, for wave functions, and even for transverse mechanical oscillations. To

visualize geometric phase, we look at the Figure 87.

The left image of Figure 87 can be seen as paper strip or a leather belt folded in space, with a bright and a dark coloured side. It is not a surprise that the orientation of the strip at the end differs from the start. Imagine to follow the strip with the palm of your hand flat on it, along its three-dimensional path. At the end of the path, your arm is twisted. This twist angle is the *geometric phase* induced by the path.

Ref. 97

Instead of a hand following the paper strip, we now imagine that a polarized light beam follows the path defined by the centre of the strip. At the bends, mirrors change the motion of the light, but at each tiny advance, the polarization remains parallel to the polarization just before. One speaks of *parallel transport*. The result for light is the same as for the belt: At the end of the path, the polarization of the light beam has been rotated. In short, *parallel transport in three dimensions results in a geometric phase.* In particular, it is thus possible to rotate the polarization of a beam of light with the help of mirrors only.

Also transverse mechanical oscillations work in this way. When a Foucault pendulum oscillates, its path – a segment of a circle due to the rotation of the Earth – is three-dimensional. The direction of oscillation – akin to the polarization of the light or the orientation of the paper strip – changes along the path.

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Since wave functions in quantum mechanics are also described by a transverse phase, they show similar effects when the follow three-dimensional paths. The Aharonov-Bohm effect is an example for a situation where a three-dimensional path leads to phase change.

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The other, right-hand drawing in Figure 87, illustrating the so-called *sphere of directions*, shows how to calculate the angle of rotation due to a specific path. *The geometric phase turns out to be the solid angle enclosed by the path.* In short, the geometric phase angle is given by the enclosed solid angle. With this result, the geometric phase has no mysteries any more. (For paths that are not closed on the sphere of directions, the calculation can still be carried out by suitably closing the path on the sphere.) A pretty case is the experiment in which polarized light is fed into a helically coiled optical fibre. In this case, the geometric phase is fixed by the length of the fibre and the pitch length of the helix. Effects of the geometric phase have also been observed in molecules, in nuclei, neutron beams, in interferometers of all kind, in particle accelerators, in gyroscopes, in general relativity and in many other settings.

Challenge 145 ny

Ref. 98

Historically, the geometric phase has been discovered independently by many people in different fields of physics. The researcher who understood its general importance in quantum physics was Michael Berry in 1983, but the phase was known in quantum physics, optics and mechanics long before, among others through the work in nuclear physics by Christopher Longuet-Higgins in the 1950s, through the work on light by the young genius Shivaramakrishnan Pancharatnam also in the 1950s, through the work on molecules by Alden Mead in the 1970s, and, of course, through the mentioned Foucault pendulum from 1851. But also the errors in the south-pointing carriage, which we mentioned before, are due to the geometric phase. Following Michael Berry, the phenomenon is now called the *geometric phase*. Older expressions, such as adiabatic phase, topological phase, quantal phase, Berry's phase and various other terms are not used any more.

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After this excursion, here is a challenge of the real world. What is the smallest number of mirrors needed in a device to change the polarization of a light beam that exits the device in the same direction as it came in?

Challenge 146 s

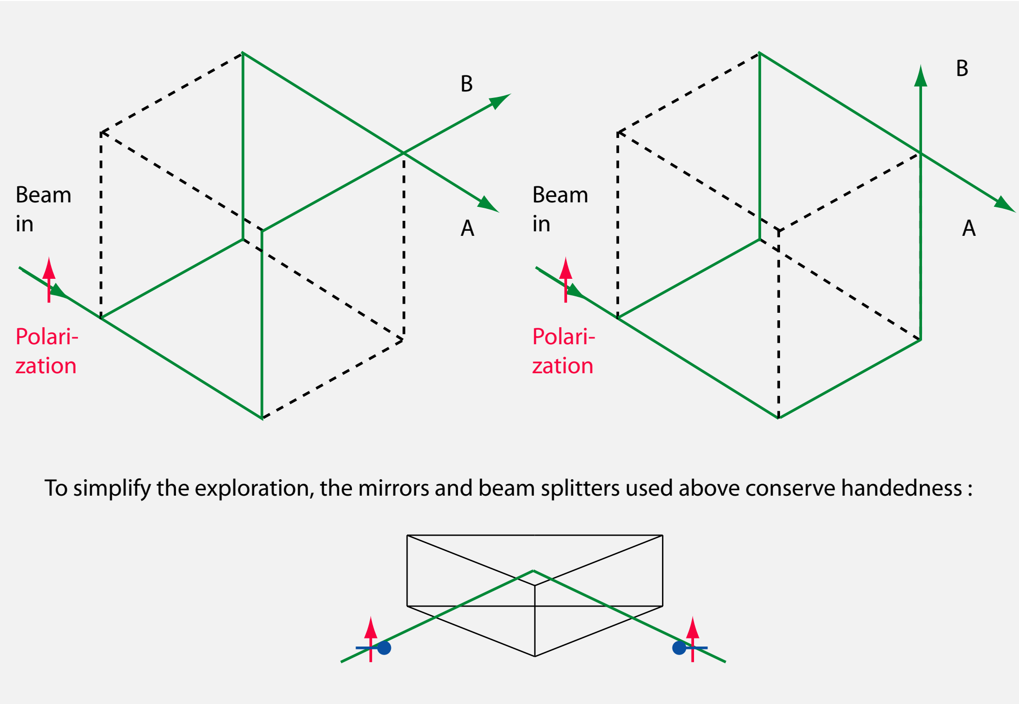

FIGURE 88 Two different three-dimensional interferometers, with all edges of equal lengths, the mirrors/beam splitters used, and their outputs A and B. Where does the light exit?

* *

An interferometer is a device that uses the interference of light to study the properties of a light beam. A common interferometer, the Mach-Zehnder interferometer, is shown in Figure 86. If all sides have equal length, light interferes constructively in the output direction A and destructively in the other output direction B. Thus light exits in direction A.

Ref. 99 Only in the 1990s people started asking what would happen in *three-dimensional* interferometers, such as the one shown in Figure 88. To clarify the situation, a few points are necessary. First, we need to specify the polarization of the light used, and recall that only light of the same polarization can interfere. Secondly, to simplify the discussion, we assume that the mirrors are of a special type (namely corner cubes based on total refraction) so that, in contrast to usual mirrors, they *conserve* polarization. Thirdly, we assume that all edges have equal length. Can you deduce which exits are bright in the two cases of Figure 88? Challenge 147 s

* *

In regions of destructive interference one finds so-called *phase singularities*. If the interfering light is white, such regions are not black but show, if the intensity is amplified, fascinating colour patterns. These colours, predicted in the 1970s, were found experimentally a few decades later. They follow an universal blue-orange patterns. Ref. 100

* *

Maxwell's equations of the electromagnetic field are 150 years old. Is all about them known? Probably not. For example, only in the 1990s Antonio Rañada discovered that the equations have solutions with *knotted* field lines. The most spectacular solutions so far have been published by Arrayás and Trueba. More such surprising results are probably waiting to be found. Ref. 101

Summary on light

In summary, radio waves, infrared light, visible light, ultraviolet light, X-rays and gamma rays are electromagnetic waves. Their dispersion relation in vacuum is $\omega = ck$, where the phase velocity $c = 299\,792\,458$ m/s is a universal constant, an invariant. Electromagnetic waves carry energy, linear momentum and angular momentum, and move faster than any material object. In vacuum, the phase velocity is also the group and the signal velocity. In addition, the speed of electromagnetic waves c is the (local) limit energy speed in nature.

CHAPTER 4

IMAGES AND THE EYE – OPTICS

Optics is the field that explores the production of images. In particular, optics is the study and use of light *production*, of light *transport*, and of light and image *detection*. With this definition of optics, we note directly that classical electrodynamics can describe only the transport of light. The production and the detection of light are always quantum effects. Every lamp is a device based on quantum physics. Every detector of light, including the eye, is based on quantum physics. Therefore, in this chapter we mainly explore the motion of light and the way it forms images, and give only a short introduction into light sources and the eye. Ref. 102

WAYS TO ACQUIRE IMAGES

Acquiring images is an important part of modern society. The quality of images depends on the smart use of optics, electronics, computers and materials science. Despite the long history of optics, there are still new results in the field. Images, i.e., two or three-dimensional reproductions of a physical situation, can be taken by at least six groups of techniques:

— *Photography* uses a light source, lenses and film – or another large area detector. Photography can be used in reflection, in transmission, with phase-dependence, with various illuminations, and with light sources and detectors for various wavelengths.
— *Optical microscopy* uses a light source, magnifying lens systems and film (or another large area detector). If the illumination is through the sample, in transmission, one speaks of *bright-field microscopy*. (Variations use coloured or polarizing filters.) If the illumination is from the side, one speaks of *oblique microscopy*. If the illumination is confined to an outer ring of light, one speaks of *dark-field microscopy*. An even more elaborate illumination system, using plane waves, allows *phase-contrast microscopy*. (It was invented by Frits Zernike in the 1930s and earned him the Nobel Prize in Physics in 1953.) If one splits a polarized illumination beam into two components that pass the sample at close (but not identical) locations, and then recombines them afterwards, one speaks of *differential interference contrast microscopy*. If a sample is treated with a fluorescent dye, the illuminating light is filtered out, and only the fluorescence is observed, one speaks of *fluorescence microscopy*. The image quality of expensive microscopes can be further improved with the help of a computer, with the help of deconvolution techniques. Ref. 103
— *Telescopy* is used most of all in geodesy and astronomy. The most advanced astronomical telescopes can compensate star images for the effects of the turbulence of

FIGURE 89 An X-ray photographic image of a ten-year old boy with polydactyly (© Drgnu23).

the atmosphere; they can also take images at various wavelengths, ranging from radio frequencies, infrared, visible, ultraviolet to X-rays. Simple telescopes are lens-based; high-performance telescopes are usually mirror-based. X-ray telescopes have to be operated outside the atmosphere, to avoid absorption by air, for example on rockets, satellites or high-altitude balloons. They are all mirror based.

— *Scanning techniques* acquire images point by point through the motion of the detector, the light source or both. There are numerous scanning microscopy techniques: *confocal laser scanning microscopy*, the fibre-based *near-field scanning optical microscopy*, and combinations of them with fluorescence techniques or various deconvolution techniques. Many of these scanning microscopy techniques allow resolutions much lower than the wavelength of light, a feat that is impossible with conventional microscopic techniques. Scanning techniques are also used in special fields of photography.
— *Tomography*, usually performed in transmission, uses a source and a detector that are rotated together around an object. This technique, effectively a specialized scanning technique, allows imaging cross sections of physical bodies. For example, light tomography is a promising technique, without any health risk, for breast cancer detection.
— *Holography* uses lasers and large area detectors and allows taking three-dimensional images of objects. Such images seem to float in space. Holography can be used in reflection or in transmission.

Each image acquisition method can be used with radio waves, infrared light, visible light, ultraviolet light, X-rays or with gamma rays. In fact, these techniques can even be used with electron beams; one then speaks of electron optics. In all imaging methods, the race is twofold: progress aims for images with the highest resolution possible and for images with the shortest shutter times possible. Short shutter times allow to produce films. We start our overview of imaging techniques with the most important tool: light sources.

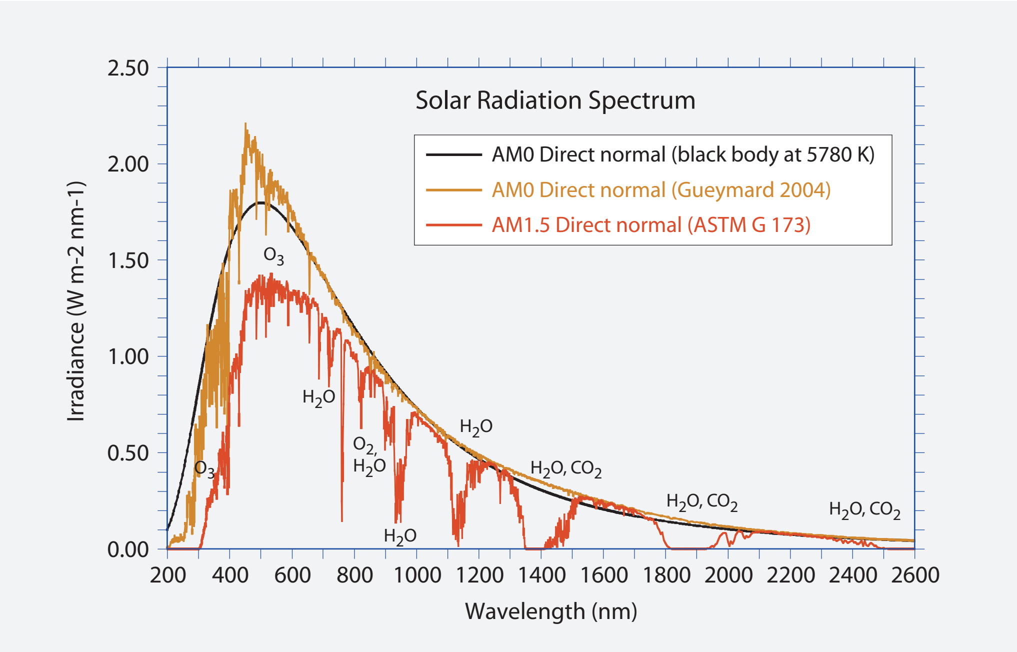

FIGURE 90 A black body spectrum at 5780 K, the solar spectrum *above* the atmosphere in direction of the Sun, with 1350 W/m^2, and the spectrum with 1.5 air masses, or atmospheric thicknesses, in between, with 844 W/m^2. The latter roughly describes the spectrum of a typical sunny day at sea level. The gases responsible for the absorption bands are also shown (© Chris Gueymard).

Light sources

Without radiation sources, there would be no images. All imaging techniques need sources of radiation. In the domain of visible light optics, the most important light sources of visible and infrared light are *hot* objects, such as candles, the Sun or flash-lamps. Physically speaking, these light sources are approximations of black bodies. Let us see why they are used. *Cold* light sources, such as light emitting semiconductor diodes, fireflies or lasers, are explored later on.

Why can we see each other? Black bodies and the temperature of light

Physicists have a strange use of the term 'black'. A body that glows perfectly is called a *black body*. In this domain, 'perfect' means that the surface of the body has *no* effect on its colour.

▷ A ***black body*** is a body that absorbs all radiation impinging on it.

In other words, a black body is a body without reflection or transmission of radiation. Black bodies are an idealization; above all, they are only black at low temperature. With increasing temperature, black bodies glow or shine in black, brown, red, orange, yellow, white or light blue.

The essence of black bodies is that the colour they have, i.e., the light they radiate, is independent of the surface. Black bodies are thus *ideal* in this sense. Real bodies, which do show surface effects, can be classified by their *emissivity*. The emissivity gives the degree to which a body approaches a black body. Mirrors have emissivities of around 0.02, whereas black soot can have values as high as 0.95. Practically all bodies at everyday temperature are not black bodies: their colour is not determined by emission, but mostly by the absorption and reflection of light at their surface.

Black bodies, as the section on quantum theory will show, have *smooth* light emission spectra. An example for a spectrum of a black body, and for a spectrum of a real body – in this case, the Sun – is shown in Figure 90.

Black bodies are also used to define the colour *white*. What we commonly call *pure white* is the colour emitted by the Sun. The sun is not a good black body, as Figure 90 shows (its effective temperature is 5780 K). Because of these problems, pure white is now defined as the colour of a black body of 6500 K, e.g. by the Commission Internationale d'Eclairage. Hotter black bodies are bluish, colder ones are yellow, orange, red, brown or black. The stars in the sky are classified in this way.

Ref. 104
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Black bodies are thus bodies that glow perfectly. Most real bodies are only rough approximations of black bodies, even at temperatures at which they shine yellow light. For example, the tungsten in incandescent light bulbs, at around 2000 K, has an emissivity of around 0.4 for most wavelengths, so that its spectrum is a corresponding fraction of that of black body. (However, the glass of the light bulb then absorbs much of the ultraviolet and infrared components, so that the final spectrum is not at all that of a black body.)

Black body radiation has two important properties: first, the emitted light power increases with the fourth power of the temperature. With this relation alone you can check the temperature of the Sun, mentioned above, simply by comparing the size of the Sun with the width of your thumb when your arm is stretched out in front of you. Are you able to do this? (Hint: use the excellent approximation that the Earth's average temperature of about 14.0°C is due to the Sun's irradiation.)

Challenge 148 d
Ref. 105

The precise expression for the emitted energy density u per frequency ν can be deduced from the radiation 'law' for black bodies discovered by Max Planck*

$$u(\nu, T) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/kT} - 1} . \tag{76}$$

He made this important discovery, which we will discuss in more detail in the quantum part of our mountain ascent, simply by comparing this curve with experiment. The new constant h is called the *quantum of action* or *Planck's constant* and turns out to have the value $6.6 \cdot 10^{-34}$ Js, and is central to all quantum theory, as we will find out. The other

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* Max Planck (b. 1858 Kile, d. 1947 Göttingen), professor of physics in Berlin, was a central figure in thermodynamics. He discovered and named *Boltzmann's constant* k and the *quantum of action* h, often called Planck's constant. His introduction of the quantum hypothesis gave birth to quantum theory. He also made the works of Einstein known in the physical community, and later organized a job for him in Berlin. He received the Nobel Prize for physics in 1918. He was an important figure in the German scientific establishment; he also was one of the very few who had the courage to tell Adolf Hitler *face to face* that it was a bad idea to fire Jewish professors. (He got an outburst of anger as answer.) Famously modest, with many tragedies in his personal life, he was esteemed by everybody who knew him.

Ref. 106

constant Planck introduced, the Boltzmann constant k, appears as a prefactor of temperature all over thermodynamics, as it acts as a conversion unit from temperature to energy.

Challenge 149 e The radiation 'law' gives for the total emitted energy density the expression

$$u(T) = T^4 \frac{8\pi^5 k^4}{15 c^3 h^3} \tag{77}$$

Challenge 150 ny from which equation (85) is deduced using $I = uc/4$. (Why?)

The second property of black body radiation is the value of the peak wavelength, i.e., the wavelength emitted with the highest intensity. This wavelength determines the colour

Challenge 151 ny of a black body; it is deduced from equation (76) to be

$$\lambda_{\max} = \frac{1}{T}\frac{hc}{4.956\,k} = \frac{2.90\,\mathrm{mm\,K}}{T} \quad \text{but} \quad \hbar\nu_{\max} = T \cdot 2.82\,k/h = T \cdot 5.9 \cdot 10^{10}\,\mathrm{Hz/K}\,. \tag{78}$$

Either of these expressions is called *Wien's colour displacement rule* after its discoverer.* The colour change with temperature is used in optical thermometers; this is also the way the temperatures of stars are measured. For 37°C, human body temperature, it gives a peak wavelength of 9.3 μm or 115 THz, which is therefore the colour of the bulk of the radiation emitted by every human being. (The peak wavelength does not correspond to

Challenge 152 s the peak frequency. Why?) On the other hand, following the telecommunication laws of many countries, any radiation emitter needs a licence to operate; it follows that strictly in Germany only dead people are legal, and only if their bodies are at absolute zero temperature.

We saw that a black body – or a star – can be blue, white, yellow, orange, red or brown.

Challenge 153 ny A black body is never green. Can you explain why?

Above, we predicted that any material made of charges emits radiation. Are you able to find a simple argument showing whether heat radiation is or is not this classically

Challenge 154 ny predicted radiation?

But let us come back to the question in the section title. The existence of thermal radiation implies that any hot body will cool, even if it is left in the most insulating medium there is, namely in vacuum. More precisely, if the vacuum is surrounded by a wall, the temperature of a body in the vacuum will gradually approach that of the wall.

Interestingly, when the temperature of the wall and of the body inside have become the same, something strange happens. The effect is difficult to check at home, but impres-

Ref. 107 sive photographs exist in the literature.

One arrangement in which walls and the objects inside them are at the same temperature is an *oven*. It turns out that it is *impossible* to see objects in an oven using the light coming from thermal radiation. For example, if an oven and all its contents are red hot, taking a picture of the inside of the oven (without a flash!) does not reveal anything; no

* Wilhelm Wien (b. 1864 Gaffken, d. 1928 Munich) received the Nobel Prize for physics in 1911 for the discovery of this relation.

Note that the value appearing in Wien's rule can be uniquely calculated from equation (76), but cannot be expressed as a formula. Indeed, Wien's constant contains the solution of the equation $x = 5(1 - e^{-x})$.

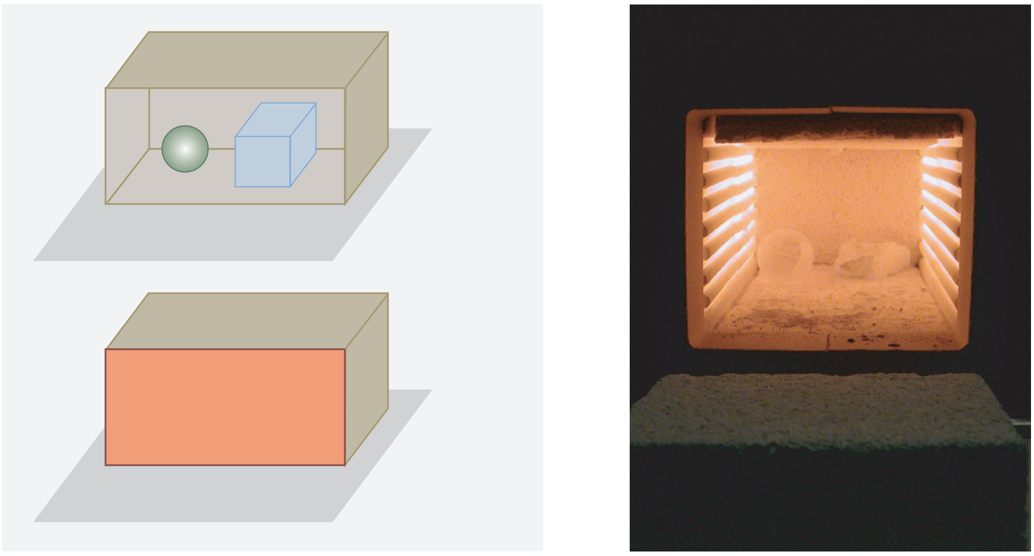

FIGURE 91 Bodies inside an oven at room temperature differ in colour, in contrast to bodies at high temperature (photo © Wolfgang Rueckner).

FIGURE 92 The last mirror of the solar furnace at Odeillo, in the French Pyrenees (© Gerhard Weinrebe).

contrast nor brightness changes exist that allow one to distinguish the objects from the walls or their surroundings. Can you explain the finding?

Challenge 155 s

In short, we are able to see each other only because the light sources we use are at a *different* temperature from us. We can see each other only because we do *not* live in thermal equilibrium with our environment.

Limits to the concentration of light

Light sources should be as bright as possible. Are there any limits? Interestingly, for black body radiation there is an important and instructive limitation.

If we build a large lens or a large curved mirror, we can collect the light of the Sun and focus it on a tiny spot. Everybody has used a converging lens as a child to burn black spots on newspapers – or ants – in this way. In Odeillo, in Spain, wealthier researchers have built a curved mirror as large as a house, in order to study solar energy use and material behaviour at high temperature. Essentially, the mirror provides a cheap way to

FIGURE 93 The solar power plant at Sanlucar la Mayor, near Seville, in Spain (© Wikimedia).

fire an oven in its focus. (And 'focus' is the Latin word for 'hearth'.)

Kids find out quite rapidly that large lenses or mirrors allow them to burn things or paper more easily than small ones. The Odeillo site shown in Figure 92 is the record holder in the quest for the larges possible collection area. Interstingly, building a larger mirror does not make much sense. Whatever its size may be, the temperature in such a set-up is *limited*: Ref. 108

▷ The effective temperature of the light in a focus *cannot* exceed the temperature of the original light source.

In all practical situations, the temperature of the light source is much higher than in the focus. The surface temperature of the Sun is about 5780 K; the highest temperature reached so far in Odeillo is about 4000 K. Are you able to show that this limitation is equivalent to the second principle of thermodynamics, as Hemholtz, Clausius and Airy showed? Challenge 156 s

In short, nature provides a *limit* to the concentration of light energy. More precisely, we can say: thermodynamics limits what can be achieved through heating with thermal light sources.

The thermodynamic limit on heating with light does not prevent people to use light concentration to gather solar energy. Experimental power plants such as the one shown

in Figure 93 are one promising way to supply energy to households when fossil fuel prices rise too much.

As we just saw, a beam of thermal light has entropy. In contrast, a laser beam only has a tiny entropy. We can also ascribe a temperature value to either beam: the temperature of a thermal beam is the temperature of the light source; the temperature of a laser beam is a 'negative' number. This makes some sense intuitively, because a laser beam is able to cool gases; more precisely, a laser beam is a non-equilibrium situation, and temperature is not defined for such cases.

In several countries, taxpayer's money is wasted in so-called *inertial confinement fusion* centres. In those centres, several powerful lasers are focused on a small sphere of material, typically, 1 mm in size; a target temperature of around 3 MK (or, equivalently, Challenge 157 s 300 eV) has been achieved. Why is this possible?

Measuring light intensity

Light sources differ in brightness. Measuring what we call 'dark' and 'bright' is somewhat involved, because light can be diffuse or directed. To achieve proper measurements, the Page 326 *SI*, the international system of units, defines a specific base unit, the candela:

▪ 'The *candela* is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \cdot 10^{12}$ hertz and has a radiant intensity in that direction of (1/683) watt per steradian.'

The candela is thus a unit for light power per (solid) angle, usually called *luminous intensity*, except that it is corrected for the eye's sensitivity: the candela measures only *visible* power per angle. The definition of the candela simply says that 683 cd = 683 lm/sr corresponds to 1 W/sr. For example, a glow worm produces 0.01 cd, a candle indeed produces around 1 cd, a car light around 100 cd, and a lighthouse around 2 Mcd. Another way to look at the candela is the following: watching a source with 1 cd from a distance of 1 m is a just bit brighter than the full moon.

Total light power, irrespective of its direction, is measured in lumen. Therefore, 683 lm = 683 cd sr corresponds to 1 W. In other words, both the lumen and the watt measure power, or energy flux, but the lumen measures only the *visible* part of the power or energy flux. This difference is expressed by adding 'luminous' or 'radiant': thus, the lumen measures *luminous* flux, whereas the Watt measures *radiant* flux.

The factor 683 appearing in the definitions is historical. An ordinary candle emits a luminous intensity of about a candela. To put this into perspective: at night, a candle can Challenge 158 e be seen up to a distance of 10 or 20 kilometres. A 100 W incandescent light bulb produces 1700 lm, and the brightest commercial light emitting diodes about 20 lm, though laboratory devices exceed 1000 lm. Cinema projectors produce around 2 Mlm, and the brightest flashes, like lightning, 100 Mlm.

The *irradiance* of sunlight is about 1300 W/m^2 on a sunny day; on the other hand, the *illuminance* is only 120 klm/m^2 = 120 klx or 170 W/m^2. A cloud-covered summer day or a clear winter day produces about 10 klx. These numbers show that most of the energy from the Sun that reaches the Earth is outside the visible spectrum.

Illuminance is essentially what we call 'brightness' in everyday life. On a glacier, near the sea shore, on the top of a mountain, or in particular weather condition the bright-

TABLE 16 Some measured illuminance values.

Observation	Illuminance
Brightness of the human body	1 plx
Faint star	0.1 nlx
Sirius	10 μlx
phot (old illuminance unit)	10 μlx
Jupiter	20 μlx
Dark, moonless night	1 mlx
Full moon	0.01 to 0.24 lx
Street at night, low traffic, poor lighting	0.1 to 3 lx
Street at night, high traffic	10 to 30 lx
For reading	50 to 100 lx
Cinema screen	100 lx
Workplace	0.2 to 5 klx
Cloudy day	1 klx
Brightest lamps, used for surgery	120 klx
Sunny day	120 klx
Film in cinema projector	5 Mlx
Painful to the eye	100 Mlx

ness can reach 150 klx. Museums are often kept dark because water-based paintings are degraded by light above 100 lx, and oil paintings by light above 200 lx. The eyes lose their ability to distinguish colours somewhere between 0.1 lx and 0.01 lx; the eye stops to work below 1 nlx. Technical devices to produce images in the dark, such as night goggles, start to work at 1 μlx. By the way, the human body itself *shines* with about 1 plx, a value too small to be detected with the eye, but easily measured with specialized apparatus. The origin of this emission is still a topic of research.

Ref. 109

The highest achieved light intensities, produced with high-power lasers, are in excess of 10^{18} W/m^2, more than 15 orders of magnitude higher than the intensity of sunlight. (How much is that in lux?) Such intensities are produced by tight focusing of pulsed laser beams. The electric field in such light pulses is of the same order as the field inside atoms; such a laser beam therefore ionizes all matter it encounters, including the air.

Challenge 159 e

Ref. 110

The *luminous density* is a quantity often used by light technicians. Its unit is 1 cd/m^2, unofficially called 1 Nit and abbreviated 1 nt. Human eyes see using *rods* only from 0.1 μcd/m^2 to 1 mcd/m^2; they see with *cones* only above 5 cd/m^2. Eyes see best between 100 and 50 000 cd/m^2, and they get completely overloaded above 10 Mcd/m^2: a total range of 15 orders of magnitude. Very few technical detectors achieve this range.

Other light and radiation sources

Apart from black bodies, many other types of light sources exist. Cold sources of light range from glowing fish to high-power lasers. They range in size from an atom to a build-

FIGURE 94 A modern picosecond pulse laser and an industrial X-ray source, both about 700 mm in size (© Time-Bandwidth, SPECS).

ing, in cost from a fraction of an Euro to hundreds of millions of Euros, and in lifetime from a fraction of a second to hundreds of years.

Lasers are important light sources for industry, medicine and research. Lasers can emit visible, infrared and ultraviolet light, continuously or as light pulses, with various powers, polarizations and beam shapes; they are explored later on in our adventure. In the domain of imaging, lasers are used in many microscopy techniques, in scanning imaging systems, in tomography and in holography.

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Sources of radio waves are common in everyday life: mobile phones, radio transmitters, tv transmitters and walkie-talkies are all sources of radio waves. They are used for imaging in magnetic resonance imaging, which allows to image the interior of the human body, and in astronomy: Since many stars are radio emitters, one can image the sky at radio wavelengths. Nowadays, radio astronomy is an important part of modern astronomy and has led to many discoveries. Radio astronomy has also been an important tool for the precision testing and confirmation of general relativity.

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On the other end of the electromagnetic spectrum, light sources that emit X-rays and gamma rays are common. They are routinely used in medicine and materials science, also for various imaging techniques.

All sources of electromagnetic radiation are potentially dangerous to humans, so that

special care has to be taken when using them. This has also led to various unfortunate developments.

Radiation as weapon

High-intensity electromagnetic radiation is dangerous. In many countries, more money is available to study assault weapons than to increase the education and wealth of their citizen. Several types of assault weapons using electromagnetic radiation are being researched. Two are particularly advanced.

The first weapon using electromagnetic radiation is a truck with a movable parabolic antenna on its roof, about 1 m in size, that emits a high power – a few kW – microwave beam at 95 GHz. The beam, like all microwave beams, is invisible; depending on power and beam shape, it is painful or lethal, up to a distance of 100 m and more. This terrible device, officially called *active denial system*, with which the operator can make many victims even by mistake, was ready in 2006. Some extreme politicians want to give it to the police. (Who expects that a parabolic antenna is dangerous?) Efforts to ban it across the world are slowly gathering momentum.

The second weapon under development is the so-called *pulsed impulse kill laser*. The idea is to take a laser that emits radiation that is not absorbed by air, steam or similar obstacles. An example is a pulsed deuterium fluoride laser that emits at 3.5 μm. This laser burns every material it hits; in addition, the evaporation of the plasma produced by the burn produces a strong hit, so that people hit by such a laser are hurt and hit at the same time. Fortunately, it is still difficult to make such a device rugged enough for practical mobile use. Nevertheless, experts expect battle lasers, mounted on trucks, to appear soon – after a number of Potemkin's versions.

In short, it is probable that radiation weapons will appear in the coming years. What the men working on such developments tell their children when they come home in the evening is not clear, though.

Images – transporting light

Every image is formed by transporting light in a useful manner along known paths. The simplest possible path is the straight line.

Making images with mirrors

Since light moves in a straight line, a flat mirror produces an image of the same size than the original. Curved mirrors can be used to enlarge, reduce and distort images. For example, expensive bed room mirrors are often slightly curved, in order to make people appear thinner.

Most human-made mirrors are made of metal, usually evaporated onto a glass substrate; in contrast, living systems cannot produce pure metals. On the other hand, in living systems, mirrors abound: they are found as the *tapetum* in the eyes, on fish scales, on bugs, etc. How does nature produce mirrors, despite lacking the ability to use pure metals? It turns out that sandwiches of different thin transparent materials – one of which is typically crystalline guanine – can produce mirrors that are almost as good as metal

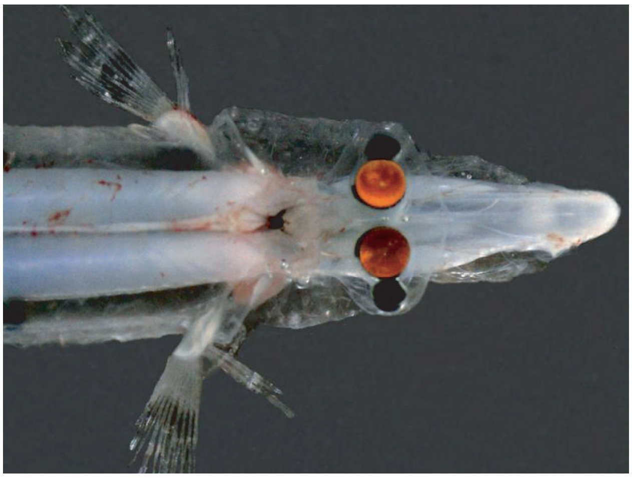

FIGURE 95 The spookfish *Dolichopteryx longipes* has orange mirrors that help him make sharp images also from the dim light coming upwards from bioluminescent lifeforms below it (© Tamara Frank).

FIGURE 96 A Wolter-type grazing incidence collector for 13.5 nm radiation built with the help of concentric mirrors (© Media Lario Technologies).

mirrors. Such mirrors are based on interference effects and are called *dielectric mirrors*. Dielectric mirrors are also used to make laser mirrors.

Image-forming mirrors are used in large telescopes, in systems for X-rays, and in medical devices used by physicians. Interestingly, also some living beings use mirrors for imaging. The most famous example is the spookfish shown in Figure 95. It is able to look up and down at the same time, and does so using mirrors attached to his eyes.

Challenge 160 s By the way, why are mirrors frequently used in telescopes, but not in microscopes?

In illumination systems, mirrors are used for the shaping of light beams in cars, in pocket lamps and in LED lamps. It might be that some deep water creatures use mirrors for similar uses – but no example is known to the author.

The most involved mirror systems to date are used in the extreme ultraviolet mask lithography systems that will be used in the future production of integrated circuits.

FIGURE 97 Light usually travels in a straight line. In the figure, a sodium frequency laser beam is used as laser guide star to provide a signal for adaptive optics in large telescopes. The laser illuminates a layer of sodium found in the atmosphere at around 90 km of altitude, thus providing an artificial star. The artificial star is used to improve the image quality of the telescope through adaptive optics. In the photograph, the images of the real stars are blurred because of the long exposure time of 3 min (photo by Paul Hirst).

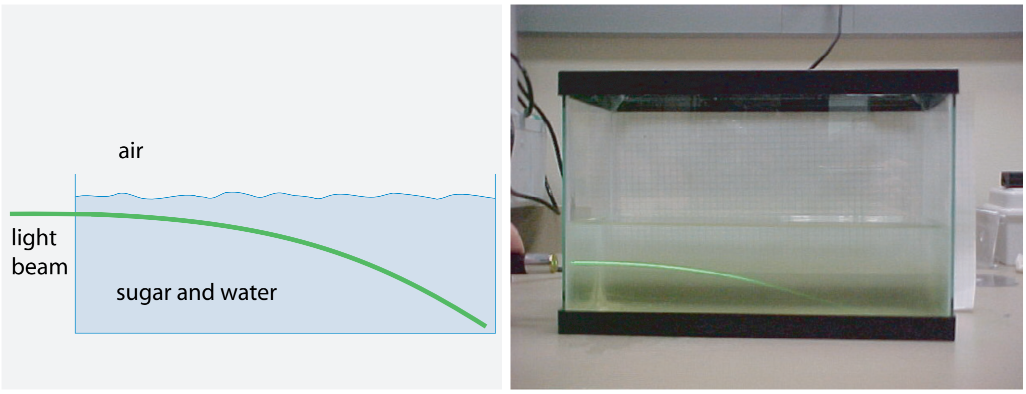

FIGURE 98 Diluted sugar syrup bends light (© Jennifer Nierer).

These systems use a wavelength of 13.5 nm, at which lenses are not available. Collimating an expanding beam thus requires many concentric mirrors, as shown in Figure 96. These optical systems are the very best that modern technology can provide; for example, the mirrors have a surface roughness below 0.4 nm. Similar optical mirror systems are also used in x-ray satellite telescopes.

Does light always travel in a straight line? – Refraction

Usually light moves in straight lines. A laser in a misty night shows this most clearly, as illustrated in Figure 97. But any laser pointer in the mist is equally fascinating. Indeed, we Page 15 use light to *define* 'straightness', as we explained in the exploration of relativity. However, there are a number of situations in which light does not travel in a straight line, and every Ref. 111 expert on motion should know them.

In diluted sugar syrup, light beams curve, as shown in Figure 98. The reason is that in such an experiment, the sugar concentration changes with depth. Are you able to explain

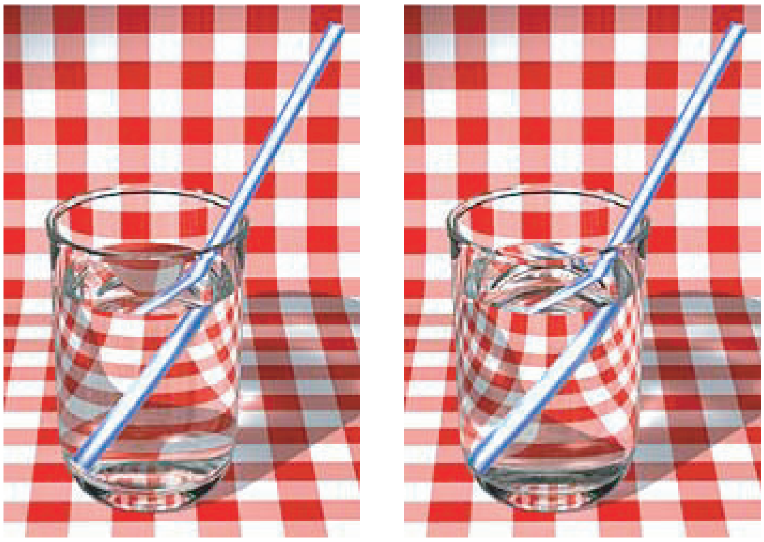

FIGURE 99 Realistic computer graphics showing the refraction in water and in diluted sugar syrup (graphics © Robin Wood). Can you tell which one is which?

Challenge 162 e

FIGURE 100 A pretty effect of refraction at the water–air interface that you can repeat at home (© Maric Vladimir).

Challenge 161 s

the syrup effect?

More detailed observation show that a light beam is bent at every material change it encounters on its path. This effect, called *refraction*, is quite common. Refraction changes the appearance of the shape of our feet when we are in the bath tub; refraction also makes aquaria seem less deep than they actually are and produces effects such as those shown in Figure 99 and Figure 100. Refraction is a consequence of the change of the phase velocity of light from material to material; all refraction effects are thus explained by Figure 101.

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Refraction can also be seen to follow from the minimization principle for the motion of light:

▷ Light always takes the path that requires the *shortest* travel time.

For example, light moves more slowly in water than in air; that is the reason for the bend illustrated in Figure 102.

The speed ratio between air and water is called the *refractive index* of water. The refractive index, usually abbreviated n, is material-dependent. The value for water is about 1.3. This speed ratio, together with the minimum-time principle, leads to the 'law' of re-

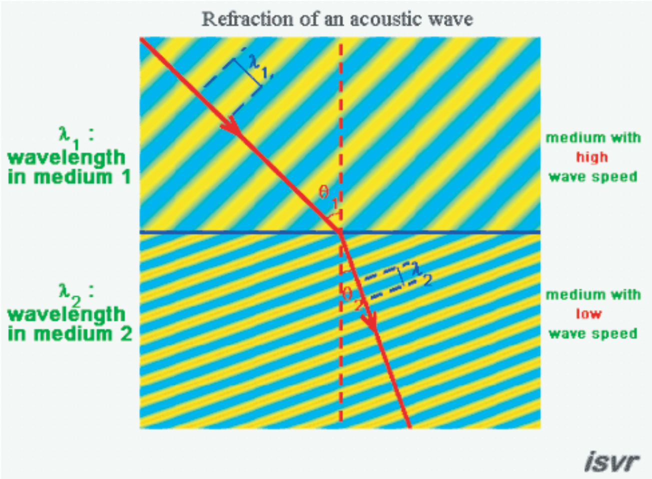

FIGURE 101 A visualisation of refraction (QuickTime film © ISVR, University of Southampton).

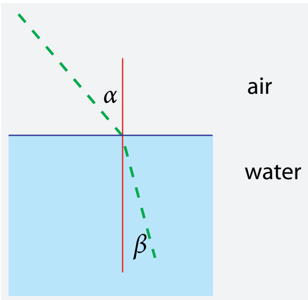

FIGURE 102 Refraction of light is due to travel-time optimization.

fraction, a simple relation between the sines of the two angles shown in Figure 102. Can you deduce the relation? In fact, the exact definition of the refractive index of a material is with respect to vacuum, not to air. But the difference is negligible, because gases are mainly made of vacuum and their index of refraction is close to one.

Challenge 163 s

In many fluids and solids, light signals move more slowly than in vacuum; also the (different) phase and group velocities of light inside materials are regularly *lower* than c, the light speed in vacuum. We discussed the difference between these speeds above. For such 'normal' materials, the refractive index n, the ratio of c to the phase velocity inside the material, is larger than 1. The refractive index is an important material property for the description of optical effects. For example, the value for visible light in water is about 1.3, for glasses it is around 1.5, and for diamond 2.4. The high value is one reason for the sparkle of diamonds cut with the 57-face *brilliant* cut.

Page 129

The refractive index also depends on wavelength; this effect, called *dispersion*, appears in most materials. Prisms make use of dispersion in glass to split white or other light into its constituent colours. Also diamond, and in particular the brilliant cut, works as a prism, and this is the second reason for their sparkle.

In contrast to 'normal' materials, various materials have refractive indices that are lower than 1, and thus phase velocities larger than c. For example, gold has a refractive

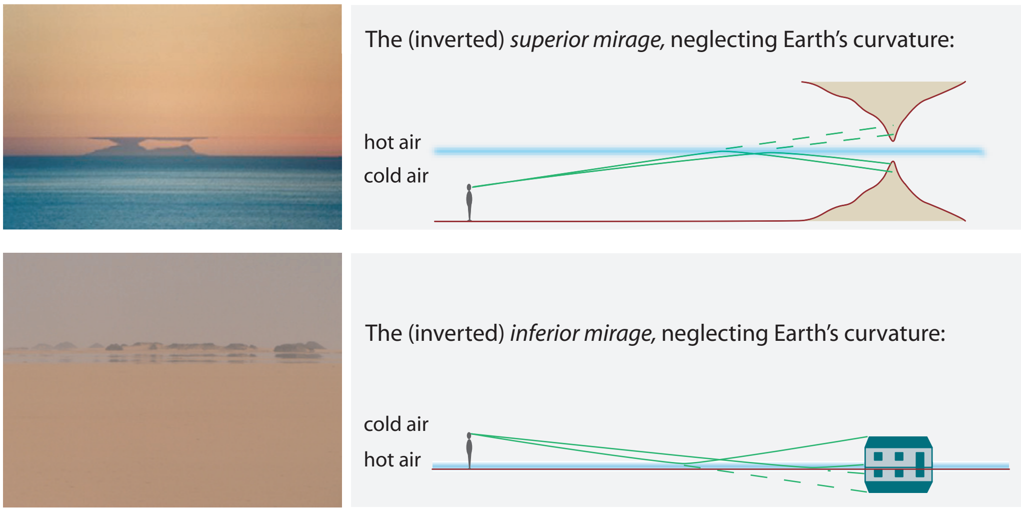

FIGURE 103 The basis of mirages is an effective reflection due to refraction in a hot air layer; it can lead to spectacular effects, such as the inverted superior mirage (top left and right) and the inferior image (bottom left and right) (photographs © Thomas Hogan and Andy Barson).

index of around 0.2 for visible light, and thus a phase velocity of around $5c$ for such waves. In fact, almost all materials have refractive indices below 1 for *some* wave frequencies, Ref. 112 including table salt.

In short, refraction of light, the change of the direction of light motion, is due to different phase velocities of light in different materials. *Material changes bend light paths.* Refraction is so common because it is extremely rare to have different adjacent materials with the same refractive index.

Gases have refractive indices close to the vacuum value 1. Nevertheless, also gases lead to refraction. In particular, the refractive index of gases depends on temperature. In air of varying temperature, refraction leads to curved light paths and produces a well-known effect: the *mirage*, also called *fata morgana*. Figure 103 shows photographs of a *superior* mirage, which relies on an *inversion layer* in the atmosphere *above* the object and the observer, and a *inferior* mirage, due to a hot layer of air *below* the observer, just above Ref. 113 the ground. Inferior mirages are also regularly seen on hot highways. All mirage types are due to refraction; their detailed appearance depends on the given temperature profile in the air, and the relative heights of the observer, the inversion layer and the observed object. Often, the curvature of the Earth also plays a role.

Above all, refraction is used in the design of *lenses*. With glass one can produce precisely curved surfaces that allow us to *focus* light. All focusing devices, such as lenses, can be used to produce images. The two main types of lenses, with their focal points and the images they produce, are shown in Figure 104; they are called *converging lenses* and *divergent lenses*. When an object is more distant from a single converging lens than its focus, the lens produces a *real* image, i.e., an image that can be projected onto a screen. In all other cases single converging or diverging lenses produce so-called *virtual images*: such images can be seen with the eye but not be projected onto a screen. For example, when an object is put between a converging lens and its focus, the lens works as a *magni-*

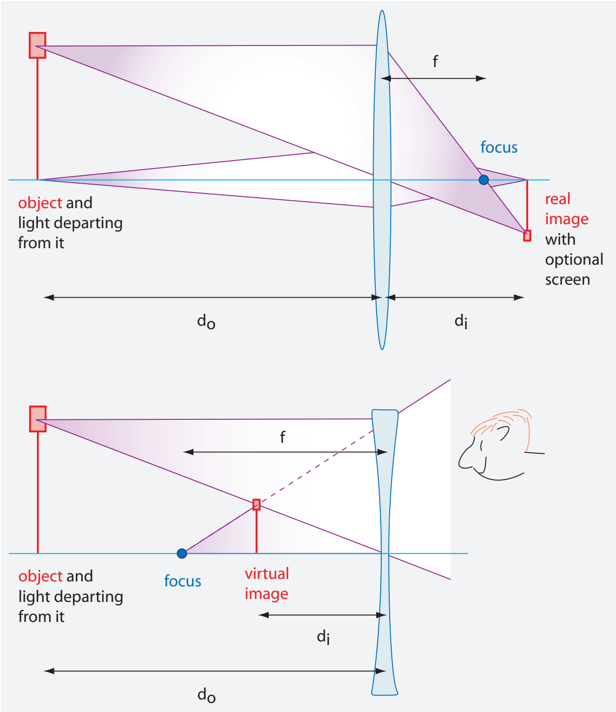

FIGURE 104 A real image produced by a converging lens (if used in the way shown) and the virtual image produced by a diverging lens.

fying glass. Figure 104 also allows one to deduce the *thin lens formula* that connects the lengths d_o, d_i and f. What is it? Challenge 164 s

Even though glasses and lenses have been known since antiquity, the Middle Ages had to pass by before two lenses were combined to make more elaborate optical instruments. The various effects that can be observed with one or two lenses are shown in Figure 105. Ref. 114 The *telescope* was invented – after a partial success in Italy by Giambattista della Porta – just before 1608 in the Netherlands. The most well-known of at least three simultaneous inventors was the lens grinder Johannes Lipperhey (b. *c.* 1570 Wesel, d. 1619 Middelburg) who made a fortune by selling his telescopes to the Dutch military. When Galileo heard about the discovery, he quickly took it over and improved it. Already in 1609, Galileo performed the first astronomical observations; they made him world-famous. The *Dutch telescope* design has a short tube yielding a bright and upright image, and its magnification is the ratio of the focal distances of the two lenses. Challenge 165 e It is still used today in opera glasses. Over the years, many other ways of building telescopes have been developed; in particular, many modern high-performance telescopes use mirrors. Ref. 115 Since mirrors are cheaper and easier to fabricate for high-precision imaging, most large telescopes have a mirror instead of the first lens.

By the way, telescopes also exist in nature. Most spiders have several types of eyes, and some up to 6 different pairs. For example, the jumping spider genus *Portia (Salticidae)* has two especially large eyes, made to see distant objects, which have two lenses behind each other; the second lens and the retina behind it can be moved with muscles, so that such spiders can effectively point their telescope in different directions without moving

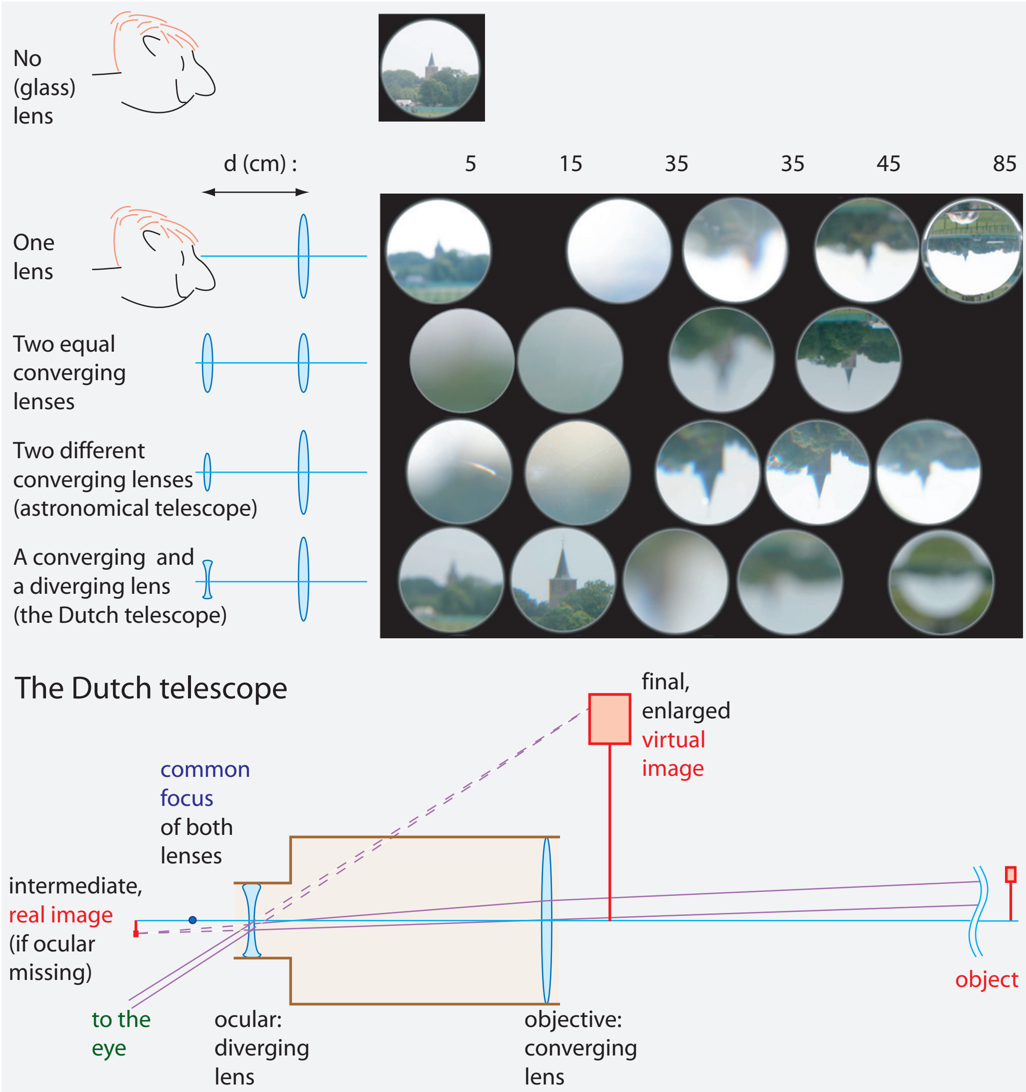

FIGURE 105 Lens refraction is the basis of the telescope: above, the experiments with lenses that lead to the development of the telescope: the object to watch compared with the images produced by a single converging lens, by two equal converging lenses, by two different converging lenses in the astronomical telescope, and by a diverging and a converging lens in the Dutch telescope, at various distances from the eye; below, the explanation of the Dutch telescope (photographs © Eric Kirchner).

their head. In order to process the input from all their eyes, jumping spiders need a large brain. In fact, about 50 % of the body mass of jumping spiders is brain mass.

Another way to combine two lenses leads to the *microscope*. Can you explain to a non-physicist how a microscope works? Werner Heisenberg almost failed his Ph.D. exam because he could not. The problem is not difficult, though. Indeed, the inventor of the microscope was an autodidact of the seventeenth century: the technician Antoni van Leeuwenhoek (b. 1632 Delft, d. 1723 Delft) made a living by selling over five hundred of his microscopes to his contemporaries. (This is a somewhat nasty remark: Van Leeuwen-

Challenge 166 s

FIGURE 106 The glory produced by the droplets in a cloud (© Brocken Inaglory).

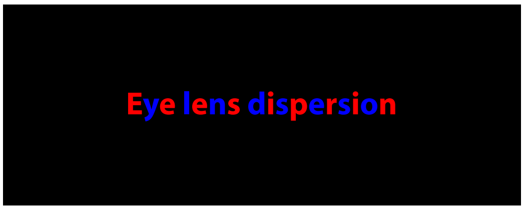

FIGURE 107 Watching this graphic at higher magnification shows the dispersion of the human eye: the letters float at different depths.

hoek only used one lens, not two, as in the modern microscope.)

No ray tracing diagram, be it that of a simple lens, of a telescope or of a microscope, is really complete if the eye, with its lens and retina, is missing. Can you add it and convince yourself that these devices really work? Challenge 167 ny

As mentioned, refraction is usually colour-dependent. For this reason (and also in order to compensate the other lens imaging errors called *aberrations*) microscopes or photographic cameras have *several* lenses, made of different types of glass. The different glass types compensate dispersion, which otherwise yields coloured image borders. The colour dependence of refraction in water droplets is also the basis of the rainbow, as shown below, Page 121 and refraction in ice crystals in the atmosphere is at the basis of the halos, the Sun pillars and the many other light patterns often seen around the Sun or the Moon in cold weather. Ref. 116

Also the human eye shows colour-dependent refraction, i.e., dispersion. Fortunately, the effect is small. Indeed, for the working of the eye, the curvature of the cornea is more important than the refractive power of the lens, because the lens is embedded in a medium with nearly the same index of refraction, thus limiting the effects of refraction. The small effects of colour-dependent refraction is not corrected in the eye, but in the brain. Therefore, the dispersion of the eye lens can be noticed if this correction by the brain is prevented, for example when red or blue letters are printed on a black back-

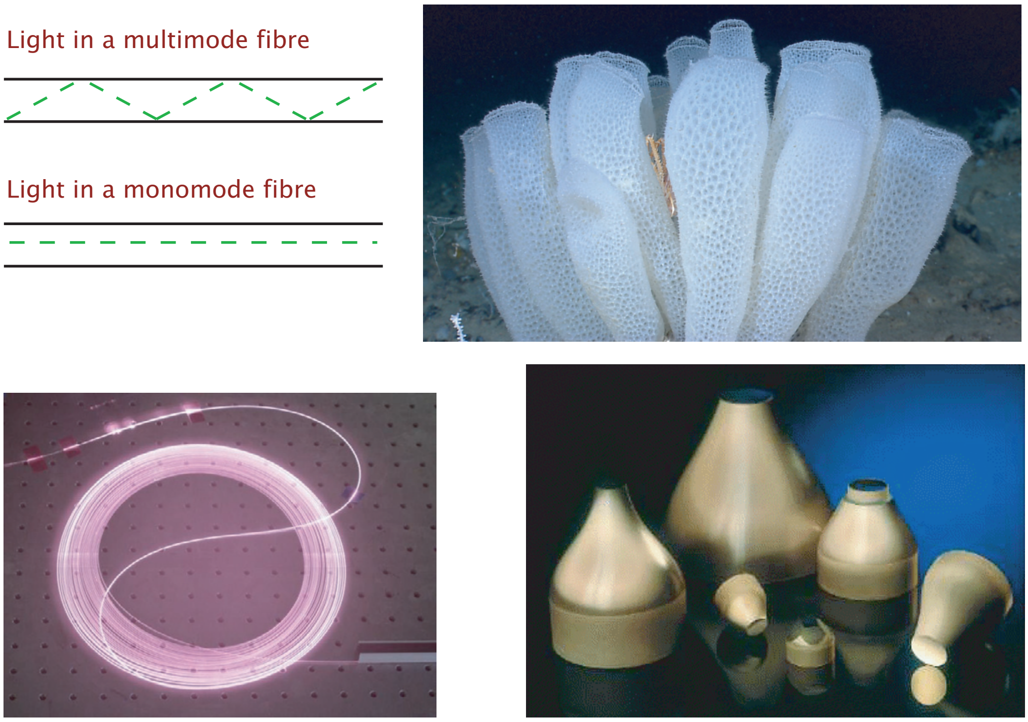

FIGURE 108 Optical fibres: the working principle of the two extreme fibre types, the astonishing marine sponge *Euplectella aspergillum* (height about 30 cm) that contains silica optical fibres with lenses at the end and synthesized at water temperature to help symbiotic algae, a modern fibre laser used in material processing and in medicine, and, glued together in large numbers, fibre tapers to change image sizes (maximum diameter about 20 cm) (© NOAA, Hochschule Mittweida, Schott).

ground, as shown in Figure 107. We get the impression that the red letters float in front Challenge 168 s of the blue letters. Can you explain how dispersion leads to this floating effect?

Bending light with tubes – fibre optics

Another way to bend light, also based on refraction, is used by many animals and by many technical devices: the *optical fibre*. Optical fibres are based on total internal reflection; an overview of their uses is given in Figure 108.

In nature, optical fibres appear in at least three systems. In insect eyes, such as the eyes Page 185 of the house fly or the eye of a honey bee, the light for each image pixel is transported along a structure that works as a conical optical fibre. In certain sea animals, such as the Ref. 117 glass sponge *Euplectella aspergillum* and a number of other sponges, actual silica fibres are used to provide structural stability and to transport light signals to photodetectors. Finally, all vertebrate eyes, including the human eye, contain a large number of optical fibres above the retina, to avoid the image problems that might be caused by the blood Ref. 118 vessels, which lie *above* the retina in all vertebrate eyes. By the way, the frequently heard Ref. 119 claim that the white hair of polar bears works as optical fibres for UV light is *false*.

In technical applications, optical fibres are essential for the working of the telephone network and the internet, for signal distribution inside aeroplanes and cars, for the trans-

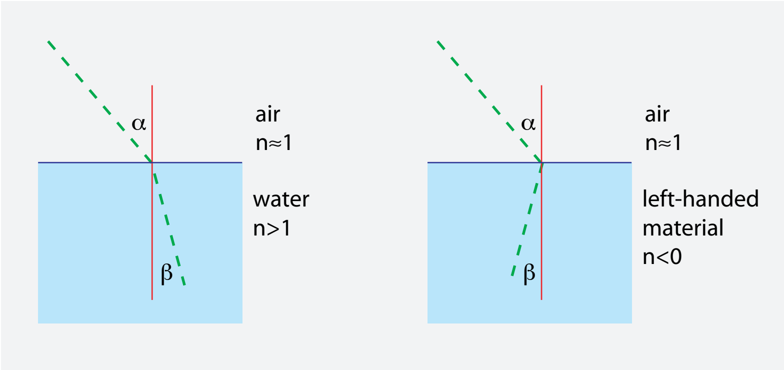

FIGURE 109 Positive and negative indices of refraction n.

port of laser light for medical uses, for high-power lasers and in many other settings. Hollow glass fibres are successfully used for the guiding of X-rays in X-ray imaging systems.

200 years too late – negative refraction indices

In 1967 the Soviet physicist Victor Veselago made a strange prediction: the index of refraction could have *negative* values without invalidating any known 'law' of physics. A negative index means that a beam is refracted to the same side of the vertical, as shown in Figure 109. As a result, concave lenses made of such materials focus parallel beams and convex lenses disperse them, in contrast to usual lens materials.

In 1996, John Pendry and his group proposed ways of realizing such materials. In 2000, a first experimental confirmation for microwave refraction was published, but it Ref. 120 met with strong disbelief. In 2002 the debate was in full swing. It was argued that negative refraction indices imply speeds greater than that of light and are only possible for either phase velocity or group velocity, but not for the energy or true signal velocity. The conceptual problems would arise only because in some physical systems the refraction angle for phase motion and for energy motion differ.

In the meantime, the debate is over. Negative indices of refraction have indeed been observed frequently; the corresponding systems are being extensively explored all over Ref. 121 the world. Systems with negative index of refraction do exist. Following Veselago, the materials showing this property are called *left-handed*. The reason is that the vectors of the electric field, the magnetic field and the wave vector form a left-handed triplet, in contrast to vacuum and usual materials, where the triplet is right-handed. All left-handed materials have *negative* magnetic permeability μ_r and *negative* dielectric coefficient, i.e., Ref. 122 negative permittivity ε_r. However, in actual systems, these properties are only realized for a narrow range of frequencies, usually in the microwave range.

Apart from the unusual refraction properties, left-handed materials have negative phase velocities, i.e., a phase velocity opposed to the energy velocity and show a reversed Doppler effect. These properties have been confirmed by experiment. Left-handed materials should also yield obtuse angles in the Vavilov–Çerenkov effect, thus emitting Vavilov–Çerenkov radiation in the backward instead of in the forward direction, they are predicted to have an inverted *Goos-Hänchen effect* and to show a repulsive Casimir effect. However, these predictions have not been verified yet.

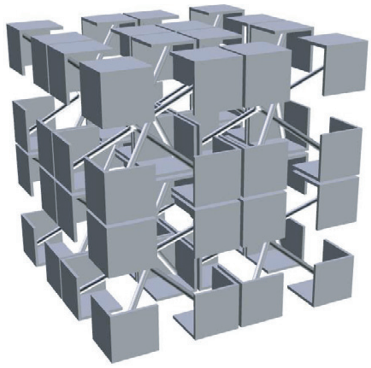

FIGURE 110 An example of an isotropic metamaterial (M. Zedler et al., © 2007 IEEE).

Most intriguing, negative index materials are predicted to allow constructing lenses that are completely flat. In addition, in the year 2000, John Pendry gained the attention of the whole physics community world-wide by predicting that lenses made with such materials, in particular for a refractive index $n = -1$, would be *perfect*, thus beating the usual diffraction limit. This would happen because such a perfect lens would also image the *evanescent* parts of the waves – i.e., the exponentially decaying ones – by amplifying them accordingly. First experiments claim to confirm the prediction. Exploration of the topic is still in full swing. Ref. 123 Ref. 122

So far, left-handed materials have been realized only for microwave and terahertz frequencies. First claims in the visible domain have been published, but have to be taken with care. It should be mentioned that one type of negative refraction systems have been known since a long time: diffraction gratings. We could argue that left-handed materials are gratings that attempt to work in all spatial directions. And indeed, all left-handed materials realized so far are periodic arrangements of electromagnetic circuits.

Metamaterials

The simplest realization of left-handed systems are metamaterials. *Metamaterials* are engineered metal-insulator structures with a periodicity below the wavelength of the radiation for which they are designed, so that the structure behaves like a homogeneous material. Metamaterials have negative or otherwise unusual permittivity or permeability properties in a certain wavelength range, usually in the microwave domain; some metamaterials are left-handed.

Ref. 124 Currently, there are two basic approaches to realize metamaterials. The first is to build a metamaterial from a large array of compact resonant substructures, such as inductor-capacitor (LC-) circuits or dielectric spheres. The second approach is to build a metamaterial from transmission lines. The latter approach has lower losses and a wider spectral range; an example for this type is shown in Figure 110. Comparing and exploring different realizations is subject of intense research. Ref. 125

Most metamaterials are conceived for microwaves or terahertz waves. Industrial applications of metamaterials are expected for antenna design; for example, an antenna dipole could be located just above a metamaterial and thus allowing to build flat directional antennas. Applications in terahertz technology might also arise.

Less serious workers in the field claim that *invisibility cloaks* can be realized with metamaterials. While this is a good marketing slogan to attract funding and get into newspapers, the dream is not realistic, due to inevitable signal losses in the materials, dispersion, refraction, finite cell size, the need for windows to observe the outside from inside and the impossibility to achieve invisibility for all wavelengths. So far, all aeroplanes that were claimed to be invisible even only for specific radar frequencies have turned out to be visible to radar after all. But sources of military funding are known to have only a distant relation to reality.

Metamaterials for sound and lower-frequency waves are also subject of research. Such acoustic or mechanical metamaterials have not found a technical application yet.

Light around corners – diffraction

Light goes around corners. This effect was called *diffraction* by Francesco Grimaldi, in his text *Physico-mathesis de lumine*, published in 1665. Grimaldi studied shadows very carefully. He found out what everybody now learns in secondary school: light goes around corners in the same way that sound does, and light diffraction is due to the wave nature of light. (Newton got interested in optics after he read Grimaldi; Newton then wrongly dismissed Grimaldi's conclusions.) Ref. 126

Because of diffraction, it is *impossible* to produce strictly parallel light beams. For example, every laser beam diverges by a certain minimum amount, called the *diffraction limit*. Maybe you know that the world's most expensive Cat's-eyes are on the Moon, where they have been deposited by the Lunokhod and the Apollo missions. Can you determine how wide a laser beam with minimum divergence has become when it arrives at the Moon and returns back to Earth, assuming that it was 1 m wide when it left Earth? How wide would it be on its return if it had been 1 mm wide at the start? In short, both diffraction and the impossibility of non-diverging beams confirm that light is a wave. Ref. 127 Challenge 169 s

Diffraction implies that there are no perfectly sharp images: there exists a *limit on resolution*. This is true for every optical instrument, including the eye. The resolution of the eye is between one and two minutes of arc, i.e., between 0.3 and 0.6 mrad. The limit is partly due to the finite size of the pupil. (That is why squinting helps to see more sharply.) In practice, the resolution of the eye is often limited by chromatic aberrations and shape imperfections of the cornea and lens. (Can you check the numbers and their interpretation by calculation? Is it true that the number of rods in the eye is tuned exactly to its resolution?) Therefore, for example, there is a maximum distance at which humans can distinguish the two headlights of a car. Can you estimate it? Challenge 170 d Challenge 171 s

Resolution limits also make it impossible to see the Great Wall in northern China from the Moon, contrary to what is often claimed. In the few parts that are not yet in ruins, the wall is about 6 metres wide, and even if it casts a wide shadow during the morning or the evening, the angle it subtends is way below a second of arc, so that it is completely invisible to the human eye. In fact, three different cosmonauts who travelled to the Moon performed careful searches and confirmed that the claim is absurd. The story is one of the most tenacious urban legends. (Is it possible to see the Wall from the space shuttle?) The largest human-made objects are the polders of reclaimed land in the Netherlands; they *are* visible from outer space. So are most large cities as well as the highways in Belgium at night; their bright illumination makes them stand out clearly from the dark side of the Ref. 128 Challenge 172 ny

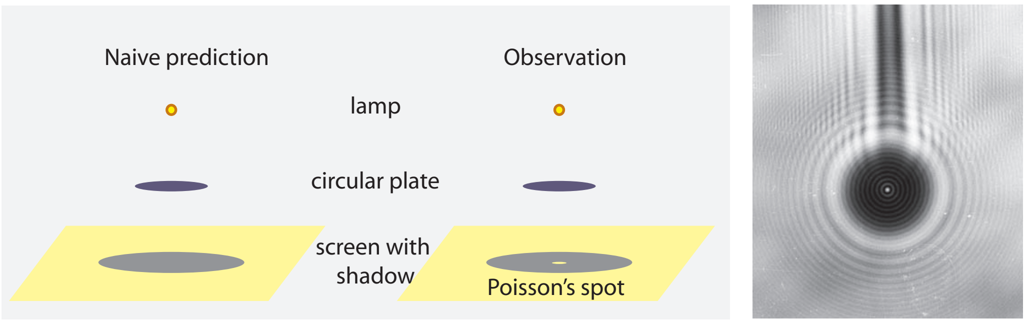

FIGURE 111 Shadows show that light is a wave: the naive expectation (left), neglecting the wave idea, and the actual observation (middle and right) of the shadow of a circular object (photo © Christopher Jones).

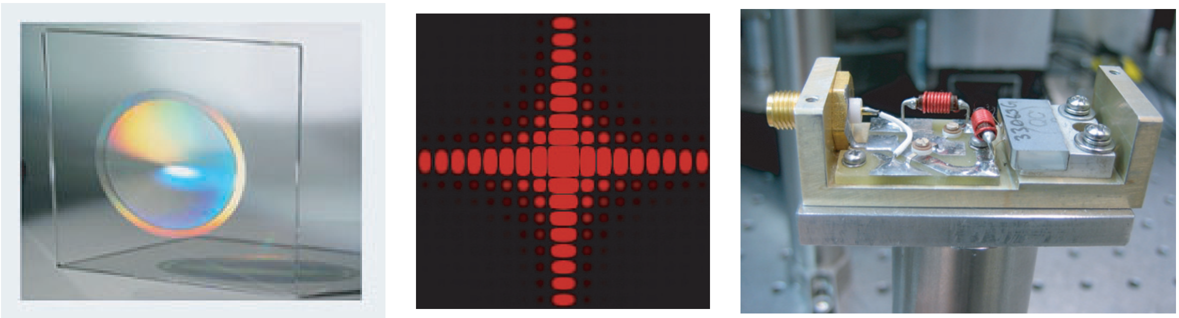

FIGURE 112 Examples of diffractive optics: a diffractive aspherical lens, the result shining a red laser through of a plastic sheet with a diffractive cross generator, and an acousto-optic modulator used to modulate laser beams that are transmitted through the built-in crystal (© Jenoptik, Wikimedia, Jeff Sherman).

Earth.

Diffraction has the consequence that behind a small disc illuminated along its axis, the centre of the shadow shows, against all expectations, a bright spot, as shown in Figure 111. This 'hole' in the shadow was predicted in 1819 by Denis Poisson (b. 1781 Pithiviers, d. 1840 Paris) in order to show to what absurd consequences the wave theory of light would lead. He had just read the mathematical description of diffraction developed by Augustin Fresnel* on the basis of the wave description of light. But shortly afterwards, François Arago actually observed Poisson's spot, converting Poisson, making Fresnel famous and starting the general acceptance of the wave properties of light.

Diffraction can also be used, in certain special applications, to produce images. A few examples of the use of diffraction in optics are shown in Figure 112. Of these, acousto-optic modulators are used in many laser systems, for example in laser shows. Also holograms, to be discussed in detail below, can be considered a special kind of diffractive

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* Augustin Jean Fresnel (b. 1788 Broglie, d. 1827 Ville d'Avray), engineer and part time physicist. The 's' in his name is silent. In 1818, he published his great paper on wave theory for which he got the prize of the French Academy of Sciences in 1819. To improve his finances, he worked in the commission responsible for lighthouses, for which he developed the well-known Fresnel lens. He died prematurely, partly of exhaustion due to overwork.

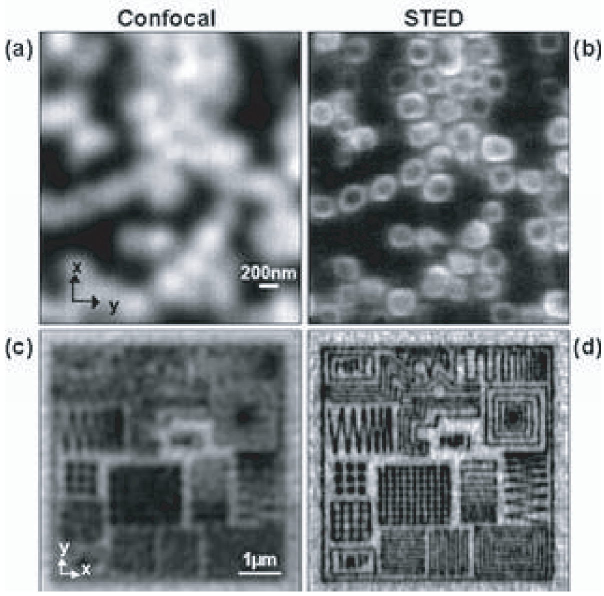

FIGURE 113 Sub-wavelength optical microscopy using stimulated emission depletion (right) compared to conventional confocal microscopy (left) (© MPI für biophysikalische Chemie/Stefan Hell).

images.

In summary, diffraction is sometimes used to form or to influence images; but above all, in every image, diffraction determines the resolution, i.e., the image quality.

Beating the diffraction limit

In all imaging methods, the race is for images with the highest resolution possible. The perfect lens mentioned above has not been realized for visible light. However, other techniques of producing images with resolutions *less* than the wavelength of light have made great progress in recent years.

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Nowadays, extraordinary images can be produced with modified commercial light microscopes. The conventional diffraction limit for microscopes is

$$d \geqslant \frac{\lambda}{2n \ \sin \alpha} , \tag{79}$$

where λ is the wavelength, n the index of refraction and α is the angle of observation. There are three main ways to circumvent this limit. The first is to work in the 'near field', where the diffraction limit is not valid, the second way is to observe and measure the diffraction effects and then to use computers to reduce the effects via image processing, the third way is to use effects that produces light emission from the sample that is smaller than the wavelength of light, and the fourth way is to use resolution in time to increase resolution in time.

A well-known near-field technique is the near-field scanning optical microscope. Light is sent through a tapered glass fibre with a small transparent hole at the end, down to 15 nm; the tip is scanned over the sample, so that the image is acquired point by point. These microscopes achieve the highest resolution of all optical microscopes. However, it

is hard to get a practical amount of light through the small hole at the end of the tip.

Many computational techniques can achieve images that achieve resolutions below the diffraction limit. The simpler types of these deconvolution microscopy techniques are already commercially available.

One of the first techniques that beat the diffraction limit by a substantial amount using a conventional microscope is *stimulated emission depletion microscopy*. Using a clever illumination system based on two laser beams, the technique allows spot sizes of almost molecular size. The new technique, a special type of fluorescence microscopy developed by Stefan Hell, uses an illuminating laser beam with a circular spot and a second laser beam with a ring-like shape. As a result of this combination, the techniques modifies the diffraction limit to

$$d \geqslant \frac{\lambda}{2n \ \sin\alpha \ \sqrt{I/I_{\text{sat}}}} , \qquad (80)$$

so that a properly chosen saturation intensity allows one to reduce the diffraction limit to arbitrary low values. So far, light microscopy with a resolution of 16 nm has been Ref. 129 performed. An example image is shown in Figure 113. This and similar techniques have galvanized the microscopy field; they are now commonplace in materials science, medicine and biology. In 2014, Stefan Hell received the Nobel Prize for Physics for his achievements.

Research in new microscopy techniques is still ongoing, also in the numerous attempts to transfer resolution in time to resolution in space. Another important domain of research is the development of microscopes that can be included in endoscopes, so that physicians can explore the human body without the need of large operations. Microscopy is still a field in full swing.

Other ways to bend light

Optical technology can be defined as the science of bending light. Reflection, refraction and diffraction are the most important methods to achieve this. But it makes sense to explore the question more generally: what other ways can be used to bend light beams?

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A further way to bend light is gravity, as discussed already in the chapters on universal gravity and those on general relativity. Since the effect of gravity is weak, it is only of importance in astronomy. Gravitational lensing is used in various projects to measure the size, mass and distance of galaxies and galaxy groups. The usually negligible effect of Vol. II, page 251 gravity between two light beams was also discussed earlier on.

In practice, there are thus no laboratory-scale methods to bend light beams apart from reflection, refraction and diffraction. All known methods are specialized cases of these three options.

An important way in which materials can be used to bend light are *acousto-optic de-* Page 163 *flectors*. They work like acousto-optic modulators, i.e., a sound wave travelling through a crystal generates a diffraction grating that is used to deflect a laser beam. Such modulators thus use diffraction to bend light.

Additional electromagnetic fields usually do not influence light directly, since light has no charge and since Maxwell's equations are linear. But in some materials the effective equations are non-linear, and the story changes. For example, in certain photorefractive

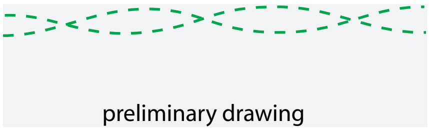

FIGURE 114 In certain materials, light beams can spiral around each other.

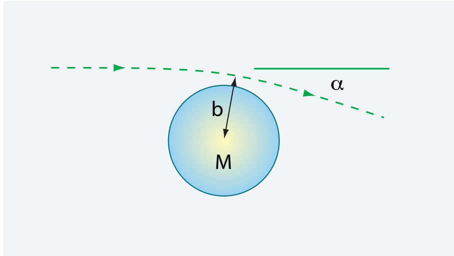

FIGURE 115 Masses bend light.

materials, two nearby light beams can even *twist* around each other, as was shown by Segev and coworkers in 1997. This is illustrated in Figure 114. This effect is thus a form of refraction.

Ref. 130

Another common way to deflect light uses its polarization. Many materials, for example liquid crystals or electro-optic materials, bend light beams depending on their polarization. These materials can be used to steer or even to block laser beams. Liquid crystal modulators and electro-optic modulators are thus based in refraction.

Scattered light also changes direction. It is debatable whether it is appropriate to call this process an example of bending of light. In any case, scattering is important: without it, we would not see almost anything around us. After all, everyday seeing is detection of scattered light. And of course, scattering is a case of diffraction.

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The next question is: what methods exist to *move* light beams? Even though photons have zero mass and electrons have non-zero mass, scanning electron beams is easily achieved with more than 1 GHz frequency, whereas scanning powerful light beams is hard for more than 10 kHz.

Moving light beams – and *laser beams* in particular – is important: solutions are the basis of a sizeable industry. Moving laser beams are used for laser treatments of the eye, for laser marking, for laser shows, for laser cutting, for barcode reading in supermarkets, for rapid prototyping, for laser sintering three-dimensional parts, for laser distance measurements, for lidar, for the mentioned microscopy techniques, and for various industrial processes in the production of electronic printed circuits, of semiconductor products, and of displays for mobile phones. Most laser scanners are based on moving mirrors, prisms or lenses, though acousto-optic scanners and electro-optic scanners, which achieve a few MHz scanning rate for low power beams, are also used in special applications. Many applications are eagerly waiting for inventions that allow faster laser scanning.

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In summary, moving light beams requires to move matter, usually in the form of mirror or lenses. Light travels in straight line only if it travels *far from matter*. In everyday life, ‘far’ simply means more than a few millimetres, because electromagnetic effects are negligible at these distances, mainly due to light’s truly supersonic speed. However, as we have seen, in some cases that involve gravitation, larger distances from matter are necessary to ensure undisturbed motion of light.

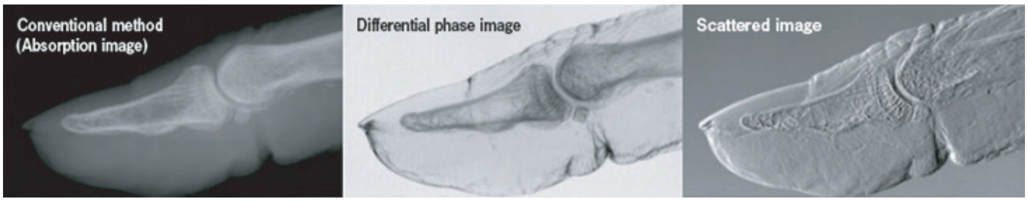

FIGURE 116 Three types of X-ray images of a thumb: the conventional image (left) and two images taken using interference effects (© Momose Atsushi).

Using interference for imaging

Page 101 As we saw above for the case of the guitar, images produced by interference can be useful. Above all, interference effects can be used to measure the deformation and the motion of objects.

Interference can also be used to enhance images. Figure 116 show the improvement that is possible when a special case of interferometer, a so-called *Talbot-Lau interferometer*, is used with X-rays. In particular, the technique increases the sensitivity of X-rays for soft tissue. Ref. 131

Interference is also at the basis of holography, an important technique to produce three-dimensional images.

How does one make holograms and other three-dimensional images?

Our sense of sight gives us an image of the world around us that includes the impression of depth. We constantly experience our environment as *three*-dimensional. Stereopsis, the experience of depth, occurs because of three main effects. First, the two eyes see *different images*. Second, the images formed in each eye are *position dependent*: when we move the head, we observe parallax effects between the bodies near and for from us. Third, for different distances, our eyes needs to *focus differently* and to *converge* more or less strongly, depending on the position of the object.

A usual paper photograph does not capture any of these three-dimensional effects: a paper photograph corresponds to the picture taken by one eye, from one particular spot and at one particular focus. In fact, all photographic cameras are essentially copies of a single, static eye with fixed focus. Challenge 173 e

Any system trying to produce the perception of depth for the observer must include at least one of the three three-dimensional effects just mentioned. In fact, the third effect, varying focus with distance, is the weakest one, so that most systems concentrate on the other two effects, different images for the two eyes, and an image that depends on the position of the head. Stereo photography and stereo films extensively use the first effect by sending two different images to the eyes, sometimes with the help of coloured glasses. Also certain post cards and computer screens are covered by thin cylindrical lenses that allow sending two different images to the two eyes, thus generating an impression of depth.

But obviously the most spectacular depth effect is obtained whenever position-dependent images can be created. Modern virtual reality systems produce this effect us-

FIGURE 117 The highest-quality holograms available in the world at present are produced by Yves Gentet and can be found on his website www.ultimate-holography.com. They are Denisjuk holograms. The viewer is tricked into thinking that there are real butterflies behind the glass pane. (© Yves Gentet).

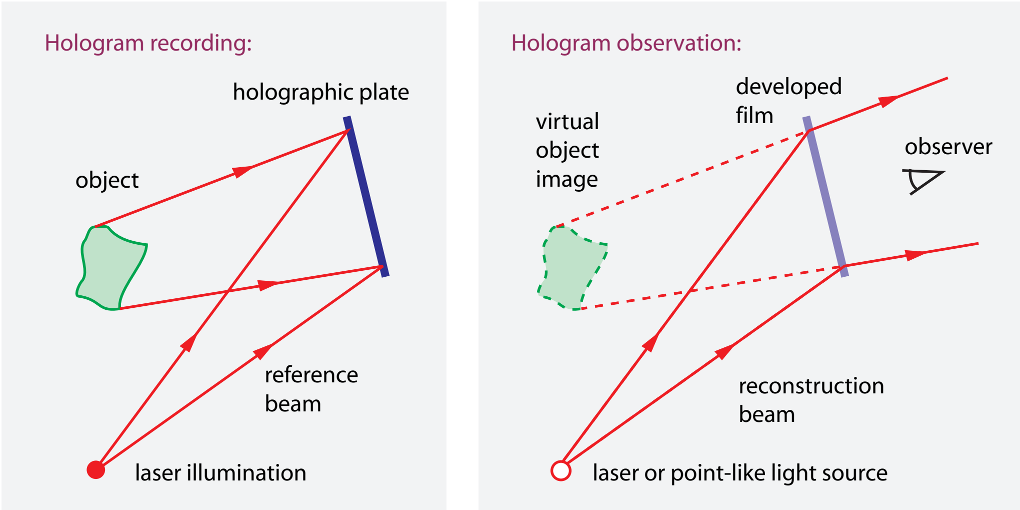

FIGURE 118 The recording (left) and the observation (right) of a monochromatic hologram (in this case, in transmission). True colour holograms use three lasers, for red, green and blue.

ing a sensor attached to the head, and creating computer–generated images that depend on the head's position. However, such systems are limited to computer graphics; they are not able to reproduce reality.

The only method that achieves all three depth effects is *holography*. The resulting images are called *holograms*. An example of a hologram is shown in Figure 117. Even though a hologram is only a film with a thickness of a fraction of a millimetre, the observer has the impression that there are objects behind it. Depending on the details of the geometry, objects can also seem to float in front of the film.

A hologram reproduces all data that is seen from any point of a region of space. A *hologram* is thus a stored set of position-dependent pictures of an object. In a first step, a hologram is captured by storing amplitude *and phase* of the light emitted or scattered by an object, as shown in Figure 118 and Figure 120. To achieve this storage of the whole light field, the object is illuminated by *coherent* light,* such as light from a laser, and the interference pattern between the illumination and the scattered light is stored; usually it is stored in a photographic film. The procedure is shown schematically in Figure 118. In a second step, illuminating the developed film by coherent light – from a laser or a lamp that is as point-like as possible – allows one to see a full three-dimensional image. In particular, due to the reproduction of the situation, the image appears to float in free space.

A few examples of holograms are shown in Figure 119. Holograms were developed in 1947 by the famous physicist Dennis Gabor (b. 1900 Budapest, d. 1979 London), who received the 1971 Nobel Prize for physics for this work. The beauty of Gabor's invention is that it was mainly theoretical, since lasers were not yet available at the time.

* Generally speaking, two light beams or two parts of one light beam – or other waves – are called *coherent* if they have constant phase difference and frequency. In practice, due to ubiquitous disturbances, this only happens over a certain finite volume, which is then called the *volume of coherence*. Coherence enables and is required for interference.

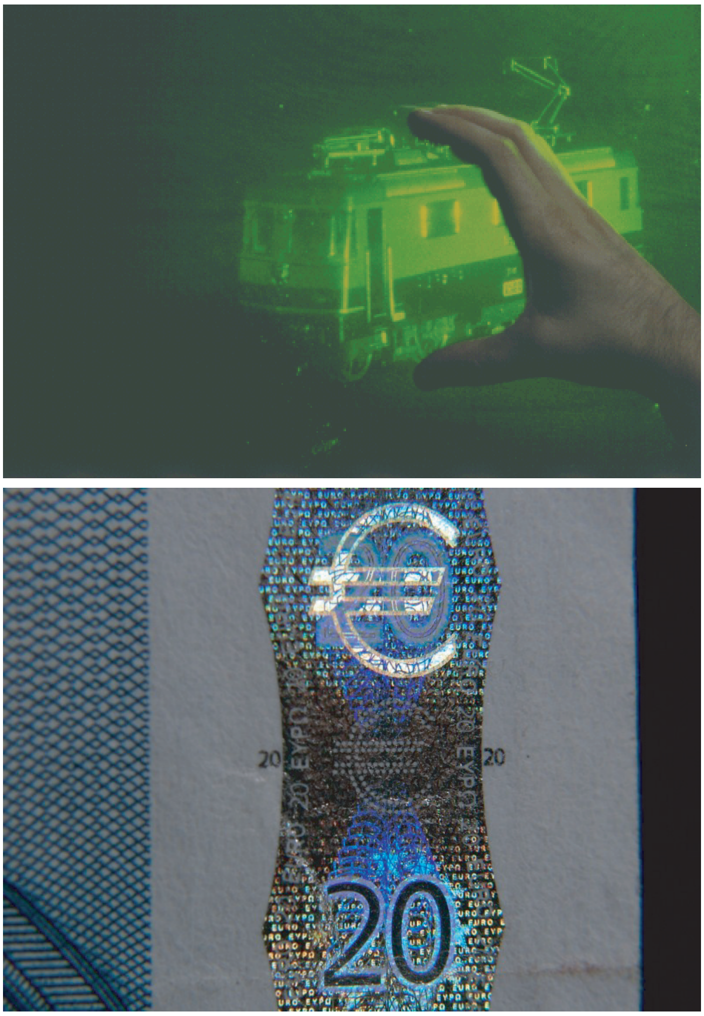

FIGURE 119 A hologram of a train and the reflection hologram on a Euro bill (© Anonymous, Hans-Ulrich Pötsch).

Holograms can be *transmission* holograms, like those in seen in museums, or *reflection* holograms, like those found on credit cards or currency bills. Holograms can be laser holograms and white light holograms. Most coloured holograms are rainbow holograms, showing false colours that are unrelated to the original objects. Real colour holograms, made and rendered with three different lasers, are possible but expensive.

Holograms are based on interference. Interference images can also be used in other ways. By a double illumination at two different times, one obtains a so-called *interferogram*, which allows visualizing and measuring the deformation of an object. Interferograms are used to observe and measure deformation, oscillation or temperature effects.

Is it possible to make *moving* holograms? Yes; however, the technical set-ups are still subject of research. So far, such systems exist only in a few laboratories (for example, www.optics.arizona.edu/pstg/index.html) and are expensive. By the way, can you describe how you would distinguish a high quality moving hologram from a real body

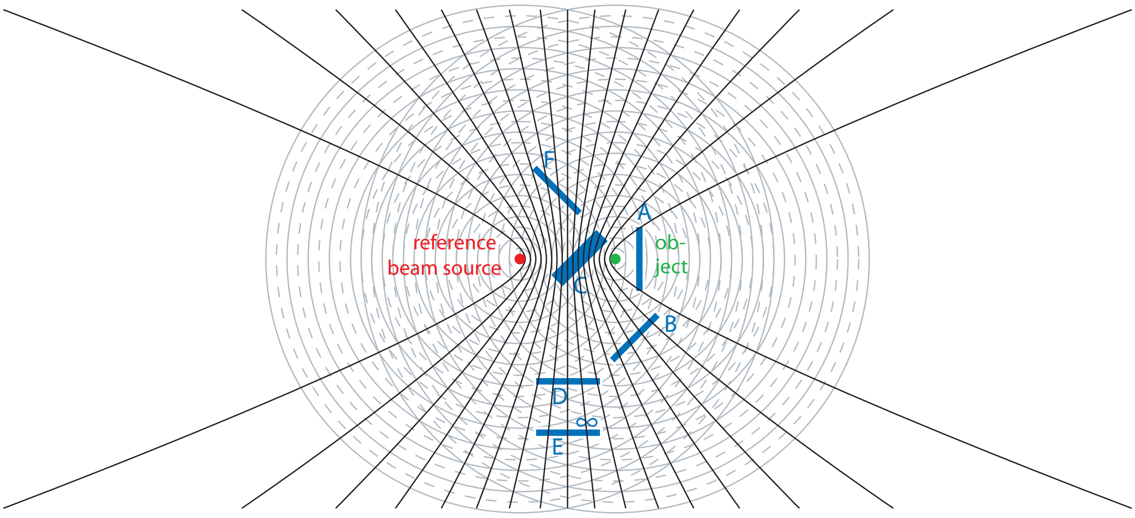

FIGURE 120 Different types of holograms arise through different relative position of object (green), holographic plate (blue) and reference beam (red). Situation A denotes a thin inline transmission hologram as proposed by Gabor, B a thin offline transmission hologram following Leith and Upatnieks, C a thick reflection hologram, or white light hologram, following Denisjuk, D a Fourier hologram at large distance, E Fraunhofer hologram at infinite distance and F two-dimensional hologram with inverted wave train (© DGH).

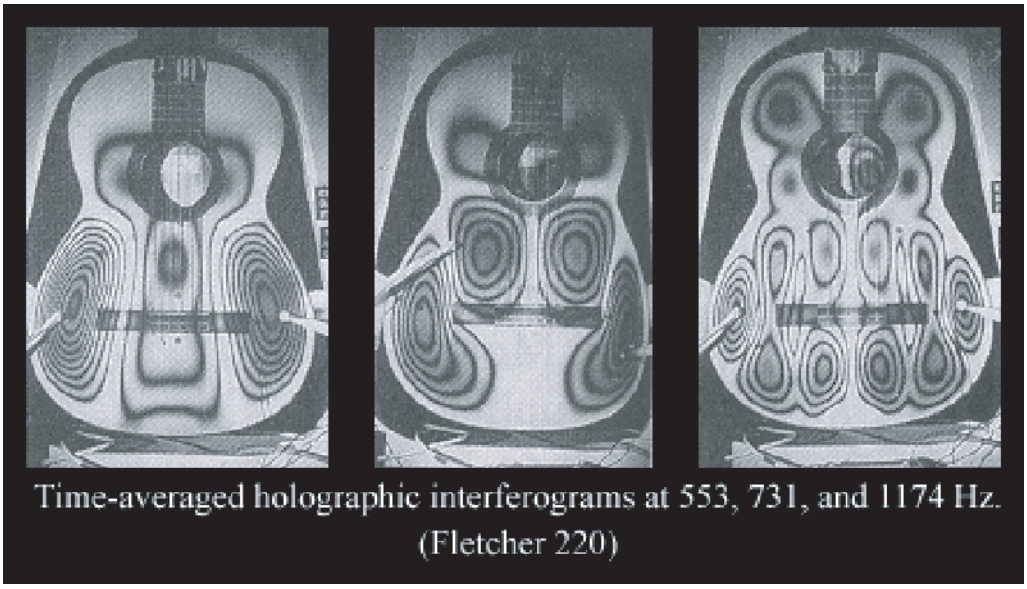

FIGURE 121 Interferograms of a guitar (© Wikimedia).

Challenge 174 s without touching it?

In the beginning of the computer industry, the aim of display makers was to produce *photo-realistic* displays, i.e., displays that could not be distinguished from a photograph. This aim has become reality. In 2012, a technology visionary proposed that the next aim of the industry should be to produce *window-realistic* displays, i.e., displays that cannot

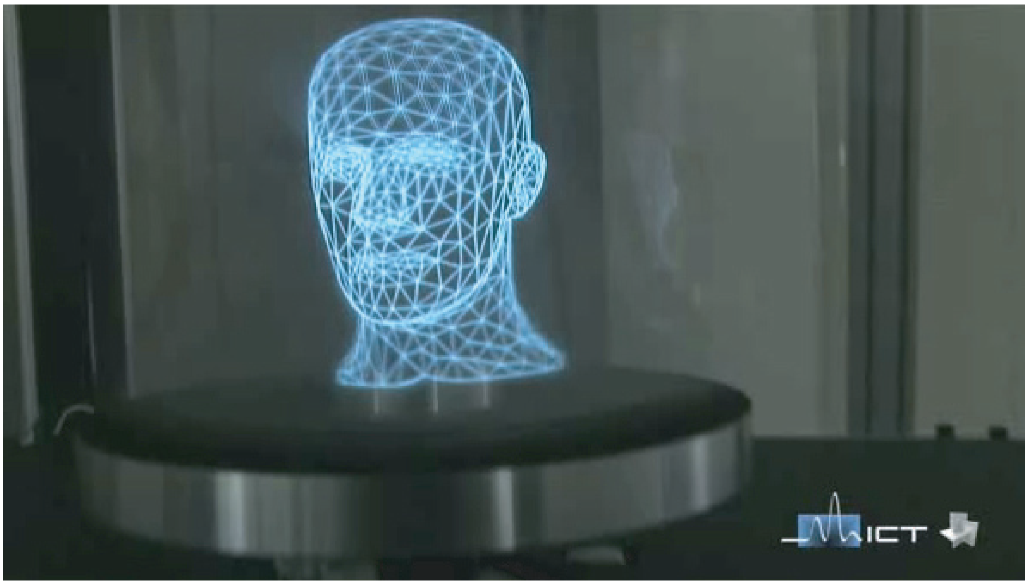

FIGURE 122 A three-dimensional image system based on a rotating mirror, from the University of Southern California, at gl.ict.usc.edu/Research/3DDisplay (© USC Stevens Institute for Innovation).

be distingusihed from a window. This should include the three-dimensionality of everything that is shown inside such a display. Will such a display ever be possible? Challenge 175 d

Not all three-dimensional images are holograms. Using rotating displays, rotating mirrors or rotating screens, it is possible to produce stunning three-dimensional images. An impressive example of such technology demonstrators is presented in Figure 122. Can you deduce why it was not a commercial success? Challenge 176 e

Images through scanning

When images are produced using lenses or mirrors, all the pixels of an image are produced in parallel. In contrast, in scanning techniques, images are constructed serially, pixels after pixels. Even though scanning is intrinsically slower than any parallel technique, it has its own advantages: scanning allows imaging in three dimensions and achieving resolutions higher than the diffraction limit. Scanning techniques are mainly used in microscopy.

The most famous scanning technique does not use light rays, but electrons: the *scanning electron microscope*. As shown in Figure 124, such microscopes can produce stunning images. However, the images produced are two-dimensional.

A typical example for a modern three-dimensional imaging technique based on light is *confocal laser scanning microscopy*. The technique is based on eliminating all light signals that are outside the focus of the microscope. The technique allows taking a picture of a more or less transparent specimen at a specified depth below its surface, up to a maximum depth of about 500 μm. Confocal microscopes are now available from various manufacturers.

An example of a technique for high-resolution is *multiphoton microscopy*. In this technique, the fluorescence of a specimen is excited using two or three photons of longer

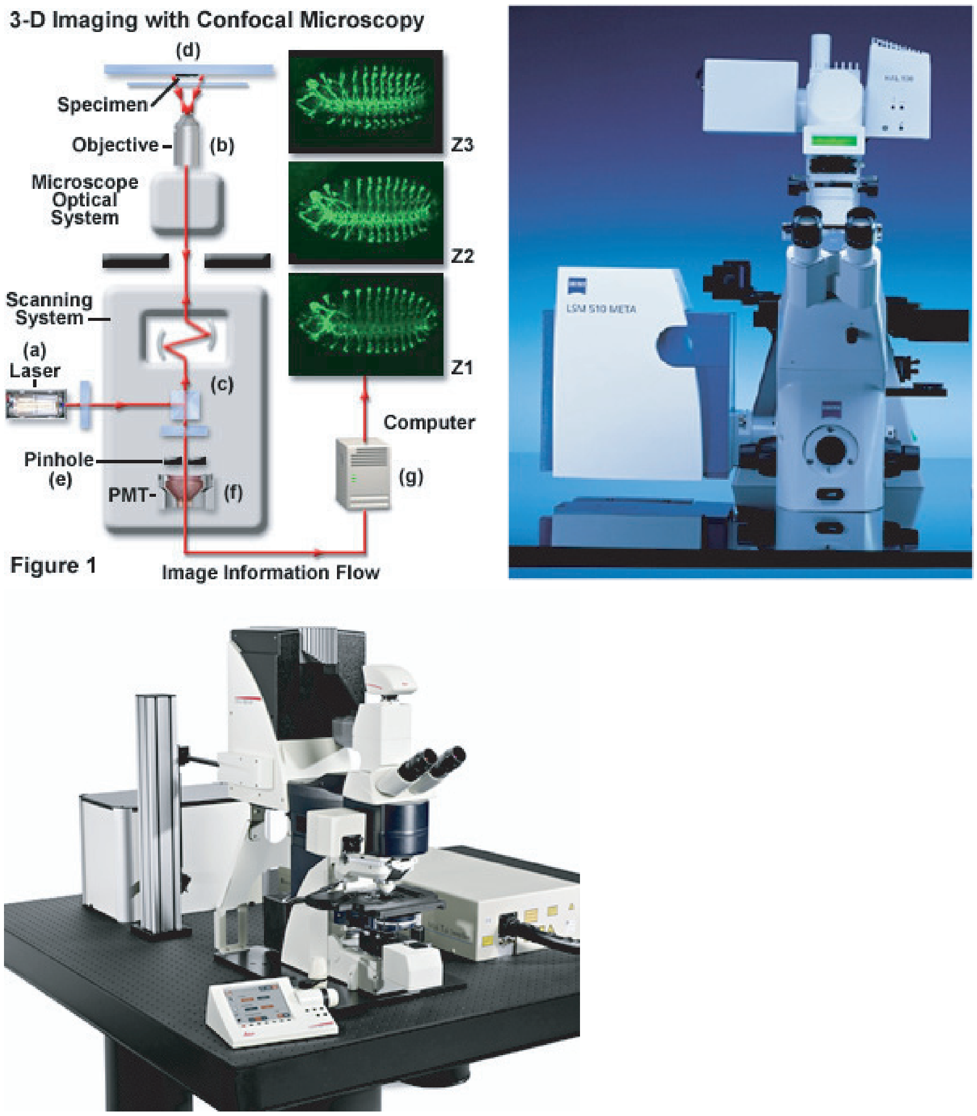

FIGURE 123 Two scanning imaging techniques: confocal laser scanning microscopy and multiphoton microscopy (© Nikon, Carl Zeiss).

FIGURE 124 A modern scanning electron microscope, and an image of pollen – field size about 0.3 mm – showing the resolution and the depth of field achievable with the technique (© Zeiss, Wikimedia).

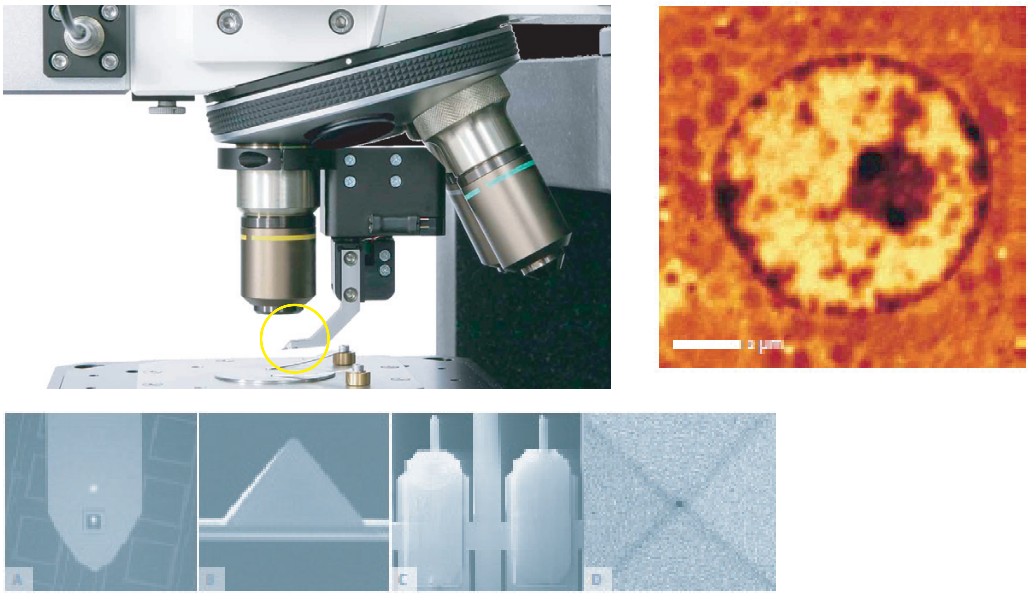

FIGURE 125 A scanning near-field optical microscope (SNOM) combined with an optical microscope, the details of the scanning probe, and an image of a liver cell nucleus produced with it (© WITec).

wavelengths. Like all fluorescence techniques, the image is produced from the fluorescent light emitted by certain chemical substances found in living organisms. In contrast to usual fluorescence microscopy, multiphoton imaging is based on a nonlinear effect, so that the emission region in extremely narrow and therefore high resolution is achieved.

For the highest possible optical resolution, *scanning near-field optical microscopy* is unsurpassed. Usually, a tiny optical probe is scanned across the surface, as shown in Figure 125. By working in the near field, the diffraction limit is circumvented, and resolution in the nanometre range becomes possible.

Another group of scanning microscopes also use electromagnetism to produce highest resolution images, though they do not use light. The most famous examples are the *scanning tunnelling microscope* or STM, the *atomic force microscope* or AFM and the *magnetic force microscope* or MFM. These instruments, though small and easy to build, have revolutionized material science in the last decades, because they allow to achieve atomic resolution in air on a normal laboratory table.

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In summary, technological advances nowadays allow sophisticated imaging systems based on scanning, in particular in the field of microscopy. Since the field is still in flux, scanning techniques are expected to yield even more impressive results in the coming years. This progress in scanning techniques reminds one of the past progress of a further type of imaging principle that reconstructs images in an even more involved way: tomography.

Tomography

A spectacular type of imaging has become possible only after high-speed computers became cheap: *tomography*. In tomography, a radiation source rotates around the object

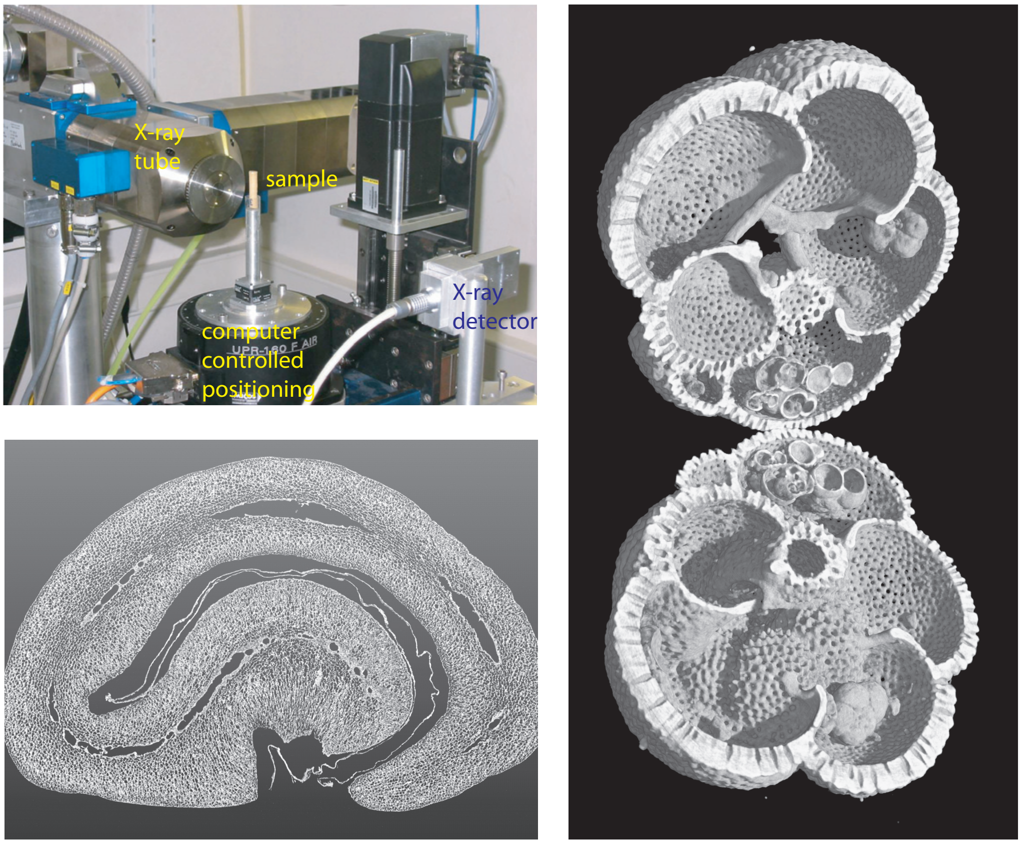


FIGURE 126 A set-up for high-resolution X-ray tomography, and two examples of images produced with it: a cross-section of a coffee bean (lower left) with a size of 8 mm, and a three-dimensional reconstruction of the exoskeleton of a foraminiferan, with a diameter of only 0.5 mm (© Manuel Dierick).

to be imaged; the radiation that is scattered and/or transmitted is detected, and with sophisticated computer programming, a cross section of the object is reconstructed. Three-dimensional reconstructions are also possible. Tomography can be performed with any type of radiation that can be emitted in sufficiently well-defined beams, such as gamma rays, X-rays, light, radio waves, electron beams, neutron beams, sound and even earthquakes. X-ray tomography is a standard method in health care; visible light tomography, which has no side effects on humans, is being developed for breast tumour detection. Additional specialized techniques are electrical resistivity tomography, magnetic induction tomography and cryo-electron tomography.

In several types of tomography, the resolution achieved is breath-taking. An example for modern high-resolution *X-ray tomography* of really small objects is shown in Figure 126. An example of X-ray tomography of a large object is shown in Figure 127. Building a set-up that produces such images is a large project and an impressive feat. Also magnetic resonance imaging, widely used in health care to image the interior of the human body, is a type of tomography, based on radio waves; it will be presented later on Vol. V, page 156 in our journey. Various types of tomographic systems – including *opto-acoustic tomography* based on sound produced by pulsed light, positron emission tomography, optical

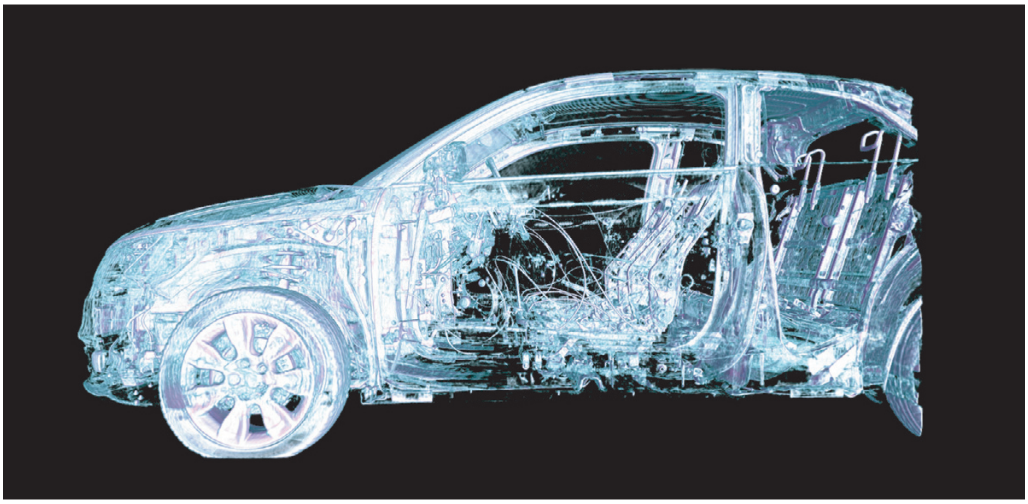


FIGURE 127 An X-ray CT image of a modern passenger car, with a resolution of less than 1 mm (© Fraunhofer IIS).

coherence tomography and the common sonography – also allow the production of film sequences.

An example of a technique that allows both three-dimensional imaging and high-resolution is *optical coherence tomography*. The technique is free of danger for the specimen, allows a depth of a few millimetres in animal or human tissue, and allows resolutions down to 500 nm. Modern systems allow imaging of 10 GVoxel/s and more, so that films of biological processes can be produced in vivo, such as the blood flow in a human finger. Using the Doppler effect, the direction of the blood flow can also be determined. Page 186 OCT is commonly used in ophthalmology; OCT is also being researched for applications in dermatology. *Endoscopic* OCT, i.e., performing OCT through a small catheter inserted into the human body, will become an important tool in oncology and cardiology in the near future. OCT is also being used in material research to image turbid media or to produce topographic profiles.

An unusual imaging method is *muon tomography*, an imaging method that uses the muons in cosmic rays to detect heavy metals in boxes, luggage and trucks. This method is particularly interesting for searching for hidden heavy metals, such as plutonium, which scatter muons much more strongly than other materials such as iron.

THE EYE AND THE BRAIN: BIOLOGICAL IMAGE ACQUISITION AND PROCESSING

Image processing systems acquire images and then exctract information from them. In technical image processing systems, the acquisition occurs with a camera and the extraction is realized with software running on a computer. An interesting image processing system is built into each of us: the combination of eye and brain. The eye and the brain are involved devices. We start by exploring the construction and performance of our eyes.

FIGURE 128 A limitation of the eye (see text).

Do we see what exists?

Sometimes we see *less* than there is. Close your left eye, look at the white spot in Figure 128, bring the page slowly towards your right eye, and pay attention to the middle lines. At a distance of about 15 to 20 cm the middle line will seem uninterrupted. Why? Challenge 177 s

Look with one eye at a full computer screen that is blinking blue and black, at a rate of once or twice a second. Now look at the same blinking screen through a blue filter (a Balzers K45 or a Kodak BG12 filter). You will see a spot. Why? Ref. 132 Challenge 178 s

Sometimes we see *more* than there is, as Figures 129 and 130 show. The first figure shows that parallel lines can look skewed, and the second show a so-called *Hermann lattice*, named after its discoverer.* The Hermann lattice of Figure 130, discovered by Elke Lingelbach in 1995, is especially striking. Variations of these lattices are now used to understand the mechanisms at the basis of human vision. For example, they can be used to determine how many light sensitive cells in the retina are united to one signal pathway towards the brain. The illusions are angle dependent because this number is also angle dependent. Ref. 133

Our eyes also 'see' things *differently:* the retina sees an *inverted* image of the world. There is a simple method to show this, due to Helmholtz.** You need only a needle and a piece of paper, e.g. this page of text. Use the needle to make two holes inside the two letters 'oo'. Then keep the page as close to your eye as possible, look through the two holes towards the wall, keeping the needle vertical, a few centimetres behind the paper. You will see two images of the needle. If you now cover the *left* hole with your finger, the

* Ludimar Hermann (b. 1838 Berlin, d. 1914 Königsberg) was an important physiologist. The lattices are often falsely called 'Hering lattices' after the man who made Hermann's discovery famous.

** See Hermann von Helmholtz, *Handbuch der physiologischen Optik*, 1867. This famous classic is available in English as *Handbook of Physiological Optics*, Dover, 1962. Physician, physicist and science politician, born as Hermann Helmholtz (b. 1821 Potsdam, d. 1894 Charlottenburg), was famous for his works on optics, acoustics, electrodynamics, thermodynamics, epistemology and geometry. He founded several physics institutions across Germany. He was one of the first to propagate the idea of conservation of energy. His other important book, *Die Lehre von den Tonempfindungen*, published in 1863, describes the basis of acoustics and, like the Handbook, is still worth reading.

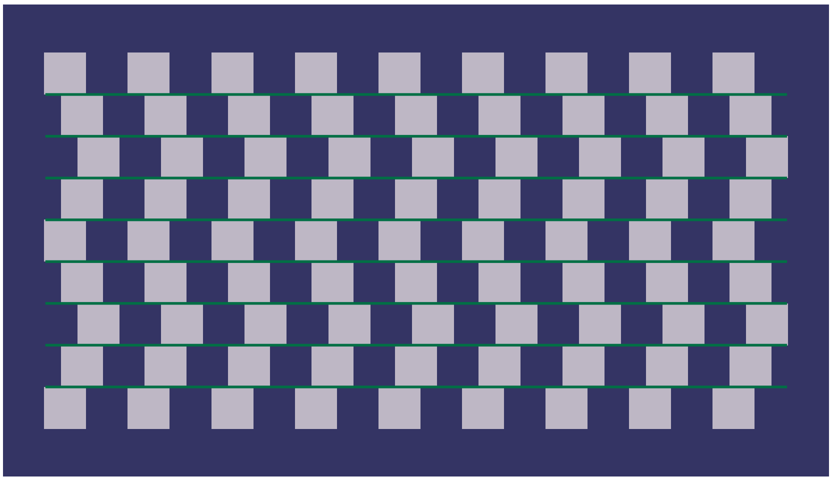


FIGURE 129 What is the angle between the thin lines between the squares?

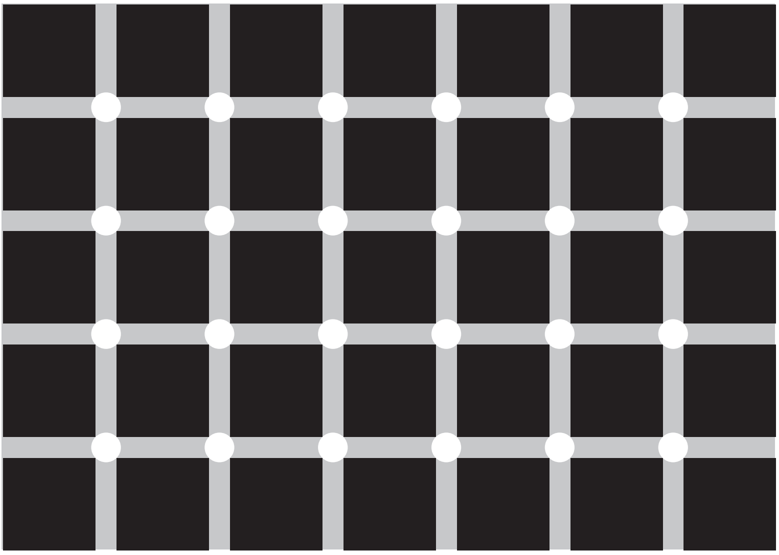


FIGURE 130 The Lingelbach lattice: do you see white, grey, or black dots at the crossings?

right needle will disappear, and vice versa. This shows that the image inside the eye, on the retina, is inverted. Are you able to complete the proof?

Challenge 179 ny

An urban legend, spread by many medical doctors and midwives to this day, claims that newborn babies see everything upside down. Can you explain why this idea is wrong?

Challenge 180 s

Two additional experiments can show that retinas acquire inverted images. If you push very lightly on the *inside* of your eye (careful!), you will see a dark spot appear on the *outside* of your vision field. And if you stand in a dark room and ask a friend to look at a burning candle, explore his eye: you will see three reflections: two upright ones, reflected from the cornea and from the lens, and a dim third one, *upside-down*, reflected from the retina.

FIGURE 131 An example of an infrared photograph, slightly mixed with a colour image (© Serge Augustin).

FIGURE 132 How the appearance of a sunflower changes with wavelength: how it looks to the human eye, how it might look to a bird, and how it looks in the ultraviolet (© Andrew Davidhazy).

Our eyes do not produce a faithful image of nature: they have a limited wavelength sensitivity. This sensitivity peaks around 560 nm; outside the red and the violet, our eyes does not detect radiation. We thus see only part of nature. As a result, infrared photographs of nature, such as the one shown in Figure 131, are interesting because they show us something different from what we see usually. The same happens to ultraviolet Ref. 134 photographs, as shown in Figure 132. Also images of the sky differ with wavelength; the website www.chromoscope.net shows this in detail.

The eye sees most sharply in the region of the fovea. But the highest light sensitivity is not in that region. As a result, we often do not see faint stars at night when we look directly at them, but see them when we look *next* to them. This effect is due to the peculiar distribution of rods, which has its peak density 20°away from the axis of sharpest vision.

Several other optical illusions are found throughout this text. In summary, we have to be careful whenever we maintain that seeing means observing. Our sense of vision is limited. Are there other limitations of our senses which are less evident? Our adventure

will indeed uncover several of them. But let us now turn to see what the eye *can* do.

The human eye

The human eye is a so-called *camera eye*. Like a photographic camera, and in contrast to insect eyes and other compound eyes, the vertebrate camera eye works by producing an image of the outer world on a surface consisting of light sensors, the retina. The retina covers more than half of the inside of the eye ball, whose typical diameter in an adult is about 16.7 mm. The pupil has a diameter between 2 mm – below which one gets problems with diffraction – and 7 mm – for which lens aberrations are just acceptable. The image on the retina has low image distortion, low chromatic aberrations (about 1 dioptre between red and blue) and low coma; the eye achieves this performance by using an deformable aspheric gradient-index lens and a cornea whose shape is always near the ideal shape within 30 μm – an extremely good value for a deformable body. The eye, together with the brain, also has a powerful autofocus – still not fully understood – and an excellent motion compensation and image stabilization system built in. A section of this amazing device is shown in Figure 133.

The retina is an outgrowth of the brain. It contains 120 million *rods*, or black and white pixels, and 6 million *cones*, or colour pixels. Each pixel can detect around 300 to 500 intensity levels (9 bit). The eye works over an intensity range of 8 to 10 orders of magnitude; the involved mechanism is incredibly complex, takes place already inside the receptors, involves calcium ions, and is fully known only since a few years. The region of highest resolution, the fovea, has an angular size of about 1°. The resolution of the eye is about 1′. The integration time of the retina is about 100 ms – despite this, nothing is seen during the saccades. The retina itself is 200 μm thick and is transparent: this means that all cables leading to the receptors are transparent as well.

The retina has very low energy consumption and uses a different type of neurons that usual nerves: instead of using spikes, the neurons in it use electrotonic potentials, not the action potentials or spikes used in most other nerves, which would generate interferences that would make seeing impossible. In the fovea, every pixel has a connection to the brain. At the borders of the retina, around 10 000 pixels are combined to one signal channel. (If all pixels were connected 1 to 1 to the brain, the brain would need to be as large as a typical classroom.) As a result, the signals of the fovea, whose area is only about 0.3 % of the retina, use about 50 % of the processing in the brain's cortex. To avoid chromatic aberrations, the fovea has no blue receptors. The retina is also a graphic preprocessor: it contains three neuronal layers that end up as 1.3 million channels to the cortex, where they feed 5 million axons that in turn connect to 500 million neurons.

The compression methods between the 125 million pixel in the retina and the 1.3 million channels to the cortex is still subject of research. It is known that the signals do not transport pixel data, but data streams processed in about a dozen different ways. The streams do not carry brightness values, but only contrasts, and they do not transmit RGB values, but colour differences. The streams carry motion signals in a compressed way and the spatial frequency data is simplified. Explorations have shown how the ganglions in
Ref. 135 the retina provide a navigational horizon, how they detect objects moving against the background of the visual field, and how they subtract the motion of the head. The coming years and decades will provide many additional results; several data channels between

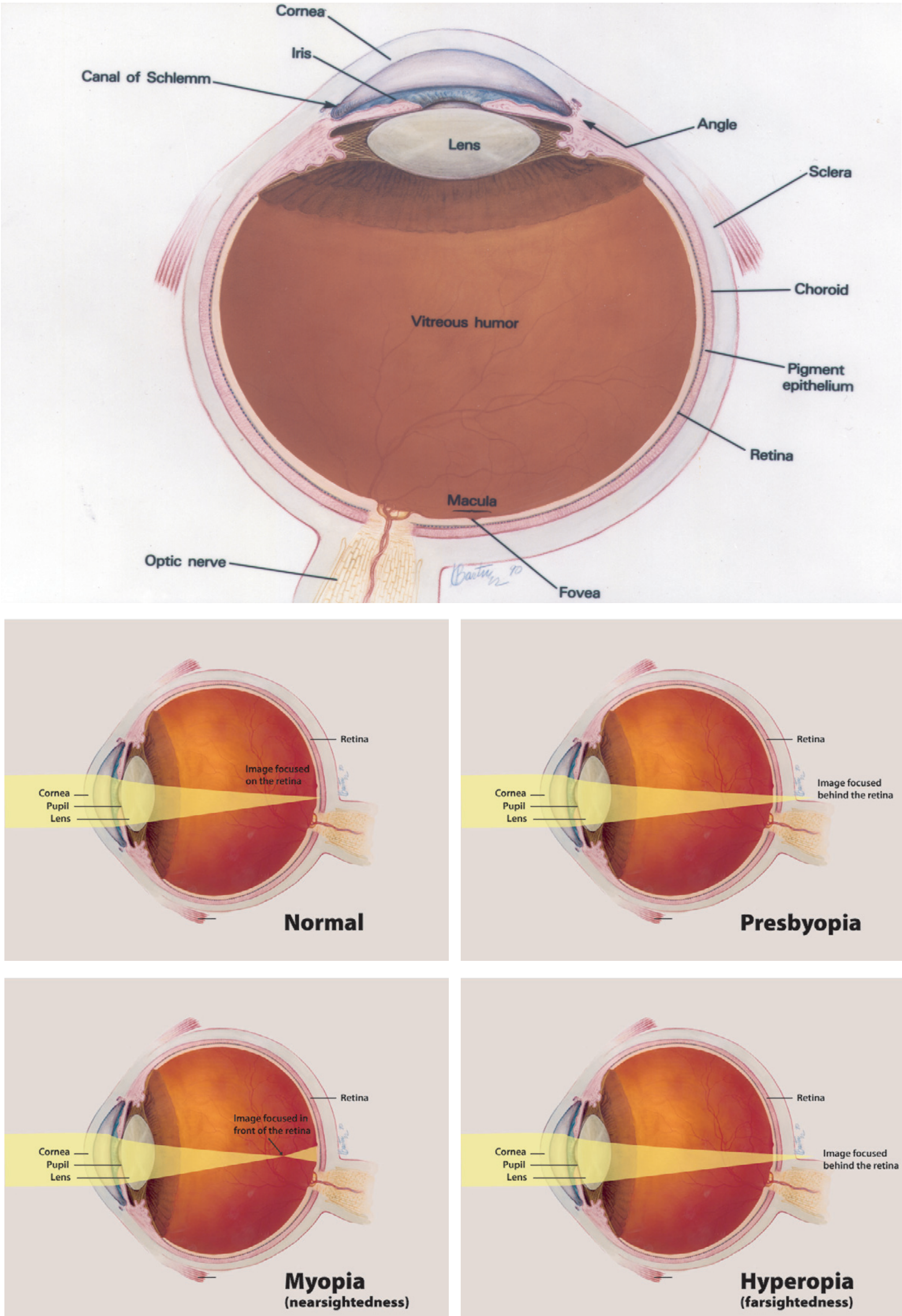


FIGURE 133 Top: a simplified cross section of the human eye; bottom: the comparison of the optical imaging for a healthy eye and for the most common eye problems, myopia, hyperopia and presbyopia (© NEI at NIH).

the eye and the brain are still unknown.

Apart from rods and cones, human eyes also contain a third type of receptor. This receptor type, the *photosensitive ganglion cell* or *intrinsically photosensitive retinal ganglion cell*, has only been discovered in the early 1990s, sparking a whole new research field. Photosensitive ganglion cells are sensitive mainly to blue light, use melanopsin as photopigment and are extremely slow. They are connected to the suprachiasmatic nucleus in the brain, a small structure of the size of a grain of rice that controls our circadian hormone cycle. For this reason you should walk a lot outside, where a lot of blue light is available, in order to reset the body's clock and get rid of jet-lag. Photosensitive ganglion cells also produce the signals that control the diameter of the pupil.

Ref. 136

It is worth recalling that drawings such as the one of Figure 133 are *simplified*. They do not show the structures in the transparent part of the eye, the vitreous body, such as the hyaloid canal, which plays an important role during the growth of the eye in the embryo stage. In fact, the growth of the eye inside the womb is even more amazing than its actual function – but this story is outside the scope of this text.

Interestingly, the eye is the part that moves most frequently in the human body – more than the heart. It is estimated that the eye performs 200 million saccades every year – without counting the tiny eye movements. Therefore, the motion and lubrification mechanisms of the eye are especially involved.

Human versus other eyes

The human eye and many other animal eyes are better devices than most modern photographic or video cameras. Not only does it have more pixels than most cameras, it is also insensitive to pixel errors, to the blood vessels in front of the sensors. No camera covers the same range of intensity variation. No human-made camera has a lens system of comparable quality or capabilities: the large viewing angle, the low field distortions – also due to the spherical shape of the retina – and the low chromatic aberrations. No technical autofocus system, image stabilizer or motion compensation system matches that of the eye.

One limitation of the eye is its speed. The human eye produces an effective number of 30 images per second and up to 120 images per second under the most ideal conditions; dogs and birds achieve twice the basic rate and insects about ten times as much. Modern video cameras can produce more than 10 000 images per second. When developing the eye, evolution has traded speed for resolution. To achieve high resolution, the eye continuously performs small movements, called *micronystagmus*. In detail, the eye continuously oscillates around the direction of vision with around 40 to 50 Hz; it constantly averages an image pixel over 30 to 50 receptors, but the exact sharpening mechanism is not clear yet. This motion increases the effective number of pixels, avoids issues with dead pixels and also allows the rods and cones to recharge.

All vertebrate eyes have *rods*, the pixel types that produce black and white images at night. Additionally, the retina of the human eye contains three types of cones, for the colours red, green and blue. As mentioned, much better eyes are found in birds, many reptiles and fish: they have four or more types of cones, built-in colour filters and an ultraviolet-transparent lens. The fourth type of cones and the special eye lens make the eyes of birds and reptiles sensitive to near-ultraviolet light; birds use their ultraviolet

sense to find food and to distinguish males from females. Indeed, most birds whose males and females look the same to humans differ markedly in the ultraviolet.

Birds and reptiles also have coloured oil droplets built into the top of their cones, with each cone type containing a different oil colour. These droplets act as colour filters. In this way, the spectral resolution of their cones is much sharper than in mammals. The sense of colour in birds is much more evolved than in humans – it would be fascinating to watch the world with a bird's eye. Birds are the best colour seers overall. They have cone receptors for red, blue, green, ultraviolet, and, depending on the bird, for up to three more sets of colours.

Eagles and a number of other birds (but not many) also have a better eye resolution than humans. They achieve this in two ways. First, their photoreceptors are small; in other words, their pixel size is the smallest known with respect to the eye diameter, with only 1.6 μm. Secondly, the eye includes bones. These bones fix the relative position of lens and retina, like a rigid camera body. With these technical solutions, the eye of the eagle is clearly better than that of humans.

In the course of evolution, the eye of mammals lost two types of cones that were part of the vertebrate heritage, and were left with only two types of cones. The (Old-World) primates later regained one type, in order to distinguish more clearly tree fruit, which are so important as food for the primate brain, from the surrounding leaves. But despite this change, primates never reached the capability of the best bird's eyes. Thus, of all mammals, only primates can see full *colours* as human do. Bulls for example, don't; they cannot distinguish red from blue.

Usual humans are thus *trichromatic*: they have three types of cones that detect red, green and blue. However, around 1 % of women are (somewhat) *tetrachromatic*. This is possible because humans can have two different red pigments. The red pigment details are encoded on the X chromosome. Now, in some women, the two X chromosomes code for two different red pigments. In a part of these women, both pigments are found in the cones of their eyes. These women thus seem to have something like RR'GB eyes. Tests showed that they can distinguish more red shades than men and than most other women. Ref. 137

Every expert of motion should also know that the highest sensitivity of the human eye does *not* correspond to the brightest part of sunlight. This myth has been spread around the world by the numerous textbooks that have copied from each other. Depending on whether frequency or wavelength or wavelength logarithm is used, the solar spectrum peaks at 500 nm, 880 nm or 720 nm. The human eye's spectral sensitivity, like the completely different sensitivity of birds or frogs, is due to the chemicals used for detection. In short, the human eye can only be understood by a careful analysis of its particular evolutionary history. Ref. 138

Camera eyes are found in all vertebrates. Mammals have eyes similar to ours, with a flexible lens; in contrast, snakes have eyes with rigid lenses that are moved with respect to the retina in order to put images into focus. Camera eyes evolved independently several times in other animal groups. Most known are the cephalopods, such as the octopus, and indeed, the largest eyes known, up to 30 cm in diameter, are from animals of this group. Camera eyes are also found in some spiders, in snails and in a number of other groups.

By the way, the human eye–brain system processes colours mainly around the direction of gaze. This allows a fun trick: if a vision system follows the direction of your gaze, it can command a computer display to show colours only in the display region at which

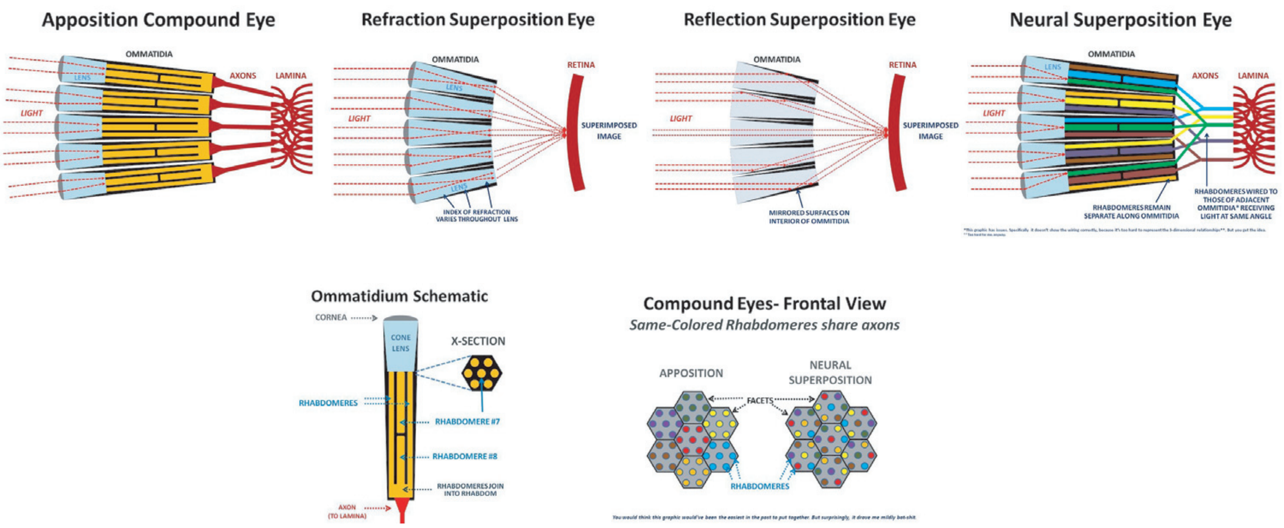


FIGURE 134 Compound eyes: the apposition compound eye found in bees and dragonflies, the refraction superposition eye of moths, the reflection superposition eye of lobsters, (not shown: the parabolic superposition eye of certain crabs) and the neural superposition eye of the house fly (© Watcher, from watchingtheworldwakeup.blogspot.com).

you are looking to, and to leave the rest of the picture in black and white. If the command system is fast enough, you get the impression that the whole picture is coloured, whereas every bystander sees that the picture is mainly black and white, and just shows colours in a spot that is constantly moving around.

The most common eyes in nature are not camera eyes, but *compound eyes*, as found in bees, dragonflies or house flies. Compound eyes have one lens for each axon. These units are usually hexagonal in shape are called *ommatidia* and typically contain a handful of photoreceptors that are connected to the outgoing axon. An ommatidium is a tiny eye; depending on the species, a compound eyes consist of at least a hundred and at most 30 000 ommatidia (for some dragonflies). Many compound eyes are also tetra- or pentachromatic. Compound eyes have low resolution – is suspected that no insect can see the stars – but the have a number of advantages. Compound eyes need no focussing mechanism, can cover a large field of view, and above all, they are extremely fast. These advantages are so interesting that compound-eye-style electronic cameras are also being explored as alternatives to usual, one-lens-plus-one-sensor cameras.

Using ideas from insect eyes is also interesting for other uses. For example, modern technology provides the possibilities to think anew how a microscope should look like. Figure 135 shows a microscope that is in fact an array of thousands of tiny microscopes.
Ref. 139 The lenses produce images on a CMOS imaging chip with 16 megapixel.

In summary, the microscopic structures inside the eye are important and fascinating. But here we face a question.

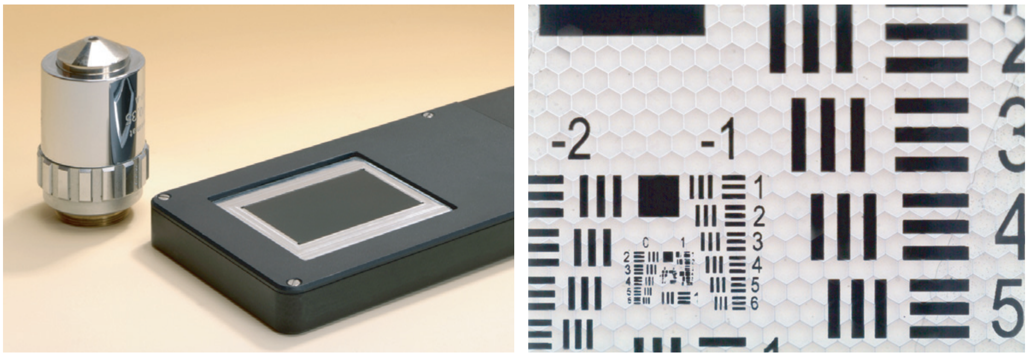


FIGURE 135 A flat microscope based on stacked microlens arrays – in front of a conventional objective – and an image it produces (© Frank Wippermann).

FIGURE 136 Top: an image of the front of the human eye acquired by optical coherence tomography, showing the cornea, the iris and the lens. Bottom: a typical apparatus used by ophthalmologues (© XXXX, Heidelberg Engineering).

How can we make pictures of the inside of the eye?

When we look through a small hole towards a bright surface, we can see the blood vessels in our eye. In particular, we can see that the fovea has no blood vessels at all. But how can we observe other people's microscopic eye structure?

Imaging the details inside of a living eye is not easy. The retina is far away from the surface of the eye, so that a normal microscope cannot be used. In addition, the continuous motions of the lens and of the eye itself disturb any imaging system. Finally, two separate developments changed the situation in the 1990s.

Page 177 The first breakthrough in eye imaging was the technique, mentioned above, of *optical*

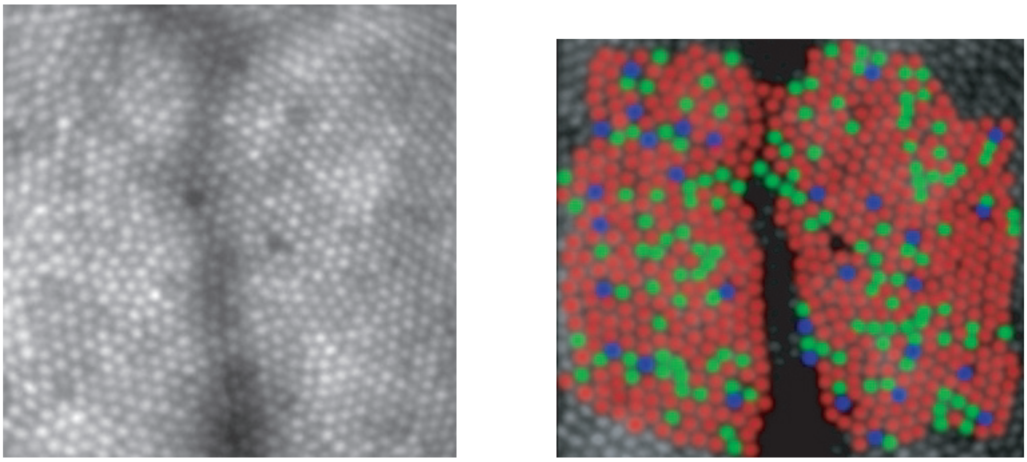


FIGURE 137 A high quality photograph of a live human retina, including a measured (false colour) indication of the sensitivity of each cone cell (© Austin Roorda).

coherence tomography. This imaging method uses a scanned low-power laser beam and allows imaging scattering media up to a depth of a few millimetres with a resolution of the order of a few µm. This microscopy technique, developed in the 1990s, allows observing in detail the retina of the human eye and the region below it; it also allows cross sections of the cornea and the lens. Through the detailed pictures it provides in a few milliseconds, shown in Figure 136, optical coherence tomography allows extremely precise diagnoses; it has profoundly changed modern ophthalmology.

Optical coherence tomography also allows imaging the skin to a depth of about 8 mm; this is already improving skin cancer diagnosis. In the future, the technique will also simplify cancer diagnosis for gynaecologists and otolaryngologists. Endoscopic systems are also being developed. Optical coherence tomography is becoming standard also in various industrial applications.

The second breakthrough in eye imaging was the technique of *adaptive optics*, a technique, also used in astronomy, that continuously and quickly changes the shape of the imaging lens. The most beautiful pictures so far of a *living* human retina, such as that of Figure 137, were made by the group of David Williams and Austin Roorda at the University at Rochester in New York using this modern technique. They used adaptive optics in order to compensate for the shape variations of the lens in the eye of the patient.

Ref. 140

The human eye produces the sensation of colour by averaging the intensity arriving at the red, blue and green sensitive cones. This explains the possibility, mentioned above, of getting the same impression of colour, e.g. yellow, either by a pure yellow laser beam, or by a suitable mixture of red and green light.

Page 121

But if the light is focused on to one cone only, the eye makes mistakes. Using adaptive optics it is possible to focus a red laser beam such that it hits a green cone only. In this case, something strange happens: even though the light is *red*, the eye sees *green* colour!

Incidentally, Figure 137 is quite puzzling. In the human eye, as in all vertebrate eyes, the blood vessels are located *in front* of the cones. Why don't they appear in the picture? And why don't they disturb us in everyday life? (The picture does not show the other type of sensitive light cells, the *rods*, because the subject was in daylight; rods come to

Challenge 181 s

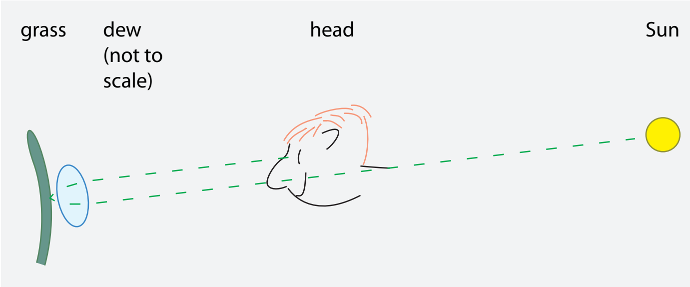


FIGURE 138 The path of light for the dew on grass that is responsible for the aureole or Heiligenschein, and a photo showing that it is seen only around one's own head (© Bernt Rostad).

the front of the retina only in the dark, and then produce black and white pictures.)

Ref. 141 In summary, evolution has provided us with an observations system that has amazing properties. Take good care of your eyes.

How to prove you're holy

Light reflection and refraction are responsible for many striking effects. The originally Indian symbol for a holy person, now used throughout most of the world, is the *aureole*, also called *halo* or *Heiligenschein*: a ring of light surrounding the head. You can easily observe it around your own head. You need only to get up early in the morning and look into the wet grass while turning your back to the Sun. You will see an aureole around your shadow. The effect is due to the morning dew on the grass, which reflects the light back predominantly in the direction of the light source, as shown in Figure 138. The fun part is that if you do this in a group, you will see the aureole around only *your own* head. Ref. 142

Retroreflective paint works in the same way: it contains tiny glass spheres that play the role of the dew. A large surface of retroreflective paint, a traffic sign for example, can also show your halo if the light source is sufficiently far away. Ref. 143 Also the so-called 'glow' of the eyes of a cat at night is due to the same effect; it is visible only if you look at the cat with a light source behind you. By the way, do Cat's-eyes work like a cat's eyes? Challenge 182 s

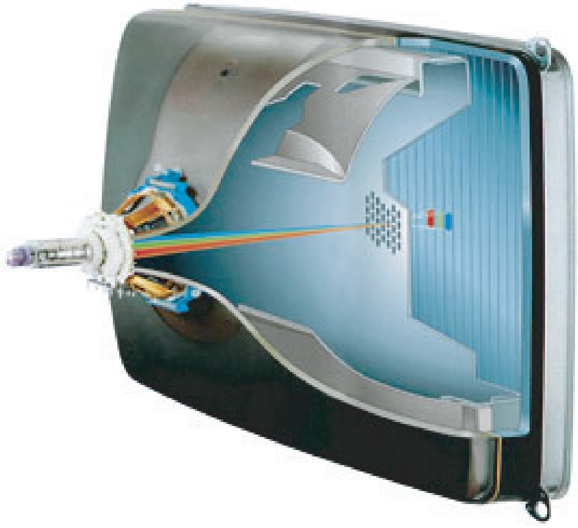


FIGURE 139 A cathode ray tube in older televisions: the first way – now obsolete – to produce changing colour images using electric signals. Television tubes emit an electron beam, deflect it, and generate light by electroluminescence on a coloured screen covered with patterned phosphors.

DISPLAYING IMAGES

Systems that display images are of importance in technical devices and, to a smaller degree, in nature. In nature, these displays are of two types: The first type is used by squids living in shallow water: they are able to produce moving colour patterns on their skin, and they use these patterns to confuse prey. The second type is found in the deep sea, where there is no ambient light: there, many living beings produce moving light displays to attract prey or to confuse predators.

In short, images can be generated by changing surface colours – passive displays – or by emitting light. Also human-made systems can divided into these two classes.

At present, the most common passive displays are liquid crystal displays – or LCDs – and electronic ink displays. The former are used in watches and mobile phones, the latter in electronic book readers.

The most common light emitting displays are the dated cathode ray tube, plasma displays, the light emitting diode displays and projection displays. These displays are used mostly in entertainment devices.

Hopping electrons and the biggest disappointment of the television industry

It is well known that when an electric field in a vacuum points along a glass surface, electrons can *hop* along the glass surface. The general effect is shown in Figure 140; usually, the effect is unwelcome. Among others, the hopping effect is responsible for sparks in vacuum systems that contain high voltage. To avoid the effect, the glass insulators on high voltage lines have complex shapes.

When this effect was studied in more detail, it turned out that reasonably low electric fields are sufficient to create sizeable electric hopping currents in hollow glass tubes with an internal diameter around a millimetre. The low elctric field can also lead elec-

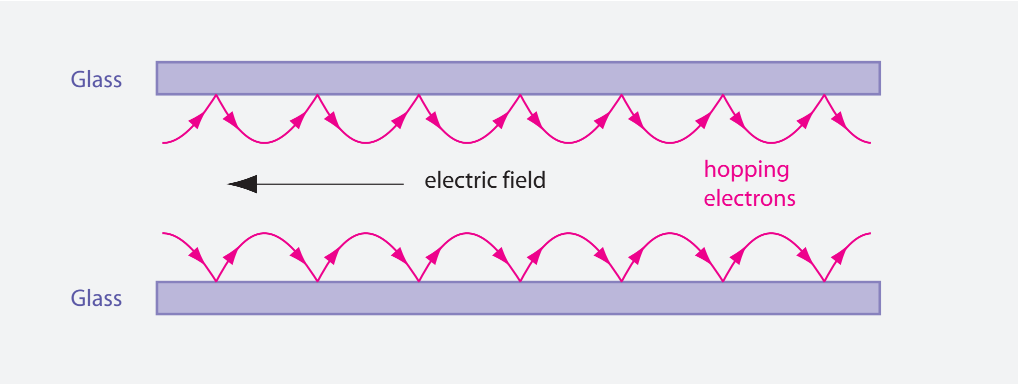


FIGURE 140 Free electrons can hop along a glass wall.

tron around bends and corners. Furthermore, electric switches that change the hopping direction can be constructed. In short, the hopping effect can be used to make extremely cheap flat television displays of high image quality. The idea is to put an array of electron sources – essentially sharp metal tips – at the start of many closeby glass channels. Each channel transports the emitted electrons along a line of the display. Making use of suitable switches at each pixel, the electrons are made to hit phosphorescent colour emitters. These are the same pixels that were used in the then common – bulky and heavy – television tubes and that are used in flat plasma displays. Since the hopping effect also works around bends and corners, and since it only needs glass and a bit of metal, the whole system can be made extremely thin, and lightweight; moreover, the machines are cheap, the yield is high and the production cost is low. Already in the early 1990s, the laboratory samples of the electron hopping displays were spectacularly good: the small displays were brighter, sharper and cheaper than liquid crystal displays, and the large ones brighter, sharper and cheaper than plasma displays. Affordable flat television was on the horizon.

Ref. 144

Then came the disappointment. The lifetime of the displays was only of the order of a few hundred hours. The limitation was due to the necessity to use helium inside the display, which cannot be contained inside a vacuum system for a long time. Despite the most intense material research, achieving a higher lifetime turned out to be impossible. All tricks that were tried did not help. Despite all their fantastic properties, despite huge investments in the technology, despite the best material researchers working on the issue, electron hopping displays could not be brought to market. Not a single display was ever sold.

Challenges and fun curiosities about images and the eye

An image sensor does not need a lens. The temple viper (or Wagler's pit viper) has two infrared sensors – one is shown in Figure 141 – with a resolution of 40 times 40 pixels each, and it just has a hole instead of a lens. The pit viper uses these sensors to catch mice even in the dark. The working of this infrared sensor has been explored and simulated by several research groups. It is now known how the sensor acquires the data, how the snake brain reconstructs the image, and how it achieves the high resolution.

Ref. 145

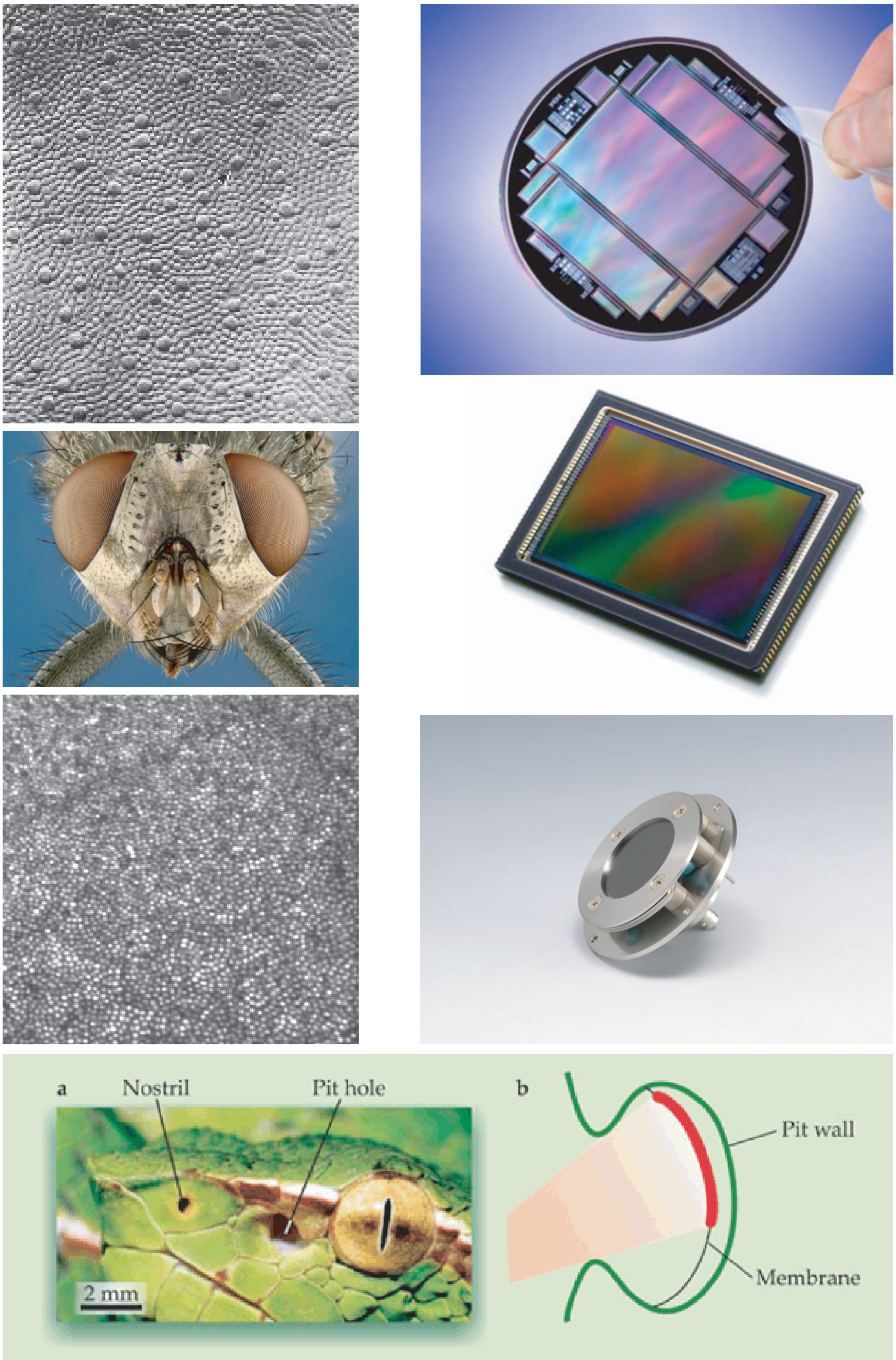


FIGURE 141 A collection of image sensors – thus of pixel systems: A cat's retina, a CCD sensor still on a wafer, the eye of a house fly, a CMOS sensor, a human retina, a multichannel plate, and a temple viper's infrared pit (© Wikimedia, Austin Roorda, Hamamatsu Photonics, Guido Westhoff/Leo van Hemmen).

* *

The simplest imaging system are eye glasses. A child that has no proper glasses misses an important experience: seeing the stars. Such a child will not understand the famous statement by Immanuel Kant: 'Two things fill the mind with ever new and increasing admiration and awe, the more often and persistently thought considers them: the starred sky above me and the moral law inside me.' Always be sure that children can see the stars.

Two lenses of 40 cents each are sufficient to change the life of a child or that of an adult. See the website www.onedollarglasses.org for an effective way to do it across the world.

* *

Challenge 183 ny How does the eye correct pixel (photoreceptor) failure? How many pixels are bad in a typical eye?

* *

Infrared light can be seen, if it is of sufficient intensity. (Never try this yourself!) People who observed such light sources – semiconductor lasers, for example – saw it as a white spot with some red borders.

* *

Among vertebrates, the largest eye is the eye of the blue whale; it has a diameter of 55 mm. (Only squids have larger eyes.) The smallest vertebrate eye seems to be that of juvenile *Brookesia micra*, a small chameleon whose head is half the size of the head of a match and whose eye is around 0.3 mm in diameter.

The eye is a wonderful organ. To learn more about it, read the beautiful book Simon Ings, *The Eye – A Natural History*, Bloomsbury, 2007.

* *

In many applications, it is important to avoid reflections. Anti-reflection coatings are used on the glass of shop-windows, and in lens systems that need to work in dim conditions, when light is scarce. Also living beings have anti-reflection coatings; the eyes of moths are famous for appearing black also in bright daylight, because they do not reflect light. Various companies are trying to reproduce this so-called *moth-eye effect* in commercial applications.

* *

Modern technology allows producing microscopes at low cost. For a fascinating example, Ref. 146 see the 1 Euro microscope that can be folded from a sheet of paper, embedded with some additional devices, and is shown in Figure 142. The device is used by holding it in front of the eye or by holding it in front of a lamp and observing the projected image on a screen.

* *

If a sufficient number of images is available, it is possible to identify the camera that produced them. Every camera has a specific image noise pattern; by extracting it through clever averaging, computer software that processes camera images is able to support police investigations.

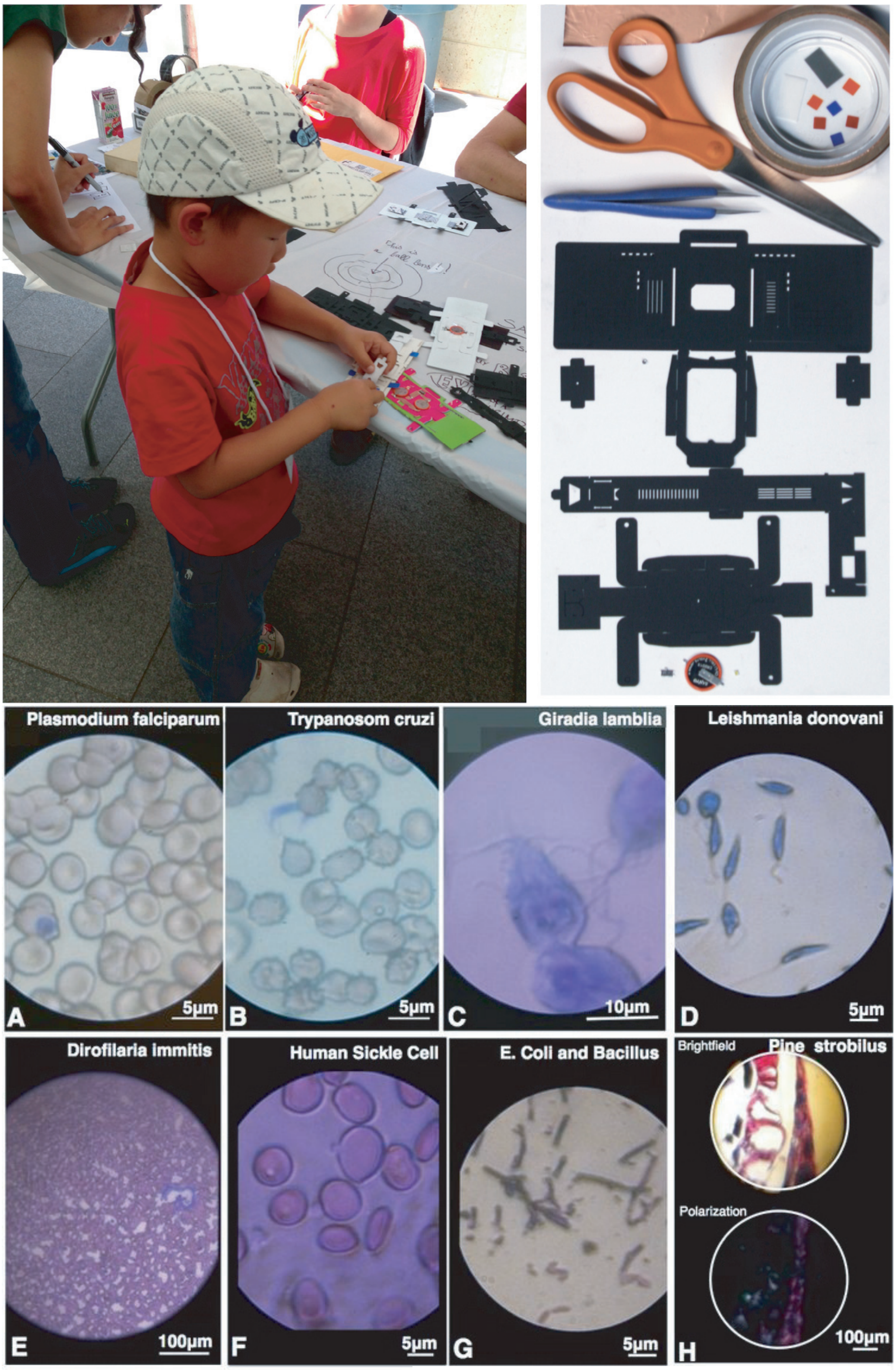


FIGURE 142 Top: the production and the parts of a flat microscope for medical use in developing countries made from sheet paper; bottom: the images it produces (© Foldscope team at www.foldscope.com).

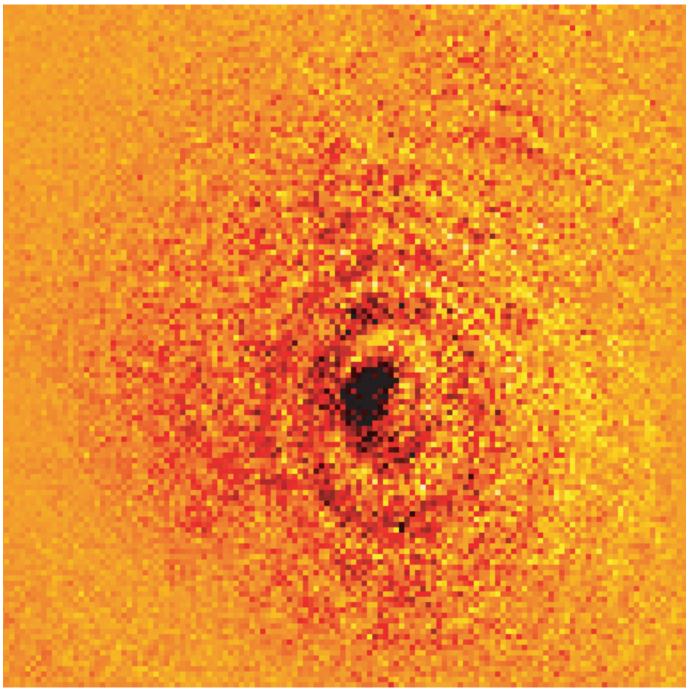


FIGURE 143 The shadow of a single ytterbium ion levitated in an ion trap and illuminated with a laser; picture size is about 16 μm in both directions (© Dave Kielpinski).

* *

Ref. 147 Mirages often have surprising effects. In 1597, a group of sailors were stranded on Novaya Zemlya during the winter. On 24 January they saw the Sun – roughly two weeks before it should be visible there. Such an unusual sighting is now called a Novaya Zemlya effect.

* *

Challenge 184 s It is possible to measure the width of a hair with a laser pointer. How?

* *

Ref. 148 Modern imaging techniques allow high sensitivity and high spatial resolution. As shown in Figure 143, using a Fresnel lens, a cooled CCD sensor and a laser as a light source, it is even possible to photograph the shadow of a single floating ion.

* *

An important device in medicine is the *endoscope*. An endoscope, shown in Figure 144, allows looking into a body cavity through a very small hole. It is a metal tube, typically with a diameter of around 5 mm and a length of 300 mm. How would you build one? Challenge 185 e (The device must resist at least 150 disinfection cycles in an autoclave; each cycle implies staying at 134°C and 3 bar for three hours.) Made of a sequence of carefully designed cylinder lenses, endoscopes allow surgeons to watch the inside of a human body through a tiny hole, thus avoiding large cuts and dangerous operations. Endoscopes have saved many lives, and their production and development employs a large industry.

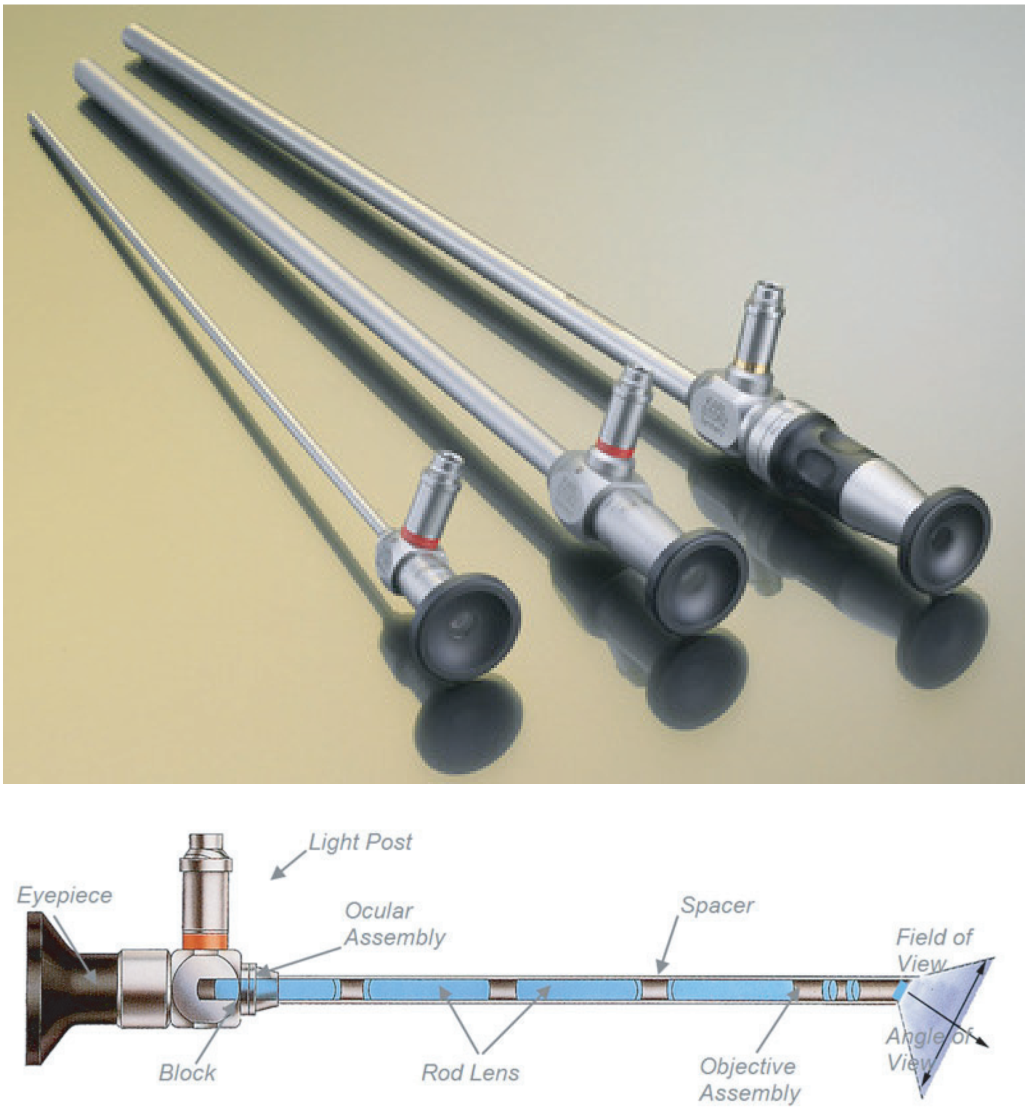


FIGURE 144 The endoscope invented by Hopkins, in which rod lenses allow large field of view and high brightness – the more so the higher the glass/air ratio is (© Karl Storz).

* *

Challenge 186 s The Sun is visible to the naked eye only up to a distance of 50 light years. Is this true?

* *

Ref. 149 Challenge 187 s Grass is usually greener on the other side of the fence. Can you give an explanation based on observations for this statement?

* *

Challenge 188 s It is said that astronomers have telescopes so powerful that they could see whether somebody would be lighting a match on the Moon. Can this be true?

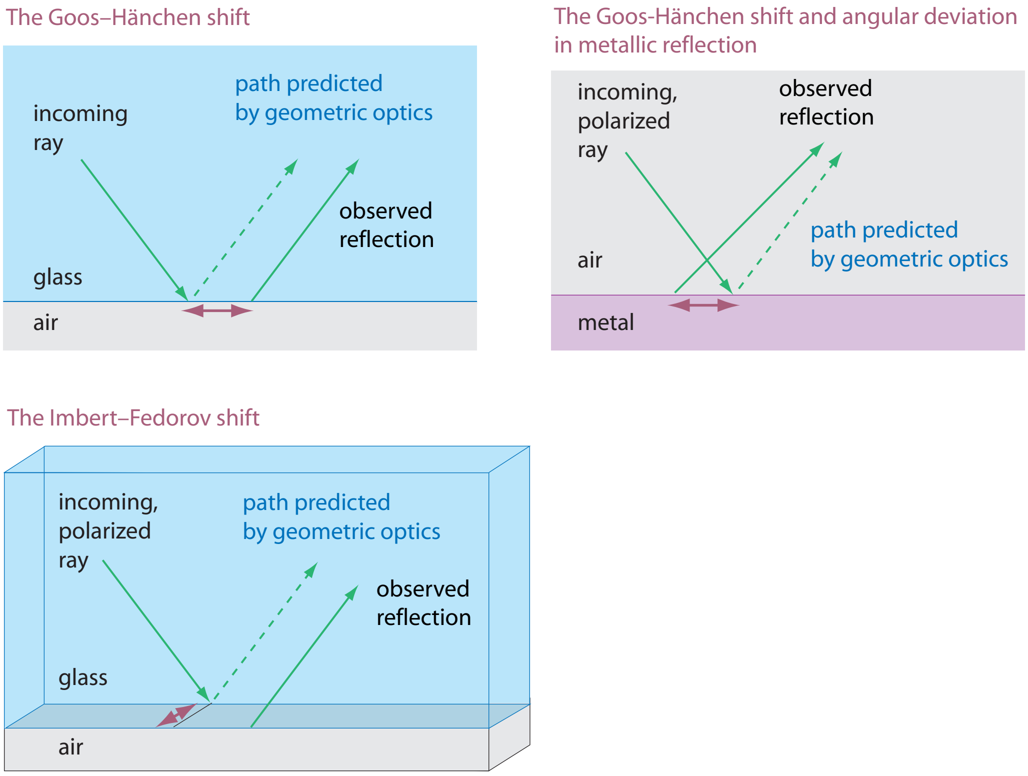


FIGURE 145 The Goos-Hänchen shift and other deviations from geometric reflection: in total reflection, the reflected light beam is slightly displaced from its naively expected position; in metallic reflection, even more deviations are observed.

* *

Total refraction is an interesting phenomenon it itself; but its details are even more fascinating. In 1943 Fritz Goos and Hilda Hänchen showed that the reflected beam is slightly shifted; in other words, the reflected beam is effectively reflected by a plane that lies slightly *behind* the material interface. This so-called *Goos-Hänchen shift* can be as large as a few wavelengths and is due travelling evanescent waves in the thinner medium.

Ref. 150

In fact, recent research into this topic discovered something even more interesting. When reflection is explored with high precision, one discovers that no reflected light ray is exactly on the position one expects them: there is also a lateral shift, the *Imbert–Fedorov shift*, and even the angle of the reflected ray can deviate from the expected one. The fascinating details depend on the polarization of the beam, on the divergence of the beam and on the material properties of the reflecting layer. These observations can be seen as higher-order effects of quantum field theory; their details are stilla topic of research.

* *

Materials that absorb light strongly also emit strongly. Why then does a door with dark paint in the sun get hotter than a door that is painted white? The reason is that the emission takes place at a much lower wavelength than that of visible light; for everyday sit-

FIGURE 146 How natural colours (top) change for three types of colour blind: deutan, protan and tritan (© Michael Douma).

uations and temperatures, emission is around 10 μm. And at that wavelength, almost all paints are effectively black, with emissivities of the order of 0.9, irrespective of their colour. And for the same reason, when you paint your home radiator, the colour is not important.

* *

Ref. 151 Challenge 189 s When two laser beams cross at a small angle, they can form light pulses that seem to move faster than light. Does this contradict special relativity?

* *

Challenge 190 s Colour blindness was discovered by the great scientist John Dalton (b. 1766 Eaglesfield, d. 1844 Manchester) – on himself. Can you imagine how he found out? It affects, in all its forms, one in 20 men. In many languages, a man who is colour blind is called *daltonic*. Women are almost never daltonic or colour blind, as the property is linked to defects on Ref. 152 the X chromosome. If you are colour blind, you can check to which type you belong with the help of Figure 146. (The X chromosome is also at the origin of the rare tetrachromatic women mentioned above.) Page 184

* *

Artificial colour blindness is induced by certain types of illumination. For example, violet light is used to reduce intravenous drug consumption, because violet light does not allow finding veins under the skin.

Artificial contrast enhancement with illumination is also useful. Pink light is used by beauticians to highlight blemishes, so that the skin can be cleaned as well as possible. In 2007, the police officer Mike Powis in Nottingham discovered that this 'acne light' could be used to reduce the crime rate; since acne is not fashionable, pink light deters youth from gathering in groups, and thus calms the environment where it is installed.

Yellowish light is used by by supermarkets to increase their sales of fruit and vegetables. In yellow light, tomatoes look redder and salad looks greener. Check by yourself: Challenge 191 e you will not find a single supermarket without these lights installed over fruit and vegetables.

* *

Light beams, such as those emitted from lasers, are usually thought of as thin lines. However, light beams can also be *tubes*, with the light intensity lower in the centre than on the rim. Tubular laser beams, i.e., Bessel beams of high order, are used in modern research experiments to guide plasma channels and sparks.

* *

Is it possible to see stars from the bottom of a deep pit or of a well, even during the day, Challenge 192 s as is often stated?

* *

Ref. 153 Humans are the *only* primates that have *white* eyes. All apes have *brown* eyes, so that it is impossible to see in which direction the are looking. Apes make extensive use of this impossibility: they often turn their head in one direction, pretending to look somewhere, but turn their eyes in another. In other words, brown eyes are useful for deception. The same effect is achieved in humans by wearing dark sunglasses. So if you see somebody with sunglasses in a situation where there is no sunlight, you know that he or she is behaving like an ape.

Apes use this type of deception to flirt with the opposite sex without their steady partner noticing. Sunglasses are tools for the unfaithful.

* *

Challenge 193 s How can you measure the power of the Sun with your eyes closed?

* *

Even in a dark, moonless and starless night, a forest is not dark. You can see luminescent mushrooms (of which there are over 70 different species), luminescent moulds, you can see sparks when you take off your pullover or when your friend bites a mint bonbon or when you unroll a roll of adhesive tape or open a letter.

* *

Challenge 194 d How do you produce X-rays with a roll of adhesive tape?

* *

The number of optical illusions is enormous, and there are many time-wasting websites devoted to the topic. Films often use the so-called *Ames room* to transform actors into dwarfs. It is shown in Figure 147.

* *

Page 187 The eye and the brain sometimes *add* false colours, as we have seen above in the discussion of cones. Also Haidinger's brush is an example of added colours. In contrast, some-

FIGURE 147 Ames rooms in Paris and in San Francisco (© Sergio Davini, David Darling).

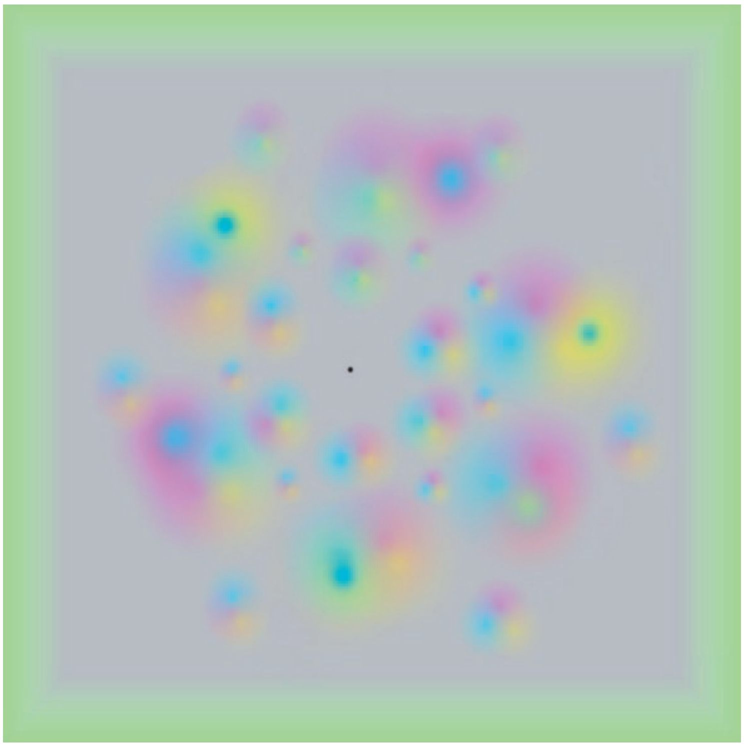


FIGURE 148 Look at the central dot for twenty seconds: the colours will disappear (© Kitaoka Akiyoshi).

times the brain and the eye make colours *disappear*, as shown in Figure 148. (The effect only works with a colour version of the figure.) The example is taken from the beautiful collection of visual illusions at www.psy.ritsumei.ac.jp/~akitaoka/color9e.html. Several related illusions, based on this one, use moving coloured dots.

* *

X-ray imaging is so impressive that it has become a form of art. One of the foremost X-ray artists is Nick Veasey, and two of his works are shown in Figure 149. Among many examples, he has even taken X-ray images of complete buses and aeroplanes.

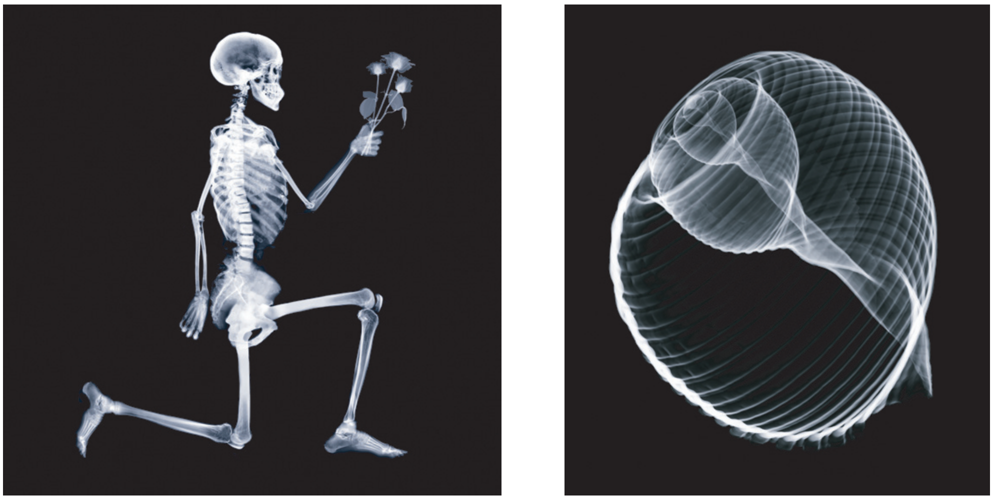


FIGURE 149 The beauty of X-rays: X-ray images of a person (taken with a corpse) and of a sea shell (© Nick Veasey).

∗∗

Lenses are important components in most optical systems. Approximately, the distance of the lens focus f, the distance of the object to be imaged o, and the distance of its image i are related by the *thin lens formula*

$$\frac{1}{f} = \frac{1}{o} + \frac{1}{i} \; . \tag{81}$$

Challenge 195 e It is not hard to deduce it with the help of raytracing.

If you ever are in the situation to design a lens, you will want to know the relation between the shape of a lens and its focal distance. It turns out that there are two types of lenses: The first type are *spherical* lenses which are easy and thus cheap to make, but whose images are not perfect. The second lens type are *aspherical* lenses, which are hard to fabricate, more expensive, but provide much better image quality. High-quality optical systems always contain aspherical lenses.

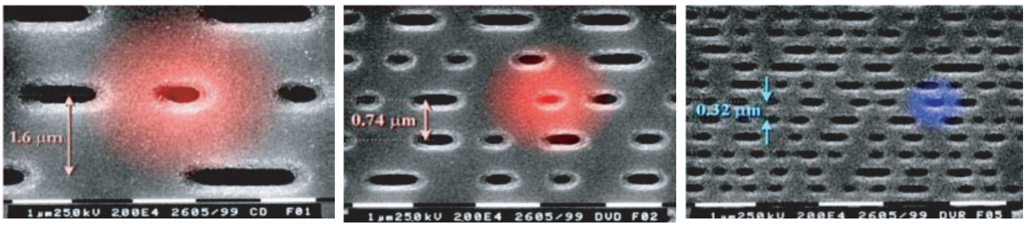
For historical reasons, most books on optics teach readers the approximate relation between the geometric radii of a thin spherical lens, its refractive index n and its focal distance:
Challenge 196 e

$$\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2}) \; . \tag{82}$$

This is called the *lensmaker formula*. Most aspherical lenses are approximately spherical, so that the formula helps as a rough first estimate also in these cases.

∗∗

Imaging is an important part of modern industry. Without laser printers, photocopying machines, CD players, DVD players, microscopes, digital photographic cameras, film and video cameras, lithography machines for integrated circuit production, telescopes,



CD
track pitch 1.6 µm
minimum pit length 0.8 µm

DVD
track pitch 0.74 µm
minimum pit length 0.4 µm

Blue Ray Disk
track pitch 0.32 µm
minimum pit length 0.15µm

FIGURE 150 Composed image of the tracks and the laser spot in a drive reading a CD, a DVD and a blue ray disc (© Wikimedia).

film projectors, our world would look much more boring. Nowadays, designing optical systems is done with the help of dedicated software packages. They allow to calculate image quality, temperature effects and mechanical tolerances with high precision. Despite the beauty of optical design, there is a shortage of experts on this fascinating field, across the world. Ref. 108

* *

Additional types of videos cameras are still being developed. Examples are time-of-flight cameras, laser scanning cameras, ultraviolet video cameras, video cameras that measure polarization and infrared video cameras. The latter cameras will soon appear in cars, in order to recognize people and animals from the heat radiation they emit and help avoiding accidents.

* *

What are the best colour images one can produce today? At present, affordable images on paper have about 400 dots/mm, or dots of about 2.5 µm. What is the theoretical maximum? You will find that several unserious research groups claim to have produced colour images with a resolution that is higher than the theoretical maximum. Challenge 197 e

* *

Ultrasound imaging is regularly used in medical applications. As mentioned earlier on, unfortunately it is not safe for imaging pregnancies. Is *ultrasound imaging*, though not an optical imaging method, a type of tomography? Vol. I, page 290 Challenge 198 e

* *

CMOS cameras, batteries and radio transmitters have become so small that they can be made into a package with the size of a pill. Such a camera can be swallowed, and with electrodes attached to the belly of a person, one can record movies of the intestine while the person is continuing its daily activities.

* *

FIGURE 151 One of the many kinds of Benham wheels. Rotating it with a top, a CD player or a drill is the simplest way to produce Fechner colours, i.e., false colours that appear from intermittent black and white patterns.

The most *common* optical systems are those found inside CD and DVD drives. If you ever have the opportunity to take one apart, do it. They are fascinating pieces of technology, in which every cubic millimetre has been optimized by hundreds of engineers. Can you imagine how a CD or DVD player works, starting from the photographs of Figure 150?

* *

The most *expensive* optical systems are not those found on espionage satellites – which can read the headlines of a newspaper from space – but those found in wafer steppers. Wafer steppers are machines used for the production of electronic integrated circuits. In such steppers, a metal mask is imaged, using light from a UV laser at 193 nm, onto a photo-resist covered silicon wafer. The optical systems used have the size of an average human, are precise within a few nanometres, and cost more than six million Euro a piece. Objectives for extreme UV will be even more expensive.

* *

You can buy transparent window panes that can be switched to translucent and back – thus from a clear glass to milk glass and back – by toggling an electrical switch. How do they work?

Challenge 199 e

* *

A rotating wheel coloured in a specific black and white pattern, such as *Benham's wheel*, will produce false colour effects in the eye. Unfortunately, a video of the effect does not work inside a pdf file such as the one of this book; instead, have a look at Kenneth Brecher's website at lite.bu.edu/vision/applets/Color/Benham/Benham.html or lite.bu.edu/vision-flash10/applets/Color/Benham/Benham.html. False colours can also be induced by flickering monochromatic images on computer screens. All these false colours are mainly due to the different response times of red, green and blue cones.

* *

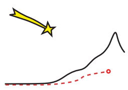
The size of the eye in mammals depends on their maximum running speed. This dependence has been verified for 50 different species. Interestingly, the correlation does not hold for the flying speed of birds.
Ref. 154

* *

Challenge 200 e
Did you ever see a shadow on a mirror or a flat water surface? Why not?

Summary on applied optics

The art and science of making images is central to modern health care, industry, science, entertainment and telecommunications. Acquiring images is in large part the result of bending light beams in predefined ways and then detecting them. All image acquisition systems, biological or human-made, are based on reflection, refraction or diffraction, combined with pixel detectors. All imaging systems that acquire or display high-quality images – biological or human-made – use clever combinations of materials science, sensors, actuators and signal processing. The field is still evolving.



Chapter 5

ELECTROMAGNETIC EFFECTS

Looking carefully, the atmosphere is full of electrical effects. The most impressive, Ref. 155 lightning, is now reasonably well understood. However, it took decades and a large number of researchers to discover and put together all the parts of the puzzle. Also below our feet there is something important going on: the hot magma below the continental crust produces the magnetic field of the Earth. Strong magnetic fields can be used for levitation. We first explore these topics and then give an overview about the many effects that fields produce.

Is lightning a discharge? – Electricity in the atmosphere

Inside a thunderstorm cloud, especially inside tall *cumulonimbus* clouds,* charges are separated by collision between the large 'graupel' ice crystals falling due to their weight Ref. 157 and the small 'hail' ice crystallites rising due to thermal upwinds. Since the collision takes part in an electric field, charges are separated in a way similar to the mechanism in the Page 19 Kelvin generator. Discharge takes place when the electric field becomes too high, taking a strange path influenced by ions created in the air by cosmic rays. (There are however, Ref. 158 at least ten other competing explanations for charge separation in clouds.) It seems that cosmic rays are at least partly responsible for the zigzag shape of lightning. For a striking example, see Figure 152.

A lightning flash typically transports 20 to 30 C of charge, with a peak current of up to 20 kA. But lightning flashes have also strange properties. First, they appear at fields around 200 kV/m (at low altitude) instead of the 2 MV/m of normal sparks. Second, lightning emits radio pulses. Third, lightning emits X-rays and gamma rays. Russian re- Ref. 159 searchers, from 1992 onwards explained all three effects by a newly discovered discharge mechanism. At length scales of 50 m and more, cosmic rays can trigger the appearance of lightning; the relativistic energy of these rays allows for a discharge mechanism that does not exist for low energy electrons. At relativistic energy, so-called runaway breakdown

* Clouds have Latin names. They were introduced in 1802 by the explorer Luke Howard (b. 1772 London, d. 1864 Tottenham), who found that all clouds could be seen as variations of three types, which he called *cirrus*, *cumulus* and *stratus*. He called the combination of all three, the rain cloud, *nimbus* (from the Latin Ref. 156 'big cloud'). Today's internationally agreed system has been slightly adjusted and distinguishes clouds by the height of their lower edge. The clouds starting above a height of 6 km are the cirrus, the cirrocumulus and the cirrostratus; those starting at heights of between 2 and 4 km are the altocumulus, the altostratus and the nimbostratus; clouds starting below a height of 2 km are the stratocumulus, the stratus and the cumulus. The rain or thunder cloud, which crosses all heights, is today called *cumulonimbus*. For beautiful views of clouds, see the www.goes.noaa.gov and www.osei.noaa.gov websites.

FIGURE 152 A rare photograph of a lightning stroke hitting a tree (© Niklas Montonen).

FIGURE 153 Cumulonimbus clouds from ground and from space (NASA).

leads to discharges at much lower fields than usual laboratory sparks. The multiplication of these relativistic electrons also leads to the observed radio and gamma ray emissions.

Incidentally, you have a 75 % chance of survival after being hit by lightning, especially

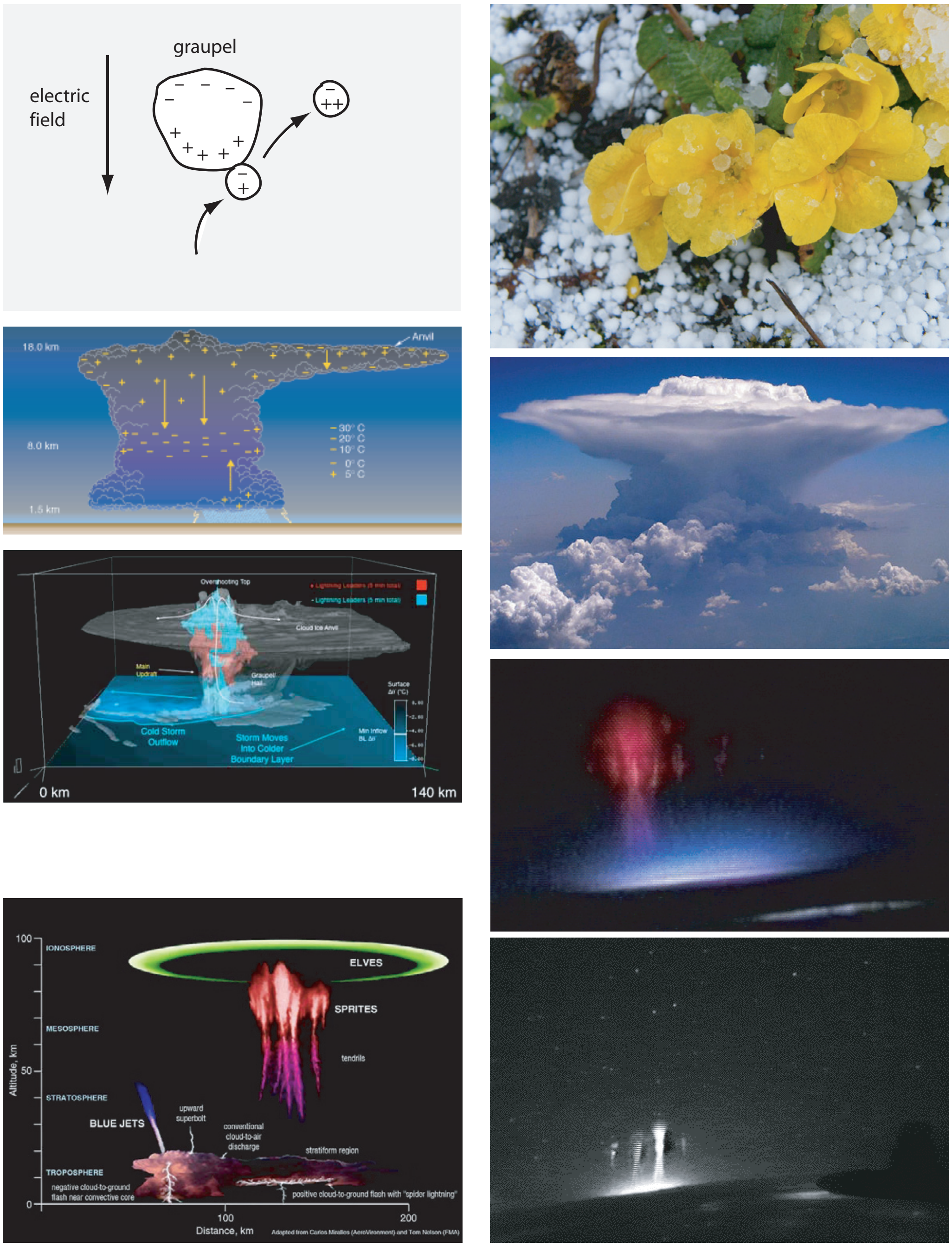


FIGURE 154 The charging and discharging of clouds: the most probable microscopic mechanism, namely charging of graupel particles by collision with ice particles, the cloud charge distribution, the three-dimensional structure and the large scale processes discovered in the past decades from aeroplanes (© nordique, NASA, NOAA).

if you are completely wet, as in that case the current will flow outside the skin. Usually, wet people who are hit lose all their clothes, as the evaporating water tears them off. Rapid resuscitation is essential to help somebody to recover after a hit.*

As a note, you might know how to measure the distance of a lightning by counting the seconds between the lightning and the thunder and multiplying this by the speed of sound, 330 m/s; it is less well known that one can estimate the *length* of the lightning bolt by measuring the *duration* of the thunder, and multiplying it by the same factor.

In the 1990s more electrical details about thunderstorms became known. Airline pilots and passengers sometime see weak and coloured light emissions spreading from the top of thunderclouds. There are various types of such emissions: blue *jets* and mostly red *sprites* and *elves*, which are somehow due to electric fields between the cloud top and the ionosphere. The details are still under investigation, and the mechanisms are not yet clear.**

Lightning is part of the electrical circuit around the Earth. This fascinating part of geophysics would lead us too far from the aim of our adventure. But every physicist should know that there is a vertical electric field of between 100 and 300 V/m on a clear day, as discovered already in 1752. (Can you guess why it is not noticeable in everyday life? And

Challenge 201 s

why despite its value it cannot be used to extract large amounts of energy?) The field is directed from the ionosphere down towards the ground; in fact the Earth is permanently negatively charged, and in clear weather current flows downwards (electrons flow upwards) through the clear atmosphere, trying to *discharge* our planet. The current of about 1 to 2 kA is spread over the whole planet; it is possibly due to the ions formed by cosmic radiation. (The resistance between the ground and the ionosphere is about 200 Ω, so the total voltage drop is about 200 kV.) At the same time, the Earth is constantly being *charged* by several effects: there is a dynamo effect due to the tides of the atmosphere and there are currents induced by the magnetosphere. But the most important charging effect is lightning. In other words, contrary to what one may think, lightning does not

Ref. 160

discharge the ground, it actually charges it up!*** Of course, lightning does discharge the cloud to ground potential difference; but by doing so, it actually sends (usually) a negative charge down to the Earth as a whole. Thunderclouds are batteries; the energy from the batteries comes from the thermal uplifts mentioned above, which transport charge *against* the global ambient electrical field.

Using a few electrical measurement stations that measure the variations of the electrical field of the Earth it is possible to locate the position of all the lightning that comes down towards the Earth at a given moment. (Distributed around the world, there are

Ref. 161

about a hundred lightning flashes per second.) Present research also aims at measuring the activity of the related electrical sprites and elves in this way.

The ions in air play a role in the charging of thunderclouds via the charging of ice crystals and rain drops. In general, all small particles in the air are electrically charged. When aeroplanes and helicopters fly, they usually hit more particles of one charge than

* If you are ever hit by lightning and survive, go to the hospital! Many people died three days later having failed to do so. A lightning strike often leads to coagulation effects in the blood. These substances block the kidneys, and one can die three days later because of kidney failure. The remedy is to have dialysis treatment.

** For images, have a look at the interesting elf.gi.alaska.edu/, www.fma-research.com/spriteres.htm and pasko.ee.psu.edu/Nature websites.

Challenge 202 s

*** The Earth is thus charged to about −1 MC. Can you confirm this?

of the other. As a result, aeroplanes and helicopters are charged up during flight. When a helicopter is used to rescue people from a raft in high seas, the rope pulling the people upwards must first be earthed by hanging it in the water; if this is not done, the people on the raft could die from an electrical shock when they touch the rope, as has happened a few times in the past.

The charges in the atmosphere have many other effects. Recent experiments have confirmed what was predicted back in the early twentieth century: lightning emits X-rays. Ref. 162 The confirmation is not easy though; it is necessary to put a detector near the lightning flash. To achieve this, the lightning has to be directed into a given region. This is possible using a missile pulling a metal wire, the other end of which is attached to the ground. These experimental results are now being collated into a new description of lightning which also explains the red-blue sprites above thunderclouds. In particular, the processes also imply that inside clouds, electrons can be accelerated up to energies of a few MeV. Ref. 163

Why are sparks and lightning blue? This turns out to be a material property: the colour comes from the material that happens to be excited by the energy of the discharge, usually air. This excitation is due to the temperature of 30 kK inside the channel of a typical lightning flash. For everyday sparks, the temperature is much lower. Depending on the situation, the colour may arise from the gas between the two electrodes, such as oxygen or nitrogen, or it may due to the material evaporated from the electrodes by the discharge. For an explanation of such colours, as for the explanation of all colours due to materials, we need to wait for the next part of our walk, on quantum theory.

But not only electric fields are dangerous. Also time-varying electromagnetic fields can be. In 1997, in beautiful calm weather, a Dutch hot air balloon approached the powerful radio transmitter in Hilversum. After travelling for a few minutes near to the antenna, the gondola suddenly detached from the balloon, killing all the passengers inside.

An investigation team reconstructed the facts a few weeks later. In modern gas balloons the gondola is suspended by high quality nylon ropes. To avoid damage by lightning and in order to avoid electrostatic charging problems all these nylon ropes contain thin metal wires which form a large equipotential surface around the whole balloon. Unfortunately, in the face of the radio transmitter, these thin metal wires absorbed radio energy from the transmitter, became red hot, and melted the nylon wires. It was the first time that this had ever been observed.

Does ball lightning exist?

For hundreds of years, people have reported sightings of so-called *ball lightning*. Usually they were noticed during thunderstorms, often after a lightning had struck. With a few exceptions, nobody took these reports seriously, because no reproducible data existed. Ref. 164

When microwave ovens become popular, several methods to produce ball-shaped discharges became known. To observe one, just stick a toothpick into a candle, light the toothpick, and put it into (somebody else's) microwave oven at maximum power. This set-up produces a beautiful ball-like discharge. However, humans do not live in a microwave oven; therefore, this mechanism is not related to ball lightning.

The experimental situation changed completely in the years 1999 to 2001. In those years the Russian physicists Anton Egorov and Gennady Shabanov discovered a way to Ref. 165 produce plasma clouds, or *plasmoids*, floating in air, using three main ingredients: water,

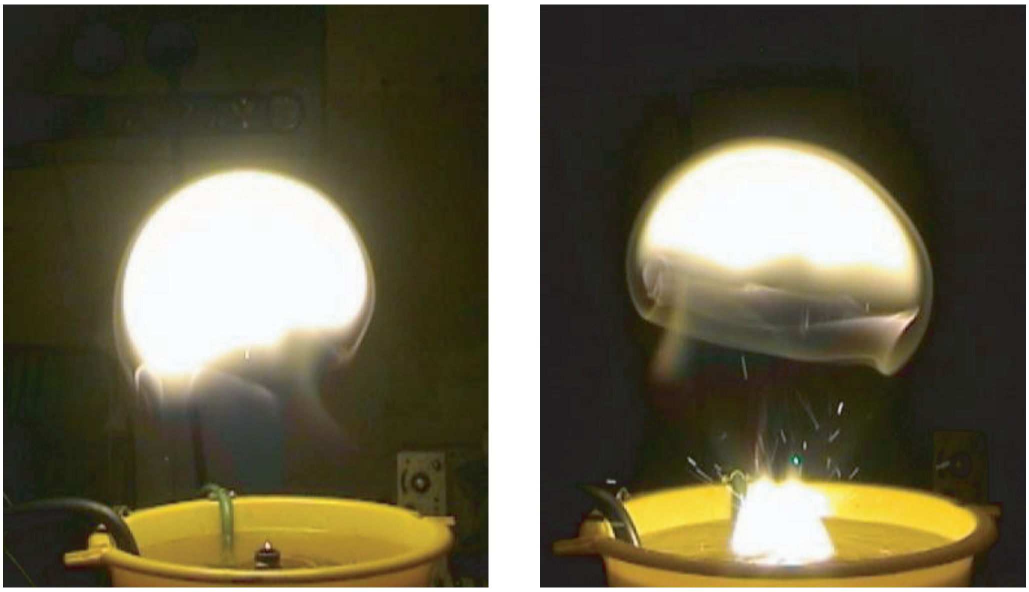


FIGURE 155 A floating plasma cloud produced in the laboratory (© Sergei Emelin and Alexei Pirozerski).

metal and high voltage. If high voltage is applied to submerged metal electrodes of the proper shape and make, plasma clouds emerge from the water, about 10 to 20 cm in size, float above the surface, and disappear after about half a second. Two examples can be seen in Figure 155.

The phenomenon of floating plasmoids is still being studied. There are variations in shape, colour, size and lifetime. The spectrum of observations and techniques will surely evolve in the coming years.

An even more astonishing effect was published in 2007. A Brazilian research team found a way to make golf-ball sized discharges that seem to roll along the floor for as long as 8 s. Their method was beautifully simple: with the help of a 25 V power supply, they passed a current of 140 A through an arc at the surface of a silicon wafer. They discovered that small silicon particles detach and move away, while being surrounded by a luminous glow. These luminous clouds can wander around the table and floor of the laboratory, until they extinguish.

Ref. 166

It seems that these phenomena could explain a number of ball lightning observations. But it is equally possible that other effects will still be discovered.

Does gravity make charges radiate?

We learned in the section on general relativity that gravitation has the same effects as acceleration. This means that a charge kept fixed at a certain height is equivalent to a charge accelerated by $9.8\,\mathrm{m/s^2}$, which would imply that it radiates electromagnetically, since all accelerated charges radiate. However, the world around us is full of charges at fixed heights, and there is no such radiation. How is this possible?

Ref. 167

The question has been a pet topic for many years. Generally speaking, the concept of radiation is not observer invariant: If one observer detects radiation, a second one does

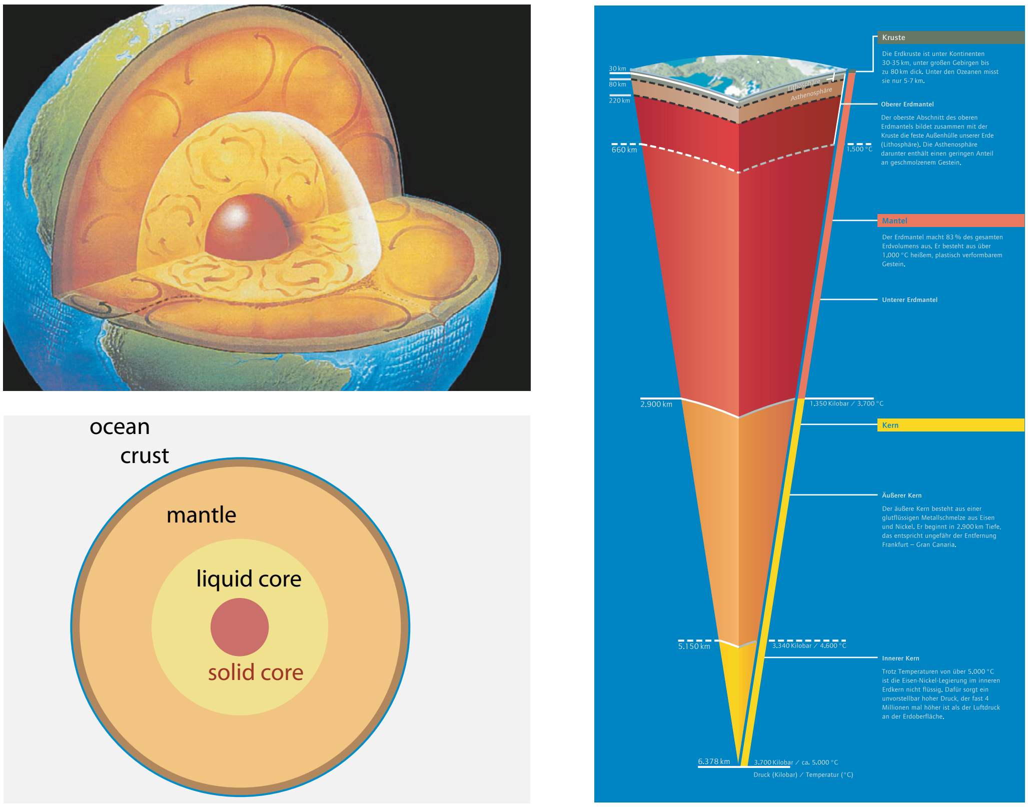


FIGURE 156 The structure of our planet (© MPI-Chemie, Mainz/GEO).

not necessarily do so as well. The exact way a radiation field changes from one observer to the other depends on the type of relative motion and on the field itself.

A detailed exploration of the problem shows that for a uniformly accelerated charge, an observer undergoing the same acceleration only detects an electrostatic field. In contrast, an inertial observer detects a radiation field. Since gravity is (to a high precision) equivalent to uniform acceleration, we get a simple result: gravity does not make electrical charges radiate for an observer at rest with respect to the charge – as is indeed observed. The results holds true also in the quantum theoretical description. Ref. 168

Planetary magnetic fields

The classical description of electrodynamics is consistent and complete; nevertheless there are still many subjects of research. We explore a few of them.

The origin of the magnetic field of the Earth, the other planets, the Sun and even of the galaxy is a fascinating topic. The way that the convection of fluids inside the planets generates magnetic fields, an intrinsically three-dimensional problem, the influence of turbulence, of non-linearities and of chaos makes it a surprisingly complex question.

The details of the generation of the magnetic field of the Earth, usually called the *geodynamo*, began to appear only in the second half of the twentieth century, when the knowledge of the Earth's interior reached a sufficient level. The Earth's interior starts below the Ref. 33

Earth's crust. The *crust* is typically 30 to 40 km thick (under the continents), though it is thicker under high mountains and thinner near volcanoes or under the oceans. As already mentioned, the crust consists of large segments, the *plates*, that float on magma and move with respect to one another. The Earth's interior is divided into the *mantle* – the first 2900 km from the surface – and the *core*. The core is made up of a liquid *outer* core, 2210 km thick, and a solid *inner* core of 1280 km radius. (The temperature of the core is not well known; it is believed to be 6 to 7 kK. Can you find a way to determine it?
Challenge 203 d The temperature might have decreased a few hundred kelvin during the last 3000 million years.)

The Earth's core consists mainly of iron that has been collected from the asteroids that collided with the Earth during its youth. It seems that the liquid and electrically conducting outer core acts as a dynamo that keeps the magnetic field going. The magnetic energy comes from the kinetic energy of the outer core, which rotates with respect to the Earth's surface; the fluid can act as a dynamo because, apart from rotating, it also *convects* from deep inside the Earth to more shallow depths, driven by the temperature gradients between the hot inner core and the cooler mantle. Huge electric currents flow in complex ways through these liquid layers, maintained by friction, and create the magnetic field. Why this field switches orientation at irregular intervals of between a few tens of thousands and a few million years, is one of the central questions. The answers are difficult; experiments are not yet possible, 150 years of measurements is a short time when compared with the last transition – about 730 000 years ago – and computer simulations are extremely involved. Since the field measurements started, the dipole moment of the magnetic field has steadily diminished, presently by 5 % a year, and the quadrupole moment has steadily increased. Maybe we are heading towards a surprise.* (By the way, the study of *galactic* magnetic fields is even more complex, and still in its infancy.)

Levitation

We have seen that it is possible to move certain objects without touching them, using a magnetic or electric field or, of course, using gravity. Is it also possible, without touching an object, to keep it fixed, floating in mid-air? Does this type of rest exist?

It turns out that there are several methods of levitating objects. These are commonly
Ref. 170 divided into two groups: those that consume energy and those who do not. Among the methods that consume energy is the floating of objects on a jet of air or of water, the floating of objects through sound waves, e.g. on top of a siren, or through a laser beam coming from below, and the floating of conducting material, even of liquids, in strong radiofrequency fields. Levitation of liquids or solids by strong ultrasound waves is presently
Ref. 171 becoming popular in laboratories. All these methods give *stationary* levitation. Another group of energy consuming methods sense the way a body is falling and kick it up again in the right way via a feedback loop; these methods are *non-stationary* and usually use magnetic fields to keep the objects from falling. The magnetic train being built in Shang-
Ref. 172 hai by a German consortium is levitated this way. The whole train, including the passengers, is levitated and then moved forward using electromagnets. It is thus possible, using magnets, to levitate many tens of tonnes of material.

* In 2005, it has been reported that the inner core of the Earth seems to rotate faster than the Earth's crust
Ref. 169 by up to half a degree per year.

For levitation methods that do *not* consume energy – all such methods are necessarily stationary – a well-known limitation can be found by studying Coulomb's 'law' of electrostatics: no static arrangement of electric fields can levitate a *charged* object in free space or in air. The same result is valid for gravitational fields and *massive* objects;* in other words, we cannot produce a local minimum of potential energy in the middle of a box using electric or gravitational fields. This impossibility is called *Earnshaw's theorem*. Speaking mathematically, the solutions of the Laplace equation $\Delta\varphi = 0$, the so-called *harmonic functions*, have minima or maxima only at the border, and never inside the domain of definition. (You proved this yourself on page 173 in volume I.) Earnshaw's theorem can also be proved by noting that given a potential minimum in free space, Gauss' theorem for a sphere around that minimum requires that a source of the field be present inside, which is in contradiction with the original assumption.

Ref. 173

We can deduce that it is also impossible to use electric fields to levitate an electrically *neutral* body in air: the potential energy U of such a body, with volume V and dielectric constant ε, in an environment of dielectric constant ε_0, is given by

$$\frac{U}{V} = -\frac{1}{2}\,(\varepsilon - \varepsilon_0)\,E^2 \quad . \tag{83}$$

Challenge 204 ny

Since the electric field E never has a maximum in the absence of space charge, and since for all materials $\varepsilon > \varepsilon_0$, there cannot be a minimum of potential energy in free space for a neutral body.**

To sum up, using static electric or static gravitational fields it is impossible to keep an object from falling; neither quantum mechanics, which incorporates phenomena such as antimatter, nor general relativity, including phenomena such as black holes, change this basic result.

For static *magnetic* fields, the argument is analogous to electrical fields: the potential energy U of a magnetizable body of volume V and permeability μ in a medium with permeability μ_0 containing no current is given by

Challenge 206 ny

$$\frac{U}{V} = -\frac{1}{2}\left(\frac{1}{\mu} - \frac{1}{\mu_0}\right)B^2 \tag{84}$$

and due to the inequality $\Delta B^2 \geqslant 0$, isolated maxima of a static magnetic field are not possible, only isolated minima. Therefore, it is impossible to levitate paramagnetic ($\mu > \mu_o$) or ferromagnetic ($\mu \gg \mu_0$) materials such as steel, including bar magnets, which are all attracted, and not repelled to magnetic field maxima.

Challenge 207 e

There are thus two ways to realize magnetic levitation: levitating a diamagnet or using a time-dependent field. Diamagnetic materials ($\mu < \mu_0$, or $\mu_r = \mu/\mu_0 < 1$) were discovered shortly after Earnshaw published his theorem, and allow circumventing it.

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* To the disappointment of many science-fiction addicts, this would even be true if a negative mass existed. And even though gravity is not really due to a field, but to space-time curvature, the result still holds in general relativity.

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** It is possible, however, to 'levitate' gas bubbles in liquids – 'trap' them to prevent them from rising would be a better expression – because in such a case the dielectric constant of the environment is higher than that of the gas. Can you find a liquid–gas combination where bubbles fall instead of rise?

Ref. 174

Challenge 205 ny

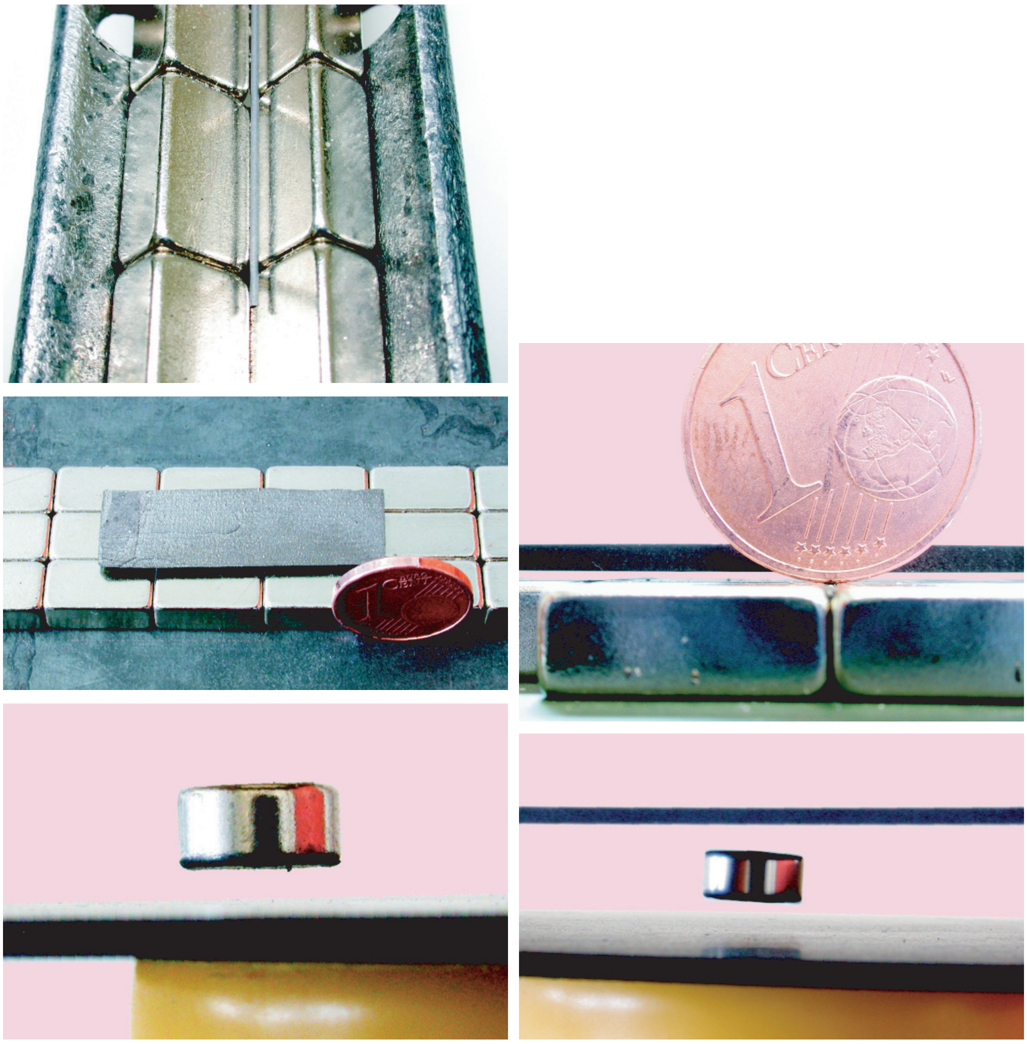

FIGURE 157 Stable diamagnetic levitation of a graphite bar over rectangular permanent magnets (above) and of two graphite plates, one seen from above and another from the side (centre); below, levitation of a 4 mm diameter NdFeB permanent magnet, above a graphite plate and between two graphite plates, near a large ring magnet (not shown) (© Joachim Schlichting from Ref. 175).

Indeed, diamagnetic materials, such as graphite or water, can be levitated by static magnetic fields because they are attracted to magnetic field minima. In fact, it is possible to levitate magnets if one uses a combination containing diamagnets. A few cases that can easily be replicated on a kitchen table are shown in Figure 157. Ref. 176 Ref. 175

Another well-known example of diamagnetic levitation is the levitation of superconductors. Indeed, superconductors, at least those of type I, are perfects diamagnets ($\mu = 0$). In some cases, superconductors can even be *suspended* in mid-air, below a magnet. Also single atoms with a magnetic moment are diamagnets; they are routinely levitated this way and have also been photographed in this state. Single neutrons, which have a magnetic dipole moment, have been kept in magnetic bottles through magnetic levitation, Ref. 177

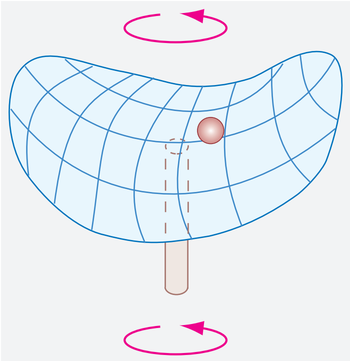

FIGURE 158 Trapping a metal sphere using a variable speed drill and a plastic saddle.

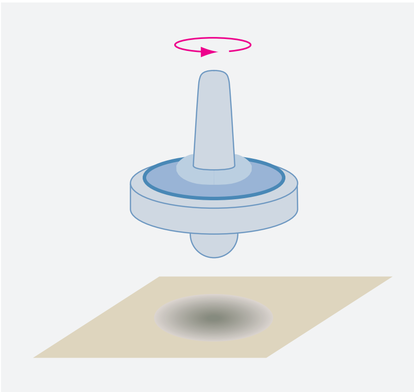

FIGURE 159 Floating 'magic' nowadays available in toy shops, left, with a spinning top and, right, with a spinning magnetic sphere levitating above a large ring magnet (© Kay Kublenz).

until they decay.

Diamagnets levitate if $\nabla B^2 > 2\mu_0 \rho g/\chi$, where ρ is the mass density of the object and $\chi = 1 - \mu/\mu_0$ its magnetic susceptibility. Since χ is typically about 10^{-5} and ρ of order 1000 kg/m^3, field gradients of about 1000 T^2/m are needed. In other words, levitation requires fields changes of 10 T over 10 cm, which is nowadays common for high field laboratory magnets. Challenge 208 ny

Recently, scientists have levitated pieces of wood and of plastic, strawberries, water droplets, liquid helium droplets as large as 2 cm, grasshoppers, fish and frogs (all alive and without any harm) using magnetic levitation. Indeed, animals, like humans, are all made of diamagnetic material. Humans themselves have not yet been levitated, but the feat, expected to require 40 T and large amounts of electrical power, is being planned and worked on. In fact, a similar feat has already been achieved: diamagnetic levitation is being explored for the levitation of passenger trains, especially in Japan, though with little commercial success. Ref. 178 Ref. 172

Time-dependent electrical or magnetic fields, e.g. periodic fields, can lead to levitation in many different ways without any consumption of energy. This is one of the methods used in the magnetic bearings of turbomolecular vacuum pumps. Also single charged particles, such as ions and electrons, are now regularly levitated with Paul traps and Pen- Ref. 170

Ref. 170 ning traps. The mechanical analogy is shown in Figure 158.

Ref. 179 Figure 159 shows a toy that allows you to personally levitate a spinning top or a spinning magnetic sphere in mid-air above a ring magnet, a quite impressive demonstration of levitation for anybody looking at it. The photo shows that is not hard to build such a Ref. 180 device yourself.

Even free electrons can be levitated, letting them float above the surface of fluid he- Ref. 181 lium. In the most recent twist of the science of levitation, in 1995 Stephen Haley predicted that the suspension height of small magnetic particles above a superconducting ring should be quantized. However, the prediction has not been verified by experiment yet.

For the sake of completeness we mention that nuclear forces cannot be used for levitation in everyday life, as their range is limited to a few femtometres. However, we will Vol. V, page 202 see later that the surface matter of the Sun is prevented from falling into the centre by these interactions; we could thus say that it is indeed levitated by nuclear interactions.

Matter, levitation and electromagnetic effects

The levitation used by magicians mostly falls into another class. When David Copperfield, a magician performing for young girls at the end of the twentieth century, 'flies' during his performances, he does so by being suspended on thin fishing lines that are Challenge 209 s rendered invisible by clever lighting arrangements. (How could one check this?) In fact, if we want to be precise, we should count fishing lines, plastic bags, as well as every table and chair as levitation devices. (Tabloid journalists would even call them 'anti-gravity' devices.) Contrary to our impression, a hanging or lying object is not really in contact with the suspension, if we look at the critical points with a microscope. The proof about lack of contact will arise in the quantum part of our walk.*

But if a lying object is not in contact with its support, why don't we fall through a table or through the floor? We started the study of mechanics by stating that a key property of matter its *solidity*, i.e., the impossibility of having more than one body at the same place at the same time. But what is the origin of solidity? Solidity is due to electricity inside matter. Again, we will be discover the details only in the forthcoming, quantum part of Vol. V, page 65 our adventure, but we can already collect the first clues at this point.

Not only solidity is due to electricity. Many other – in fact all – experiments show that matter is constituted of charged particles. Indeed, matter can be moved and influenced by electromagnetic fields in many ways. Over the years, material scientists have produced Ref. 182 a long list of such effects, all of which are based on the existence of charged constituents. Challenge 211 r An overview is given in Table 17. Can you find or imagine a new effect? For example, can electric charge change the colour of objects?

Challenge 210 ny * The issue is far from simple: which one of the levitation methods described above is used by tables or chairs?

TABLE 17 Selected matter properties related to electromagnetism, showing among other things the role it plays in the constitution of matter; at the same time a short overview of atomic, solid state, fluid and business physics.

Property	Example	Definition
thermal radiation, heat radiation or incandescence	every object	temperature-dependent radiation emitted by any macroscopic amount of matter
emissivity	all bodies	ability to emit thermal light
Interactions with charges and currents (transport-related effects)		
electrification	separating metals from insulators	spontaneous charging
triboelectricity	glass rubbed on cat fur	charging through rubbing
barometer light	mercury slipping along glass	gas discharge due to triboelectricity Ref. 183
insulation	air	no current flow below critical voltage drop
semiconductivity	diamond, silicon or gallium arsenide	current flows only when material is impure ('doped')
conductivity	copper, metals	current flows easily
superconductivity	niobium below 9 K	current flows indefinitely
ionization	fire flames	current flows easily
localization (weak, Anderson)	disordered solids	resistance of disordered solids
resistivity, Joule effect	graphite, W	heating due to current flow
thermoelectric effects at contacts: Seebeck effect, Peltier effect	ZnSb, PbTe, PbSe, BiSeTe, Bi_2Te_3, etc.	current flow due to temperature difference, cooling due to current flow
thermoelectric effect in the bulk: Thomson effect	Fe, Bi, Co, Sb, Cu, Ag, etc.	cooling due to temperature gradients
acousto-electric effect	CdS	sound generation by currents, and vice versa
magnetoresistance (several different effects)	permalloy, perovskites, metal multilayers	electrical resistance changes with applied magnetic field Ref. 184
recombination	smoke detector	charge carriers combine to make neutral atoms or molecules
annihilation	positron tomography	particle and antiparticle, e.g. electron and positron, disappear into photons
Penning effect	H, Ne, Ar	neutral metastable excited atoms ionize other atoms through collisions
Richardson effect, thermal emission	BaO_2, W, Mo, used in tv and electron microscopes	emission of electrons from hot metals
skin effect	Cu, all conductors	high current density on exterior of wire at high frequency
pinch effect	InSb, plasmas	high current density on interior of wire

TABLE 17 (Continued) Selected matter properties related to electromagnetism.

Property	Example	Definition
Josephson effect	Nb-Oxide-Nb	tunnel current flows through insulator between two superconductors
Sasaki–Shibuya effect	n-Ge, n-Si	anisotropy of conductivity due to applied electric field
switchable magnetism	InAs:Mn	voltage switchable magnetization Ref. 185
Hall effect	silicon and other semiconductors; used for magnetic field measurements	voltage perpendicular to current flow in applied magnetic field
Ettingshausen–Nernst effect	Bi	appearance of electric field in materials with temperature gradients in magnetic fields
optogalvanic effect	plasmas	change of discharge current due to light irradiation
Interactions with magnetic fields		
ferromagnetism	Fe, Ni, Co, Gd	spontaneous magnetization; material strongly attracted by magnetic fields
paramagnetism	Fe, Al, Mg, Mn, Cr	induced magnetization parallel to applied field; attracted by magnetic fields
diamagnetism	water, Au, graphite, NaCl	induced magnetization opposed to applied field; repelled by magnetic fields
magnetostriction (and the related Joule effect, Villari effect, Wiedemann effect, Matteucci effect, Barret effect and Nagaoka-Honds effect)	CeB_6, $CePd_2Al_3$, TbDyFe	change of shape or volume by applied magnetic field
magnetoelastic effect	Fe, Ni	change of magnetization by tension or pressure
acousto-magnetic effect	metal alloys, anti-theft stickers	excitation of mechanical oscillations through magnetic field
spin valve effect	metal multilayers	electrical resistance depends on spin direction of electrons with respect to applied magnetic field
Zeeman effect	atoms, e.g., Cd	change of emission frequency with magnetic field
optical orientation	paramagnetic gases	circularly polarized light and magnetic field align atomic spins due to Zeeman effect
Hanle effect	Hg, paramagnetic gases	change of polarization of fluorescence with magnetic field
Paschen–Back effect, Back–Goudsmit effect,	atomic gases	change of emission frequency in strong magnetic fields

TABLE 17 (Continued) Selected matter properties related to electromagnetism.

Property	Example	Definition
magneto-optical activity or Faraday effect or Faraday rotation	flint glass	polarization angle is rotated with magnetic field; different refraction index for right and left circularly polarized light, as in magneto-optic (MO) recording
magnetic circular dichroism	gases	different absorption for right- and left-circularly polarized light; essentially the same as the previous one
Majorana effect	colloids	specific magneto-optic effect
photoelectromagnetic effect	InSb	current flow due to light irradiation of semiconductor in a magnetic field
inverse Faraday effect	GdFeCo	switch of magnetisation by a femtosecond laser pulse
Voigt effect	vapours	birefringence induced by applied magnetic field
Cotton–Mouton effect	liquids	birefringence induced by applied magnetic field
Shubnikov–de Haas effect	Bi	periodic change of resistance with applied magnetic field
thermomagnetic effects: Ettingshausen effect, Righi–Leduc effect, Nernst effect, magneto–Seebeck effect	BiSb alloys	relation between temperature, applied fields and electric current
photonic Hall effect	CeF_3	transverse light intensity depends on the applied magnetic field Ref. 186
magnetocaloric effect	gadolinium, GdSiGe alloys	material cools when magnetic field is switched off Ref. 187
cyclotron resonance	semiconductors, metals	selective absorption of radio waves in magnetic fields
magnetoacoustic effect	semiconductors, metals	selective absorption of sound waves in magnetic fields
magnetic resonance (many types)	most materials, used for imaging in medicine for structure determination of molecules	selective absorption of radio waves in magnetic fields; includes NMR, EPR, etc.
magnetorheologic effect	liquids, used in advanced car suspensions	change of viscosity with applied magnetic fields
Meissner effect	type 1 superconductors, used for levitation	expulsion of magnetic field from superconductors

Interactions with electric fields

TABLE 17 (Continued) Selected matter properties related to electromagnetism.

Property	Example	Definition
polarizability	all matter	polarization changes with applied electric field
ionization, field emission, Schottky effect	all matter, tv	charges are extracted at high fields
paraelectricity	$BaTiO_3$	applied field leads to polarization in same direction
dielectricity	deionized water, insulators	in opposite direction
ferroelectricity	$BaTiO_3$	spontaneous polarization below critical temperature
piezoelectricity	the quartz lighter used in the kitchen, human bones, $LiNbO_3$	polarization appears with tension, stress, or pressure
electrostriction	platinum sponges in acids	shape change with applied voltage Ref. 188
pyroelectricity	$CsNO_3$, tourmaline, crystals with polar axes; used for infrared detection	change of temperature produces charge separation
electro-osmosis or electrokinetic effect	many ionic liquids	liquid moves under applied electric field Ref. 189
electrowetting	salt solutions on gold	wetting of surface depends on applied voltage
electrolytic activity	sulphuric acid	charge transport through liquid
liquid crystal effect	watch displays	molecules turn with applied electric field
electro-optical activity: Pockels effect, Kerr effect	crystalline solids ($LiNbO_3$), liquids (e.g. oil)	electric field rotates light polarization, i.e., produces birefringence
Freederichsz effect, Schadt–Helfrichs effect	nematic liquid crystals	electrically induced birefringence
Stark effect	hydrogen, mercury	colour change of emitted light in electric field
field ionization	helium near tungsten tips in field ion microscope	ionization of gas atoms in strong electric fields
Zener effect	Si	energy-free transfer of electrons into conduction band at high fields
field evaporation	W	evaporation under strong applied electric fields
Linear interactions with light		
absorption	coal, graphite	transformation of light into heat or other energy forms (which ones?) Challenge 212 s

TABLE 17 (Continued) Selected matter properties related to electromagnetism.

Property	Example	Definition
blackness	coal, graphite	complete absorption in visible range
colour	ruby	absorption depending on light frequency
metallic shine	metal, doped crystals	ability to act as 'good' mirror
chromatic dispersion	all materials	phase speed of light depends on wavelength
photostriction	PbLaZrTi	light induced piezoelectricity
photography	AgBr, AgI	light precipitates metallic silver
photoelectricity, photoeffect	Cs	current flows into vacuum due to light irradiation
internal photoelectric effect	Si p–n junctions, solar cells	voltage generation and current flow due to light irradiation
photon drag effect	p-Ge	current induced by photon momentum
transparency	glass, quartz, diamond	low reflection, low absorption, low scattering
reflectivity	metals	light bounces on surface
polarization	elongated silver nanoparticles in glass	light transmission depending on polarization angle
optical activity	sugar dissolved in water, quartz	rotation of polarization
birefringence, linear dichroism	calcite, cornea, thin polymer sheets	refraction index depends on linear polarization direction, light beams are split into two beams
circular dichroism	aminoacids, andalusite	absorption depends on circular polarization
optically induced anisotropy, Weigert effect	AgCl	optically induced birefringence and dichroism
Compton effect	momentum measurements	change of wavelength of X-rays and gamma radiation colliding with electrons
electrochromicity	wolframates	colour change with applied electric field
scattering	gases, liquids	light changes direction
Mie scattering	dust in gases	light changes direction
Raleigh scattering	sky	light changes direction, sky is blue
Raman effect or Smekal–Raman effect	molecular gases	scattered light changes frequency
switchable mirror	LaH	voltage controlled change from reflection to transparency Ref. 190
radiometer effect	bi-coloured windmills	irradiation turns mill (see page 118)
luminous pressure	*idem*	irradiation turns mill directly
solar sail effect	future satellites	motion due to solar wind
acousto-optic!effect	TeO_2, $LiNbO_3$	diffraction of light by sound in transparent materials

TABLE 17 (Continued) Selected matter properties related to electromagnetism.

Property	Example	Definition
photorefractive materials	$Bi_{12}SiO_{20}$, $LiNbO_3$, GaAs, InP	light irradiation changes refractive index
Auger effect	Auger electron spectroscopy	electron emission due to atomic reorganization after ionization by X-rays
Bragg reflection	crystal structure determination	X-ray diffraction by atomic planes
Mößbauer effect	^{57}Fe, used for spectroscopy	recoil-free resonant absorption of gamma radiation
pair creation	Pb	transformation of a photon in a charged particle–antiparticle pair
photoconductivity	Se, CdS	change of resistivity with light irradiation
optoacoustic effect, photoacoustic effect	gases, solids	creation of sound due to absorption of pulsed light; used for imaging of animal and human tissue
Light emission		
luminescence: general term for opposite of incandescence	GaAs, tv	light emission by cold matter
fluorescence	CaF_2, X-ray production, light tubes, cathode ray tubes, television tubes, dyes, coloured polymers, doped crystals	light emission during and after light absorption or other energy input
phosphorescence	$TbCl_3$, crstals doped with heavy metals	light emission due to light, electrical or chemical energy input, continuing *long after* stimulation
semiconductor luminescence	light-emissting diodes (LEDs), pointer lasers	emission of light due to electron hole recombination at p-n junctions
electroluminescence	ZnS powder	emission of light due to alternating electrical field
photoluminescence	ZnS : Cu, $SrAlO_4$: Eu, Dy, hyamine	light emission triggered by UV light, used in safety signs
chemoluminescence	H_2O_2, phenyl oxalate ester, dye solutions	chemically excited cold light emission, used in light sticks for divers and fun
bioluminescence	glow-worm, deep sea fish	cold light emission in animals, special type of chemoluminescence
triboluminescence	sugar	light emission during friction or crushing, not prectical for lighting
thermoluminescence	quartz, feldspar, metastable ion dopants in crystals	light emission during heating, often shows irradiation memory, used e.g. for archaeological dating of pottery Ref. 191

TABLE 17 (Continued) Selected matter properties related to electromagnetism.

PROPERTY	EXAMPLE	DEFINITION
sonoluminescence	air in water	light emission during cavitation
gravitoluminescence	does not exist; Challenge 213 s why?	
bremsstrahlung	X-ray generation	radiation emission through fast deceleration of electrons
Čerenkov effect	water, polymer particle detectors	light emission in a medium due to particles, e.g. emitted by radioactive processes, moving faster than the speed of light in that medium
transition radiation	any material	light emission due to fast particles moving from one medium to a second with different refractive index
Non-linear interactions with light		
laser activity, superradiation	beer, ruby, He–Ne, etc.	emission of stimulated radiation
quantum cascade laser	semiconductor multilayers	emission of stimulated infrared radiation through intersubband transitions
second, third n-th harmonic generation	$LiNbO_3$, KH_2PO_4	light partially transformed to double, threefold, n-fold frequency
phase conjugating mirror activity	gaseous CS_2, solid $Bi_{12}SiO_{20}$	reflection of light with locally opposite phase
additional optical nonlinear effects: parametric amplification, frequency mixing, saturable absorption, n-th harmonic generation, optical Kerr effect, Raman amplification, stimulated Brillouin scattering, etc.		
Interactions with vacuum		
Casimir effect	metals	attraction of uncharged, conducting bodies
General mechanical and thermal material properties		
solidity, impenetrability	floors, columns, ropes, buckets	at most one object per place at a given time
plasticity	metals	permanent deformation under stress
elasticity	solids	reversible deformation under stress
derroelasticity	Ni-Ti alloys	spontaneous strain
viscosity	liquids, solids	deformation under stress due to component motion
heat capacity and heat conductivity	silver, marble, air	ability to store and to transport disordered atomic motion
Any other everyday material property	every material	

All matter properties given in the list can be influenced by electromagnetic fields or

directly depend on them. This shows in detail:

▷ The nature of all everyday material properties is electromagnetic.

In other words, electric charges and their interactions are an essential and fundamental part of the structure of objects. The table shows so many different electromagnetic properties that the motion of charges inside each material must be complex indeed. Most effects are the topic of solid state physics,* fluid physics or plasma physics.

Solid state physics is by far the most important part of physics, when measured by the impact it has on society. Almost all its effects have applications in technical products, and give employment to many people. Can you name a product or business application for any randomly chosen effect from the table?

Challenge 214 e

In our mountain ascent however, we look at only one example from the above list: thermal radiation, the emission of light by hot bodies.

All bodies emit radiation

Earnshaw's theorem about the impossibility of a stable equilibrium for charged particles at rest implies that the charges inside matter must be *moving*. For any charged particle in motion, Maxwell's equations for the electromagnetic field show that it radiates energy by emitting electromagnetic waves. In short, we predict that all matter must radiate electromagnetic energy.

Interestingly, we know from experience that this is indeed the case. Hot bodies light up depending on their temperature; the working of light bulbs thus proves that metals are made of charged particles. *Incandescence*, as it is called, requires charges. Actually, *every* body emits radiation, even at room temperature. This radiation is called *thermal radiation*; at room temperature it lies in the infrared. Its intensity is rather weak in everyday life; it is given by the general expression

Ref. 192

$$I(T) = f T^4 \, \frac{2\pi^5 k^4}{15 c^2 h^3} \quad \text{or} \quad I(T) = f \sigma T^4 \quad \text{with} \quad \sigma = 56.7\,\mathrm{nW/K^4m^2}\,, \qquad (85)$$

where f is a material-, shape- and temperature-dependent factor, with a value between zero and one, and is called the *emissivity*. The constant σ is called the *Stefan–Boltzmann black body radiation constant* or *black body radiation constant*. A body whose emissivity is given by the ideal case $f = 1$ is called a *black body*, because at room temperature such a body also has an ideal absorption coefficient and thus appears black. (Can you see why?) The heat radiation such a body emits is called *black body radiation*.

Challenge 215 s

By the way, which object radiates more energy: a human body or an average piece of the Sun of the same mass? Guess first!

Ref. 193
Challenge 216 s

* Probably the best and surely the most entertaining introductory English language book on the topic is the one by Neil Ashcroft & David Mermin, *Solid State Physics*, Holt Rinehart & Winston, 1976.

Challenges and curiosities about electromagnetic effects

'Inside a conductor there is no electric field.' This statement is often found. In fact the truth is not that simple. First, a *static* field or a *static* charge on the metal surface of a body does not influence fields and charges inside it. A closed metal surface thus forms a shield against an electric field. Can you give an explanation? In fact, a tight metal layer is not required to get the effect; a cage is sufficient. One speaks of a *Faraday cage*. Challenge 217 s

The detailed mechanism allows you to answer the following question: do Faraday cages for gravity exist? Why?

For *moving* external fields or charges, the issue is more complex. Fields due to accelerated external charges – radiation fields – decay exponentially through a shield. Fields due to external charges moving at constant speed are strongly reduced, but do not disappear. The reduction depends on the thickness and the resistivity of the metal enclosure used. For sheet metal, the field suppression is very high; it is not necessarily high for metal sprayed plastic. Plastic shields will not necessarily protect a device from a close lightning stroke. Ref. 194

In practice, there is no danger if lightning hits an aeroplane or a car, as long they are made of metal. (There is one film on the internet of a car hit by lightning; the driver does not even notice.) However, if your car is hit by lightning in dry weather, you should wait a few minutes before getting out of it. Can you imagine why?

Faraday cages also work the other way round. (Slowly) changing electric fields that are inside a Faraday cage are not felt outside. For this reason, radios, mobile phones and computers are surrounded by boxes made of metal or metal-sprayed plastics. The metal keeps the so-called *electromagnetic smog* to a minimum.

There are thus three reasons to surround electric appliances by a grounded shield: to protect the appliance from outside fields, to protect people and other machines from electromagnetic smog, and to protect people against the mains voltage accidentally being fed into the box (for example, when the insulation fails). In high precision experiments, these three functions can be realized by three separate cages.

For purely magnetic fields, the situation is more complex. It is quite difficult to shield the inside of a machine from outside magnetic fields. How would you do it? In practice one often uses layers of so-called *mu-metal*; can you guess what this material does? Challenge 218 s

* *

Ref. 195 Researchers are trying to detect tooth decay with the help of electric currents, using the observation that healthy teeth are bad conductors, in contrast to teeth with decay. How would you make use of this effect in this case? (By the way, it might be that the totally unrelated technique of imaging with terahertz waves could yield similar results.) Challenge 219 ny

* *

Ref. 196 Human bone is piezoelectric: it produces electric signals when stressed. When we move and grow, the electric signals are used by the body to reinforce the bones in the regions that are in need. The piezoelectricity of the bones thus controls and guides their growth. This connection is also used to make fractured bones heal more rapidly: by applying *pulsed* magnetic fields to a broken bone, the healing is stimulated and accelerated. (Static magnetic fields obviously do not work for this aim.) Also teeth are piezoelectric, and the

effect plays also a role in their growth.

* *

In shops, one can buy piezoelectric devices – similar to a gas lighter – that are applied to mosquito bites and are said to reduce itching and even swelling. (Some product names are 'zanza click' and 'skeeter click') Can these claims be true?
Challenge 220 e

* *

A team of camera men in the middle of the Sahara were using battery-driven electrical equipment to make sound recordings. Whenever the microphone cable was a few tens of metres long, they also heard a 50 Hz power supply noise, even though the next power supply was hundreds of kilometres away. An investigation revealed that the high voltage lines in Europe lose a considerable amount of power by irradiation; these 50 Hz waves are reflected by the ionosphere around the Earth and thus can disturb recording in the middle of the desert. Can you estimate whether this observation implies that living directly near a high voltage line is dangerous?
Challenge 221 s

* *

When solar plasma storms are seen on the Sun, astronomers first phone the electricity company. They know that about 24 to 48 hours later, the charged particles ejected by the storms will arrive on Earth, making the magnetic field on the surface fluctuate. Since power grids often have closed loops of several thousands of kilometres, additional electric currents are induced, which can make transformers in the grid overheat and then switch off. Other transformers then have to take over the additional power, which can lead to their overheating, etc. On several occasions in the past, millions of people have been left without electrical power due to solar storms. Today, the electricity companies avoid the problems by disconnecting the various grid sections, by avoiding large loops, by reducing the supply voltage to avoid saturation of the transformers and by disallowing load transfer from failed circuits to others.

* *

If the electric field is described as a sum of components of different frequencies, its so-called *Fourier components*, the amplitudes are given by
Ref. 197

$$\hat{\boldsymbol{E}}(k,t) = \frac{1}{(2\pi)^3/2} \int \boldsymbol{E}(x,t) \mathrm{e}^{-i\boldsymbol{k}x} \,\mathrm{d}^3x \tag{86}$$

and similarly for the magnetic field. It then turns out that a Lorentz invariant quantity N, describing the energy per circular frequency ω, can be defined:

$$N = \frac{1}{8\pi} \int \frac{|\boldsymbol{E}(k,t)|^2 + |\boldsymbol{B}(k,t)|^2}{c|\boldsymbol{k}|} \,\mathrm{d}^3k \,. \tag{87}$$

Challenge 222 s
Can you guess what N is physically? (Hint: think about quantum theory.)

* *

Page 46 Faraday discovered, as told above, how to change magnetism into electricity, knowing that electricity could be transformed into magnetism. The issue is subtle. Faraday's law is not the dual of Ampère's, as that would imply the use of magnetic monopoles; neither is it the reciprocal, as that would imply the displacement current. But he was looking for a link and he found a way to relate the two observations – in a novel way, as it turned out.

Faraday also discovered how to transform electricity into light and into chemistry. He then tried to change gravitation into electricity. But he was not successful. Why not? Challenge 223 s

* *

At high altitudes (60 km to 1000 km) above the Earth, gases are partly or completely ionized; no atom is neutral. One speaks of the *ionosphere*, as space is full of positive ions and free electrons. Even though both charges appear in exactly the same number, a satellite moving through the ionosphere acquires a negative charge. Why? How does the charging stop? Vol. I, page 353 Challenge 224 s

* *

A capacitor of capacity C is charged with a voltage U. The stored electrostatic energy is $E = CU^2/2$. The capacitor is then detached from the power supply and branched on to an empty capacitor of the same capacity. After a while, the voltage obviously drops to $U/2$. However, the stored energy now is $C(U/2)^2$, which is half the original value. Where did the energy go? Challenge 225 s

* *

Challenge 226 s How can you give somebody an electric shock using a 4.5 V battery and some wire?

* *

An old puzzle about electricity results from the equivalence of mass and energy. It is known from experiments that the size d of electrons is surely smaller than 10^{-22} m. This means that the electric field surrounding it has an energy content E given by at least Ref. 198 Challenge 227 e

$$E_{\text{nergy}} = \frac{1}{2}\varepsilon_0 \int E^2_{\text{lectric field}}\, dV = \frac{1}{2}\varepsilon_0 \int_d^\infty \left(\frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\right)^2 4\pi r^2 dr$$
$$= \frac{q^2}{8\pi\varepsilon_o}\frac{1}{d} > 1.2\,\mu\text{J} . \qquad (88)$$

On the other hand, the *mass* of an electron, usually given as 511 keV/c^2, corresponds to an energy of only 82 fJ, ten million times *less* than the value just calculated. In other words, classical electrodynamics has considerable difficulty describing electrons.

In fact, a consistent description of charged point particles within classical electrodynamics is impossible. This topic receives only a rare – but then often passionate – interest nowadays, because the puzzle is solved in a different way in the quantum parts of our adventure. Ref. 199

* *

Even though the golden days of materials science are over, the various electromagnetic

properties of matter and their applications in devices do not seem to be completely explored yet. About once a year a new effect is discovered that merits inclusion in the list of
Page 216 electromagnetic matter properties of Table 17. Among others, some newer semiconductor technologies will still have an impact on electronics, such as the recent introduction of low cost light detecting integrated circuits built in CMOS (complementary metal oxide silicon) technology.

* *

The building of light sources of high quality has been a challenge for many centuries and remains one for the future. Light sources that are intense, tunable and with large coherence length or sources that emit extreme wavelengths are central to many research pursuits. As one example of many, the first X-ray lasers have recently been built; however, they are several hundred metres in size and use modified particle accelerators. The constructionof compact X-ray lasers is still many years off – if it is possible at all.

* *

In many materials, left and right circularly polarized light is absorbed differently. The effect, called *circular dichroism*, was discovered by Aimé Cotton in 1896. Since circular dichroism appears in optically active chiral molecules, the measurement of circular dichroism spectra is a simple and important method for the structure determination of biological molecules.

* *

Effects of atmospheric electricity are also observed around waterfalls. Various studies have shown that large waterfalls produce negatively charged water droplets in the air around them. It even seems that inhaling these droplets is healthy, especially for people with asthma.

* *

But maybe the biggest challenge imaginable in classical electrodynamics is to decode the currents inside the brain. Will it be possible to read our thoughts with an apparatus placed outside the head?

Challenge 228 r One could start with a simpler challenge: Would it be possible to distinguish the thought 'yes' from the thought 'no' by measuring electrical or magnetic fields around the head? In other words, is simple mind-reading possible? The answer is yes, as the feat has already been achieved. Even more, using brain imaging, it is already possible to distinguish between simple concepts that a person has in mind. Ref. 200

Page 90 As we have seen above, partial mind-reading is also possible already for motion-related tasks, including some video games.

In fact, it is now possible to use a cap with electrical contacts and use passwords that you simply think about to secure computer systems. The advantage of such a password is that it is hard to steal. (Is this system secure?) Challenge 229 s

The twenty-first century will surely bring many new results also for the mind reading of cognitive tasks. The team first performing such a feat will become instantly famous.

Chapter 6

SUMMARY AND LIMITS OF CLASSICAL ELECTRODYNAMICS

All of classical electrodynamics can be summarized in three principles. Every adventurer should know them, because they will help us later on, when we approach the top of Motion Mountain. We will discover that we can arrive at the top only if we express things as simply as possible. The three principles of classical electrodynamics are:

> ▷ Electric charges exert forces on other charges; for charges at rest, the force falls off as the inverse square of the distance.* Equivalently, charges are surrounded by an *electromagnetic field*.
>
> ▷ Electric charges are conserved.
>
> ▷ Charges move more slowly than light. Equivalently, all charged particles have mass.

From these three principles we can deduce all of electrodynamics. Electrodynamics is thus built on the definition of charge, the conservation of charge, and the invariance of the speed of light. In particular, we can deduce the following *basic statements*: Ref. 39

— The electromagnetic field is a physical observable, as shown e.g. by compass needles.
— The sources of the electromagnetic field are the (moving) charges, as shown by amber, lodestone or mobile phones.
— The electromagnetic field changes the motion of electrically charged objects via the Lorentz expression as shown, for example, by electric motors.
— The electromagnetic field can exist in empty space and moves in it as a wave, as shown, for example, by the light from the stars.
— The electromagnetic field behaves like a continuous quantity and is described by Maxwell's evolution equations, as shown, for example, by mobile phones and electric toothbrushes.

More precisely, the motion of the electric field **E** and of the magnetic field **B** is described by the Lagrangian density

$$\mathcal{L} = \frac{\varepsilon_0}{2}E^2 - \frac{1}{2\mu_0}B^2 \,. \qquad (89)$$

* Quantum theory will show that this principle, Coulomb's 'law', can be rephrased as: electric charges at rest emit virtual photons with a constant average rate.

Like for any motion described by a Lagrangian, the motion of the field is reversible, continuous, conserved and deterministic. However, there is quite some fun in the offing; even though this description is correct in everyday life, during the rest of our mountain ascent we will find that the last basic statement must be wrong: fields do not always follow Maxwell's equations. A simple example shows this.

At a temperature of zero kelvin, when matter does not radiate thermally, we have the paradoxical situation that the charges inside matter cannot be moving, since no emitted radiation is observed, but they cannot be at rest either, due to Earnshaw's theorem. In short, the simple existence of matter – with its charged constituents – shows that classical electrodynamics is wrong.

Page 216 In fact, the overview of the numerous material properties and electromagnetic effects given in Table 17 makes the same point even more strongly; classical electrodynamics can describe many of the effects listed, *but it cannot explain the origin and numerical values of any of them*. Even though few of the effects will be studied in our walk – they are not essential for our adventure – the general concepts necessary for their description will be the topic of the upcoming part of this mountain ascent, that on quantum physics.

In fact, classical electrodynamics fails in *two* domains.

Strong fields and gravitation

First of all, classical electrodynamics fails in regions with extremely strong fields. When electromagnetic fields are extremely strong, their energy density will *curve* space-time. Classical electrodynamics, which assumes flat space-time, is not valid in such situations.

The failure of classical electrodynamics is most evident in the most extreme case of all: when the fields are extremely strong, they will lead to the formation of black holes. The existence of black holes, together with the discreteness of charge, imply maximum electric and magnetic field values. These upper limits were mentioned in Table 3, which lists various electric field values found in nature, and in Table 8, which lists possible magnetic field values. Can you deduce the values of these so-called *Planck fields*?

Page 26 Page 36 Challenge 230 s

The interplay between curvature of space and electrodynamics has many aspects. For example, the maximum force in nature limits the maximum charge that a black hole can carry. Can you find the relation? As another example, it seems that magnetic fields effectively increase the stiffness of empty space, i.e., they increase the difficulty to bend empty space. Not all interactions between gravity and electrodynamics have been studied up to now; more examples should appear in the future.

Vol. II, page 103 Challenge 231 ny Ref. 201

In summary, classical electrodynamics does not work for extremely high field values, when gravitation plays a role.

Charges are discrete

Classical electrodynamics fails to describe nature correctly also for extremely weak field values. This happens also in flat space-time and is due to a reason already mentioned a number of times: *electric charges are discrete*. Electric charges do not vary continuously, but change in fixed steps. Not only does nature show a smallest value of entropy – as we found in our exploration of heat, – and smallest amounts of matter; nature also shows a smallest charge. *Electric charge values are quantized.*

Vol. I, page 369 Vol. I, page 371

In metals, the quantization of charge is noticeable in the flow of electrons. In elec-

trolytes, i.e., electrically conducting liquids, the quantization of charge appears in the flow of charged atoms, usually called *ions*. All batteries have electrolytes inside; also water is an electrolyte, though a poorly conducting one. In plasmas, like fire or fluorescent lamps, both ions and electrons move and show the discreteness of charge. Also in all known types of particle radiation – from the electron beams inside cathode ray tubes in televisions, the channel rays formed in special low-pressure glass tubes, the cosmic radiation hitting us all the time, up to the omnipresent radioactivity – charges are quantized.

In all known experiments, the same smallest value e for electric charge has been found. The most precise result is

$$e = 0.160\,217\,656\,5(35)\,\mathrm{aC}\ , \tag{90}$$

around a sixth of an attocoulomb. All observed electric charges in nature are multiples of this so-called *elementary charge*.

In short, like all flows in nature, also the flow of electricity is due to a flow of discrete particles. In fact, the nature of the charged particles differs from situation to situation: they may be electrons, ions, muons and many kind of other particles. However, the charge steps are always exactly the same. In fact, at this point of our adventure, the equality of the elementary charge for all matter particles is unexplained. We will discover the reason only at the very end of our adventure.

Above all, a smallest charge change has a simple implication:

▷ Classical electrodynamics is *wrong*.

Classical electrodynamics is just a good approximation for medium-sized field values. Indeed, a smallest charge implies that no infinitely small test charges exist. But such infinitely small test charges are necessary to *define* electric and magnetic fields. For a *finite* test charge, the disturbance of the field introduced by the test charge itself makes a precise field measurement – and thus a precise field definition – impossible. As a consequence, the values of electric and magnetic field measured with finite test charges are always somewhat fuzzy. This fuzziness is most apparent for low field values. For example, for low intensities of light, experiments detect *photons*, discrete light particles. All low light intensities are time-averages of low photon numbers; they are not continuous fields. Page 25

The lower limit on charge magnitude also implies that there is no fully correct way of defining an instantaneous electric current in classical electrodynamics. Indeed, the position and the momentum of a charge are always somewhat fuzzy, as we will find out. Vol. IV, page 168

In summary, Maxwell's evolution equations are only *approximate*. Classical electromagnetism does not work for extremely low field values, when quantum effects play a role, and does not work for extremely high field values, when gravitation plays a role. These two extreme cases will be explored in the remaining legs of our adventure, those on quantum theory and those on unification. Only some effects of the discreteness of charge can be treated in classical physics; a few instructive examples follow.

How fast do charges move?

In a vacuum, such as inside a colour television tube or inside an electron microscope, charged particles accelerated by a voltage of 30 kV move with a third of the speed of light. At higher voltage, the speed is even higher. In modern particle accelerators charges move so rapidly that their speed is indistinguishable from that of light for all practical purposes.

Challenge 232 s

Inside a metal, electric signals move with speeds of the order of the speed of light. The precise value depends on the capacity and impedance of the cable and is usually in the range $0.3c$ to $0.5c$. This high speed is due to the ability of metals to easily take in arriving charges and to let others depart. The ability for rapid reaction is due to the high mobility of the charges inside metals, which in turn is due to the properties of metallic bonds and to the small mass and size of the involved charges, the electrons.

The high signal speed in metals appears to contradict another determination. The *drift speed* v of the electrons in a metal wire, i.e., the average speed of the charges, obviously obeys

$$v = \frac{I}{Ane} , \tag{91}$$

where I is the current, A the cross-section of the wire, e the charge of a single electron and n the number density of electrons. The electron density in copper is $8.5 \cdot 10^{28}\,\mathrm{m}^{-3}$. Using a typical current of 0.5 A and a typical cross-section of a square millimetre, we get a drift speed of 0.37 µm/s. In other words, electrons move a thousand times slower than ketchup inside its bottle. Worse, if a room lamp used direct current instead of alternate current, the electrons would take several days to get from the switch to the bulb! Nevertheless, the lamp goes on or off almost immediately after the switch is activated. Similarly, the electrons from an email transported with direct current would arrive much later than a paper letter sent at the same time; nevertheless, the email arrives quickly. Why?

Water pipes show a similar effect. A long hose provides water almost in the same instant as the tap is opened, even if the water takes a long time to arrive from the tap to the end of the hose. The speed with which the water reacts, the signal speed, is given by the speed for pressure waves, or sound waves, in water. For water hoses, the signal speed is much higher than the speed of the water flow, much higher than the speed of the molecules.

Also everyday life provides us with a similar effect. Imagine a long queue of cars (representing electrons) waiting in front of a red traffic light. In an ideal world, all drivers look at the light. As soon as the light turns green, everybody starts driving. Even though the driving speed might be only 10 m/s, the speed of traffic flow onset was that of light. It is this latter speed which is the signal speed. The signal speed is much higher than the speed of the cars.

In short, inside a metal, the electrons move slowly; the speed of electrical signals is not given by the electron speed, but by the speed of electron density waves, which in turn is due to the electromagnetic field. In fact, a typical house has only an alternating current supply. In this typical case, the electrons inside the copper wires only vibrate back and forwards by a tiny distance, as you might want to check.

Challenge 233 e

Inside liquids, charges move with a different speed from that inside metals, and their

charge to mass ratio is also different. We all know this from direct experience. Our *nerves* work by using electric signals and take a few milliseconds to respond to a stimulus, even though they are (only) metres long. A similar speed is observed inside batteries. In all these systems, moving charge is transported by ions. *Ions* are charged atoms. Ions, like atoms, are large, composed and heavy entities, in contrast to the tiny and light electrons. As a result, ions move much more slowly than electrons do. Our limited reaction time is a consequence of ion motion.

In still other matter systems, charges move both as electrons and as ions. Examples are neon lamps, fire, plasmas and the Sun. This leads us to ask:

What motion occurs inside atoms?

Inside atoms, electrons behave even more strangely. We tend to imagine that they orbit the nucleus (as we will see later) at a rather high speed, as the orbital radius is so small. However, it turns out that in most atoms many electrons do not orbit the nucleus at all: many electrons have no orbital angular momentum around the nucleus. How can this be?

Worse, some electrons do have orbital momentum. But if these electrons were orbiting the atomic nucleus like planets orbit the Sun, they would move under constant acceleration. Thus they would emit electromagnetic radiation until they would fall into the nucleus. But this is not the case: atoms are stable! How can this be?

And why are all atoms of the same size anyway? Atom size should depend on the angular momentum of the electrons inside it. But what determines the orbital momentum of electrons around the nucleus?

We will discover soon that there is a smallest angular momentum in nature that fixes the size of atoms. And we will discover that moving electrons, in contrast to everyday objects, are *not* described by trajectories in space, thus allowing atoms to be stable. The strange story of atoms and their structure will be told in the quantum legs of our
Vol. IV, page 178 adventure, in the volumes following this one.

Challenges and curiosities about charge discreteness

Challenge 234 s How would you show experimentally that electrical charge comes in smallest chunks?

* *

The discreteness of charge implies that we can estimate the size of atoms by observing
Challenge 235 ny galvanic deposition of metals. How?

* *

Cosmic radiation consists of charged particles hitting the Earth. (We will discuss this
Vol. V, page 156 in more detail later.) Astrophysicists explain that these particles are accelerated by the
Ref. 202 magnetic fields around the Galaxy. However, the expression of the Lorentz acceleration shows that magnetic fields can only change the direction of the velocity of a charge, not
Challenge 236 ny its magnitude. How can nature get acceleration nevertheless?

* *

What would be the potential of the Earth in volt if we could take far away all the electrons

Challenge 237 s of a drop of water?

* *

When a voltage is applied to a resistor, how long does it take until the end value of the current, given by Ohm's 'law', is reached? The first to answer this question was Paul Drude* in the years around 1900. He reasoned that when the current is switched on, the speed v of an electron increases as $v = (eE/m)t$, where E is the electrical field, e the charge and m the mass of the electron. Drude's model assumes that the increase of electron speed stops when the electron hits an atom, loses its energy and begins to be accelerated again. Drude deduced that the average time τ up to the collision is related to the specific resistance by

$$\rho = \frac{E}{j} = \frac{E}{env} = \frac{2m}{\tau e^2 n} , \qquad (92)$$

with n being the electron number density. The right side does not depend on E any more; it is a constant. Drude had thus explained *Ohm's 'law'* $U = RI$ (or $E = j\rho$) from material properties, by assuming that resistance is due to moving electrons that continuously collide and speed up again. Inserting numbers for copper ($n = 8.5 \cdot 10^{28}\,/\mathrm{m}^{-3}$ and $\rho = 0.16 \cdot 10^{-7}\,\Omega\mathrm{m}$), we get a time $\tau = 51\,\mathrm{ps}$. This time is so short that the switch-on process can usually be neglected.

* *

Does it make sense to write Maxwell's equations *in vacuum*? Both electrical and magnetic fields require charges in order to be measured. But in vacuum there are no charges at all. And fields are defined by using infinitesimally small test charges. But, as we mentioned already, infinitesimally small charges do not exist. In fact, only quantum theory solves
Challenge 238 d this issue. Are you able to imagine how?

* *

We have seen that in cases of mid-sized fields, classical electrodynamics is a good approximation, despite charge discreteness. One practical system makes use of discrete charge but can nevertheless be described in many of its aspects with classical electrodynamics. It merits a separate discussion: our brain.

* Paul Karl Ludwig Drude (b. 1863 Braunschweig, d. 1906 Berlin), physicist, predicted with his electron gas model of metals – that the ratio between the thermal conductivity and the electric conductivity at a given temperature should be the same for all metals; this is roughly correct. Drude also conceived ellipsometry and introduced c as the symbol for the speed of light.

Chapter 7

THE STORY OF THE BRAIN

> Alles was überhaupt gedacht werden kann,
> kann klar gedacht werden.*
> Ludwig Wittgenstein, *Tractatus*, 4.116

In our quest for increased precision in the description of all motion around us, it is time to take a break, sit down and look back. In our walk so far, which has led us to investigate mechanics, general relativity and electrodynamics, we used several concepts without defining them. Examples are 'information', 'memory', 'measurement', 'set', 'number', 'infinity', 'existence', 'universe' and 'explanation'. Each of these is a common and important term. In this intermezzo, we take a look at these concepts and try to give some simple, but sufficiently precise definitions, keeping them as provocative and entertaining as possible. For example, can you explain to your parents what a concept is? Challenge 239 e

The reason for studying the definitions of concepts is simple. We need the clarifications in order to get to the top of Motion Mountain, i.e., to the full description of motion. In the past, many have lost their way because of lack of clear concepts. In order to avoid these difficulties, physics has a special guiding role. All sciences share one result: *every type of change observed in nature is a form of motion*. In this sense, but in this sense *only*, physics, focusing on motion itself, forms the basis for all the other sciences. In other words, the search for the famed 'theory of everything' is an arrogant expression for the search for a *final theory of motion*. Even though the knowledge of motion is basic, its precise description does not imply a description of 'everything': just try to solve a marriage problem using the Schrödinger equation to note the difference.

Given the basic importance of motion, it is necessary that in physics all statements on observations be as *precise* as possible. For this reason, many thinkers have investigated physical statements with particular care, using all criteria imaginable. *Physics is precise prattle by curious people about moving things.* What does precision mean? The meaning appears once we ask: which abilities does such prattle require? You might want to fill in the list yourself before reading on. Challenge 240 e

The abilities necessary for talking are a topic of research even today. The way that the human species acquired the ability to chat about motion is studied by evolutionary biologists. Child psychologists study how the ability develops in a single human being. Physiologists, neurologists and computer scientists are concerned with the way the brain

* 'Everything that can be thought at all can be thought clearly.' This and other quotes of Ludwig Wittgenstein are from the equally short and famous *Tractatus logico-philosophicus*, written in 1918, first published in 1921; it has now been translated into many other languages.

FIGURE 160 Ludwig Wittgenstein (1889–1951).

and the senses make this possible; linguists focus on the properties of the language we use, while logicians, mathematicians and philosophers of science study the general properties of statements about nature. All these fields investigate tools that are essential for the development of physics, the understanding of motion and the specification of the undefined concepts listed above. The fields structure the following exploration.

Evolution

> “A hen is only an egg's way of making another egg.”
>
> Samuel Butler, *Life and Habit.*

The evolution of the human species is the result of a long story that has been told in many excellent books. Ref. 203 A summarizing table on the history of the universe was given in the exploration of general relativity. Vol. II, page 223 The almost incredible chain of events that has lead to one's own existence includes the formation of atoms, of the galaxies, the stars, the planets, the Moon, the atmosphere, the oceans, the first cells, the water animals, the land animals, the mammals, the hominids, the humans, the ancestors, the family and finally, oneself. Ref. 204

The way the atoms we are made of moved during this sequence, being blown through space, being collected on Earth, becoming organized to form organic matter and then people, is one of the most awe-inspiring examples of motion. Remembering and meditating about this cosmic sequence of motion every now and then can be an enriching experience. Challenge 241 e

In particular, without biological evolution, we would not be able to talk about motion at all; only moving bodies can study moving bodies. Without evolution, we would have no muscles, no senses, no nerves and no brains. And without a brain, we would not be able to think or talk. Evolution was also the fount of childhood and curiosity. In this chapter we will discover that most concepts of classical physics are already introduced by every little child, in the experiences it has while growing up.

Children, laws and physics

> “Physicists also have a shared reality. Other than that, there isn't really a lot of difference between being a physicist and being a schizophrenic.”
>
> Richard Bandler

Ref. 205

During childhood, everybody is a physicist. When we follow our own memories backwards in time as far as we can, we reach a certain stage, situated before birth, which forms Ref. 206

the starting point of human experience. In that magic moment, we sensed somehow that apart from ourselves, there is something else. The first observation we make about the world, during the time in the womb, is thus the recognition that we can distinguish two parts: ourselves and the rest of the world. This distinction is an example – perhaps the first – of a large number of 'laws' of nature that we stumble upon in our lifetime. Being a physicist started back then. And it continued. By discovering more and more distinctions we bring structure in the chaos of experience. We quickly find out that the world is made of *related parts*, such as mama, papa, milk, earth, toys, etc. We divide the parts in objects and images.

Later, when we learn to speak, we enjoy using more difficult words and we call the Vol. I, page 27 surroundings the *environment*. Depending on the context, we call the whole formed by oneself and the environment together the (physical) *world*, the (physical) *universe*, *nature*, or the *cosmos*. These concepts are not distinguished from each other in this walk;* they are all taken to designate the sum of all parts and their relations. They are simply taken here to designate the *whole*.

The discovery of the first distinction in nature starts a chain of similar discoveries that continue throughout our life. We extract the numerous distinctions that are possible in the environment, in our own body and in the various types of interactions between them. The ability to distinguish is the central ability that allows us to change our view from that of the world as *chaos*, i.e., as a big mess, to that of the world as a *system*, i.e., a structured Challenge 242 s set, in which parts are related in specific ways. (If you like precision, you may ponder whether the two choices of 'chaos' and 'system' are the only possible ones.)

In particular, the observation of the differences between oneself and the environment goes hand in hand with the recognition that not only are we not independent of the environment, but we are firmly tied to it in various inescapable ways: we can fall, get hurt, feel warm, cold, etc. Such relations are called *interactions*. Interactions express the observation that even though the parts of nature can be distinguished, they cannot be Page 298 isolated. In other words, interactions describe the difference between the whole and the sum of its parts. No part can be defined without its relation to its environment. (Do you Challenge 243 e agree?)

Interactions are not arbitrary; just take touch, smell or sight as examples. They differ in reach, strength and consequences. We call the characteristic aspects of interactions *patterns of nature*, or *properties of nature*, or *rules of nature* or, equivalently, with their historical but unfortunate name, *'laws' of nature*. The term 'law' stresses their general validity; unfortunately, it also implies design, aim, coercion and punishment for infringement. However, no design, aim or coercion is implied in the properties of nature, nor is infringement possible. The ambiguous term 'law of nature' was made popular by René Descartes (b. 1596 La Haye en Touraine, d. 1650 Stockholm) and has been adopted enthusiastically because it gave weight to the laws of the state – which were far from perfect at that time – and to those of other organizations – which rarely are. The expression is an anthropomorphism coined by an authoritarian world view, suggesting that nature is

* The differences in their usage can be deduced from their linguistic origins. 'World' is derived from old Germanic 'wer' – person – and 'ald' – old – and originally means 'lifetime'. 'Universe' is from the Latin, and designates the one – 'unum' – which one sees turning – 'vertere', and refers to the starred sky at night which turns around the polar star. 'Nature' comes also from the Latin, and means 'what is born'. 'Cosmos' is from Greek κόσμος and originally means 'order'.

'governed'. We will therefore use the term as rarely as possible in our walk and it will, if we do, be always between 'ironical' parentheses. Nature cannot be forced in any way. The 'laws' of nature are not obligations for nature or its parts, they are obligations only for physicists and all other people: the patterns of nature oblige us to use certain descriptions and to discard others. Whenever one says that 'laws govern nature' one is talking nonsense (or asking for money); the correct expression is *rules describe nature*.

During childhood we learn to distinguish between interactions with the environment (or *perceptions*): some are shared with others and called *observations*, others are uniquely personal and are called *sensations*.* A still stricter criterion of 'sharedness' is used to divide the world into 'reality' and 'imagination' (or 'dreams'). Our walk will show – at the very end – that this distinction is not essential, provided that we stay faithful to the quest for ever increasing precision: we will find, surprisingly, that the description of motion that we are looking for does not depend on whether the world is 'real' or 'imagined', Vol. VI, page 384 'personal' or 'public'. The fundamental principles of motion in reality and in dreams are the same. Nevertheless, these same principles allow us to distinguish the two.

Humans enjoy their ability to distinguish parts, which in other contexts they also call *details*, *aspects* or *entities*, and enjoy their ability to associate them or to observe the *re-* Ref. 208 *lations* between them. Humans call this activity *classification*. Colours, shapes, objects, mother, places, people and ideas are some of the entities that humans discover first.

Our anatomy provides a handy tool to make efficient use of these discoveries: *memory*. It stores a large amount of input that is called *experience* afterwards. Memory is a tool used by both young and old children to organize their world and to achieve a certain security in the chaos of life.

Memorized classifications are called *concepts*. Jean Piaget was the first researcher to describe the influence of the environment on the concepts that every child forms. Step by step, children learn that objects are localized in space, that space has three dimensions, that objects fall, that collisions produce noise, etc. In particular, Piaget showed that space and time are not a priori concepts, but result from the interactions of every child with its environment.**

* A child that is unable to make this distinction among perceptions – and who is thus unable to lie – almost Ref. 207 surely develops or already suffers from *autism*, as recent psychological research has shown.

** An overview of the origin of developmental psychology is given by J. H. Flavell, *The Developmental Psychology of Jean Piaget*, 1963. This work summarizes the observations by the Jean Piaget (b. 1896 Neuchâtel, d. 1980 Geneva), the central figure in the field. He was one of the first researchers to look at child development in the same way that a physicist looks at nature: carefully observing, taking notes, making experiments, extracting hypotheses, testing them, deducing theories. His astonishingly numerous publications, based on his extensive observations, cover almost all stages of child development. His central contribution is the detailed description of the stages of development of the cognitive abilities of humans. He showed that all cognitive abilities of children, the formation of basic concepts, their way of thinking, their ability to talk, etc., result from the continuous interaction between the child and the environment.

In particular, Piaget described the way in which children first learn that they are different from the external environment, and how they then learn about the physical properties of the world. Of his many books related to physical concepts, two especially related to the topic of this walk are J. Piaget, *Les notions de mouvement et de vitesse chez l'enfant*, Presses Universitaires de France, 1972 and *Le developpement de la notion de temps chez l'enfant*, Presses Universitaires de France, 1981, this last book being born from a suggestion by Albert Einstein. These texts should be part of the reading of every physicist and science philosopher interested in these questions.

Piaget also describes how in children the mathematical and verbal intelligence derives from sensomoto-

Around the time that a child goes to school, it starts to understand the idea of *permanence of substances*, e.g. liquids, and the concept of *contrary*. Only at that stage does
Ref. 210 its subjective experience becomes *objective*, with abstract comprehension. Still later, the child's description of the world stops to be animistic: before this step, the Sun, a brook or a cloud are *alive*. In short, only after puberty does a human become ready for physics, the science of motion.

Even though everyone has been a physicist in their youth, most people stop at *Galilean* physics, where matter is approximated to be continuous and space to be flat. In the present adventure we go much further, by using all the possibilities of a toy with which nature provides us: the brain.

> Experience is the name everyone gives to their mistakes.
>
> Oscar Wilde, *Lady Windermere's Fan*.

Polymer electronics

The brain is electrical. This was definitely shown in 1924, when the neurologist Hans Berger (b. 1873 Neuses, d. 1941 Jena) recorded and named the first *electroencephalogram*. A modern electroencephalogram is shown in Figure 163.* In more detail, the brain is a flexible, metal-free, short-lived, sensitive, unreliable, electronic polymer device. Incidentally, all these properties are shared by all electronic polymer devices, whether alive or not. Higher reliability is the main reason that commercial electronics is usually silicon-based instead.

The polymer electronics that forms the brain is organized like a computer. Some de-
Ref. 211 tails are shown in Table 18, Figure 161 and Figure 162. Though the functional blocks of a brain and of a computer are astonishingly similar, the specific mechanisms they use are usually completely different.

Ref. 212 The brain consists of many parts dedicated to specific tasks and of to a general computing part, the grey matter. The division is almost fifty–fifty. Also the computing power in a modern computer is divided in this way; for example, graphics cards are often as powerful as the central processing unit.

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rial, practical intelligence, which itself stems from habits and acquired associations to construct new concepts. Practical intelligence requires the system of reflexes provided by the anatomical and morphological structure of our organism. Thus his work shows in detail that our faculty for mathematical description of the world is based, albeit indirectly, on the physical interaction of our organism with the world.

Some of his opinions on the importance of language in development are now being revised, notably
Ref. 209 through the rediscovery of the work of Lev Vigotsky, who argues that all higher mental abilities, emotions, recollective memory, rational thought, voluntary attention and self-awareness, are not innate, but learned. This learning takes place through language and culture, and in particular through the process of talking to oneself.

At www.piaget.org you can find the website maintained by the Jean Piaget Society.

* In the electric signals generated by the brain one distiguishes, *irregular signals* during data processing, *beta waves*, mainly during attention, with a frequency between 14 and 30 Hz, *alpha waves*, during relaxation, with a frequency between 8 and 13 Hz, *theta waves*, during early sleep and during rapid eye movement (REM) sleep, with a frequency between 3 and 7 Hz, and *delta waves*, during deep sleep, with a frequency between 0.5 and 2 Hz.

TABLE 18 Some aspects of the human brain.

Aspect	Details	Computer equivalent
Hardware		
Ultrashort term memory	5 to 9 concepts	cache
Hippocampus	novelty detector, spatial memory, learning	RAM and Flash memory
Amygdala	emotions, learning	priority scheduler of operating system
Ventral striatum, dopamine and opioid provider	rewards system	priority scheduler of operating system
Suprachiasmatic nucleus	day-night control	sleep controller
Neurons in cortex	women *c.* $19{\cdot}10^9$, men *c.* $22{\cdot}10^9$	hard disk and processor
Glial cells in brain	about as many as neurons	power supply, structure
Neuron number decay	women: $e^{3.05-0.00145\cdot \text{age}/a} \cdot 10^9$, men: $e^{3.2-0.00145\cdot \text{age}/a} \cdot 10^9$	hard disk scratching
Pulses exchanged between both brain halves	$4 \cdot 10^9/s$	internal bus speed
Synapses per neuron	10^4	
Total synapse connections	*c.* $2 \cdot 10^{14}$	memory cells
Input pathways from the eye	*c.* $2 \cdot 10^6$	camera wire
Input pathways from the ear	*c.* $2 \cdot 3000$	microphone line
Input pathways from skin, mouth, and nose	*c.* $0.5 \cdot 10^6$	sensor interfaces
Input signal capacity (total, 300 pulses/s per pathway)	*c.* 100 MB/s	input bandwidth
Output pathways (muscles, organs)	*c.* $1.5 \cdot 10^6$	actuator interfaces
Output signal capacity (total, 300 pulses/s per pathway)	*c.* 50 MB/s	output bandwidth
Non-serious – probably too low – estimate of the processing capacity	10 PFlop	several dozens of supercomputers
Typical mass (Einstein's brain)	1.230 kg; varies between 0.7 and 2.0 kg	1 to 5000 kg
Power consumption (average)	1600 to 2200 kJ/d or 18 to 25 W (with 750 ml/min blood supply)	20 W to 20 kW
Lifetime	130 years	often only 2 years
Size	0.14 m 0.17 m0.09 m	from a few cm^3 to 1 m^3
Software and processing		
Learning	changing synapse strength through long-term potentiation	activate, classify, store
Deep sleep and learning storage	structured writing from hippocampus to cortex	clean-up and back-up to hard disk
REM (rapid eye movement, or dream) sleep	offline processing	data compression in batch process

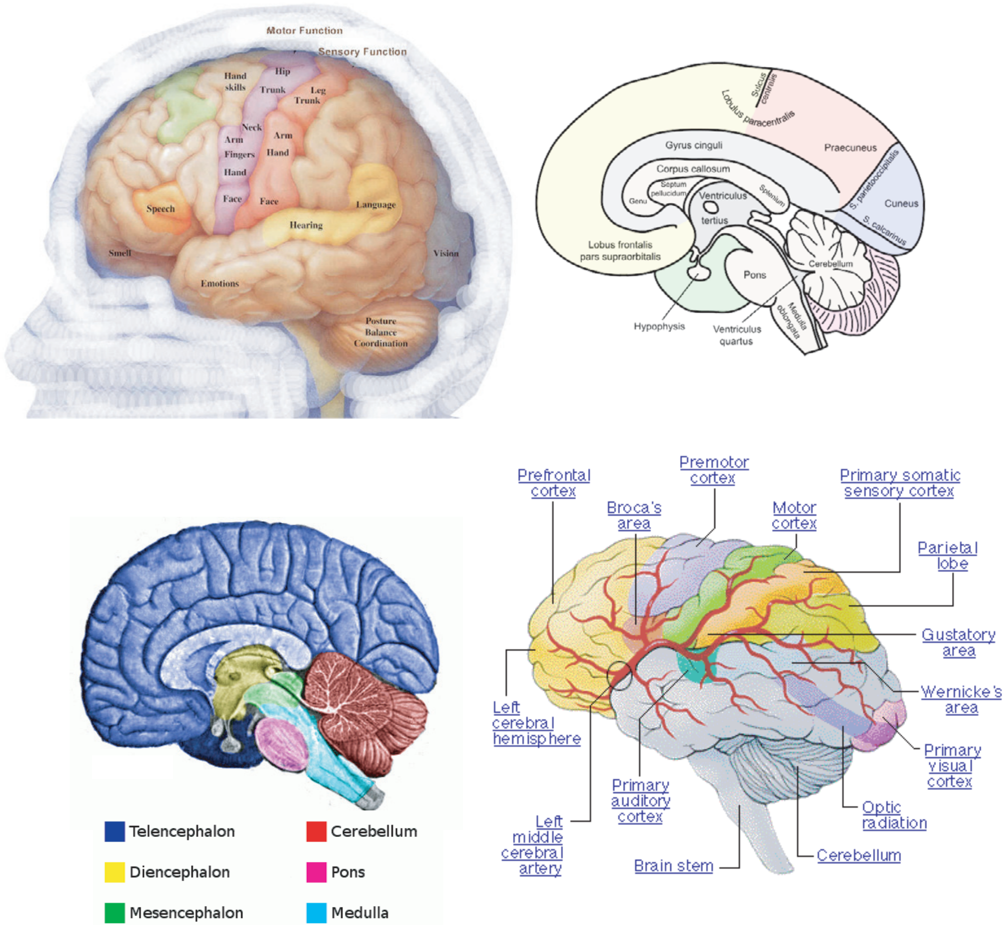

FIGURE 161 Sections and side view of the human brain, all in false colours (images WikiCommons).

In a generation or two, this section could be entitled 'how to build a brain'. Unfortunately, there is not enough knowledge yet to realize this aim. Maybe you can help in this pursuit?

Why a brain?

> "Denken ist bereits Plastik.*"
>
> Joseph Beuys.

Ref. 213

The brain exists to control the motion of an organism. The more complex the motions of an organism are, the larger its brain is. Living beings that do not move around, such as trees or dandelions, do not have a brain. Living beings that stop moving around, such as sea squirts (ascidiae or ascidiacea) digest their own brain when they attach themselves to a rock in the sea.

* 'Thinking is already sculpture.' Joseph Beuys (b. 1920 Krefeld, d. 1986 Düsseldorf) was a famous sculptor.

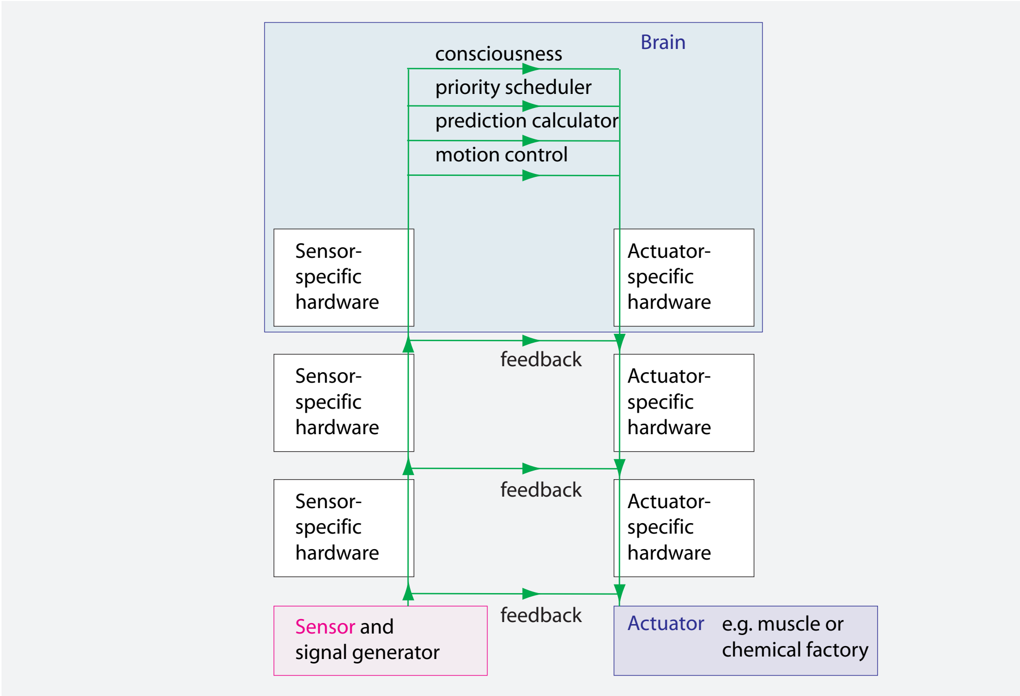

FIGURE 162 The general structure of the nervous system, with some typical feedback loops it contains and an example of its sensor-specific hardware.

The brain – together with some parts of the central nervous system – controls motion by processing the input provided by the various senses and sending the results of the processing to the various muscles in the body. Numerous observations show that sense input is processed, i.e., classified, stored and retrieved in the brain. Notably, lesions of the brain can lead to the loss of part or all of these functions. Among the important consequences of these basic abilities of the brain are thought and language. All brain abilities result from the construction, i.e., from the 'hardware' of the brain.

Ref. 214

Ref. 215

Systems with the ability to deduce classifications from the input they receive are called *classifiers*, and are said to be able to *learn*. Our brain shares this property with many complex systems; the brain of many animals, but also certain computer algorithms, such as the so-called 'neural networks', are examples of classifiers. Classifiers are studied in several fields, from biology to neurology, mathematics and computer science. All classifiers have the double ability to discriminate and to associate; and both abilities are fundamental to thinking.

Ref. 216

Ref. 217

Machine classifiers have a lot in common with the brain. As an example, following an important recent hypothesis in evolutionary biology, the necessity to cool the brain in an effective way is responsible for the upright, bipedal walk of humans. The brain, which uses around a quarter of all energy burned in the human body, needs a powerful cooling system to work well. In this, brains resemble modern computers, which usually have powerful fans or even water cooling systems built into them. It turns out that the human species has the most powerful cooling system of all mammals. An upright posture allowed the air to cool the body most effectively in the tropical environment where

Ref. 218

FIGURE 163 A modern electroencephalogram, taken at a number of positions at the head. The measured voltages are around 0.1 mV (© Wikimedia).

humans evolved. For even better cooling, humans have also no body hair, except on their head, where it protects the brain from direct heating by the Sun. The upright posture in turn allowed humans to take breath independently of their steps, a feat that many animals cannot perform. This ability increased the cooling again, and in turn allowed humans to develop speech. Speech in turn developed the brain further.

All classifiers are built from smallest classifying *units*, sometimes large numbers of them. Usually, the smallest units can classify input into only *two* different groups. The larger the number of these units, often called 'neurons' by analogy to the brain, the more sophisticated classifications can be produced by the classifier. Classifiers thus work by applying more or less sophisticated combinations of 'same' and 'different'. The distinction by a child of red and blue objects is such a classification; the distinction of compact and non-compact gauge symmetry groups in quantum theory is a more elaborate classification, but relies on the same fundamental ability.

Ref. 219

In all classifiers, the smallest classifying units *interact* with each other. Often these interactions are channelled via connections, and the set is then called a *network*. In these connections, signals are exchanged, via moving objects, such as electrons or photons. Thus we arrive at the conclusion that the ability of the brain to classify the physical world – for example to distinguish moving objects interacting with each other – arises because the brain itself consists of moving objects interacting with each other. Humans would

not have become such a successful animal species without our built-in powerful classifier. And only the motion inside our brain allows us to talk about motion in general.

Numerous researchers are identifying the parts of the brain used when different intellectual tasks are performed. Such experiments are possible using magnetic resonance Vol. V, page 156 imaging and similar imaging techniques. Other researchers are studying how thought processes can be modelled from the brain structure. Modern neurology is still making regular progress. In particular, neurologists have destroyed the belief that thinking is *more* than a physical process. This false belief results from various personal fears, as you might want to test by introspection. The fears and the belief will disappear as time goes Challenge 244 s by. How would you argue that thought is just a physical process?

Evolution developed the brain, with all its capabilities, as a tool that helps every person to find her way through the challenges that life poses. The human brain is so large because of two reasons: the sensory input is vast, and the processing is complex. More concretely, the brain is so large in order to process what we see. The amount of information provided by the eyes is indeed huge.

What is information?

> These thoughts did not come in any verbal formulation. I rarely think in words at all. A thought comes, and I may try to express it in words afterward.
>
> Albert Einstein Ref. 220

We started our adventure by stating that studying physics means to talk about motion. To *talk* is to transmit information. Can information be measured? Can we measure the progress of physics in this way? Is the universe made of information?

Information is the result of classification. A classification is the answer to one or to several yes–no questions. Such yes–no questions are the simplest classifications possible; they provide the basic *units* of classification, from which all others can be built. The simplest way to measure information is therefore to count the implied yes–no questions, the number of *bits*, leading to it. Some values are given in Table 19.

Are you able to say how many bits are necessary to define the place where you live? Obviously, the number of bits depends on the set of questions with which we start; that could be the names of all streets in a city, the set of all coordinates on the surface of the Earth, the names of all galaxies in the universe, the set of all letter combinations in the Challenge 245 s address. What is the most efficient method you can think of? A variation of the combination method is used in computers. For example, the story of the present adventure required about five thousand million bits of information. But since the amount of information in a story depends on the set of questions with which we start, it is impossible to define a precise measure for information in this way.

TABLE 19 Some measures of information.

Kind of information	Amount
Words spoken on an average day by a man	*c.* 5000
Words spoken on an average day by a woman	*c.* 7000

TABLE 19 (Continued) Some measures of information.

Kind of information	Amount
Bits processed by the ears	1 to 10 Mbit/s
Light sensitive cells per retina (120 million rods and 6 million cones)	$126 \cdot 10^6$
Bits processed by the eyes	1 to 10 Gbit/s
Words spoken during a lifetime (2/3 time awake, 30 words per minute)	$3 \cdot 10^8$
Words heard and read during a lifetime	10^9
Letters (base pairs) in haploid human DNA	$3 \cdot 10^9$
Pulses exchanged between both brain halves every second	$4 \cdot 10^9$
Bits in a compact disc	$6.1 \cdot 10^9$
Neurons in the human brain	10^{10} to 10^{11}
Printed words available in (different) books around the world (*c.* $100 \cdot 10^6$ books consisting of 50 000 words)	*c.* $5 \cdot 10^{12}$
Memory bits in the human brain	$> 10^{16}$
Image pixels seen in a lifetime ($3 \cdot 10^9$ s $\cdot (1/15$ ms$) \cdot 2/3$ (awake) $\cdot 10^6$ (nerves to the brain) Ref. 248	10^{17}
Bits of information processed in a lifetime (the above times 32)	10^{19}

The only way to measure information precisely is to take the largest possible set of questions that can be asked about a system, and to compare it with what is known about the system. In this case, the amount of unknown information is called entropy, a concept that we have already encountered. With this concept you should able to deduce yourself whether it is really possible to measure the advance of physics.

Vol. I, page 366
Challenge 246 s

Since classification or categorization is an activity of the brain and other, similar classifiers, information as defined here is a concept that applies to the result of activities by people and by other classifiers. In short, information is produced when talking about the universe.

Information is the result of classification. This implies that the universe itself is *not the same* as information. There is a growing number of publications based on the opposite of this view; however, this is a conceptual short circuit. Any transmission of information implies an interaction; physically speaking, this means that any information needs *energy* for transmission and *matter* for storage. Without either of these, there is no information. In other words, the universe, with its matter and energy, has to exist *before* transmission of information is possible. Saying that the universe *is made of* information, or that it *is* information, is as meaningful and as correct as saying that the universe is made of toothpaste.

The aim of physics is to give a *complete* classification of all types and examples of motion, in other words, to know everything about motion. Is this possible? Or are you able to find an argument against this endeavour?

Challenge 247 s

What is memory?

> The brain is my second favorite organ.
>
> Woody Allen

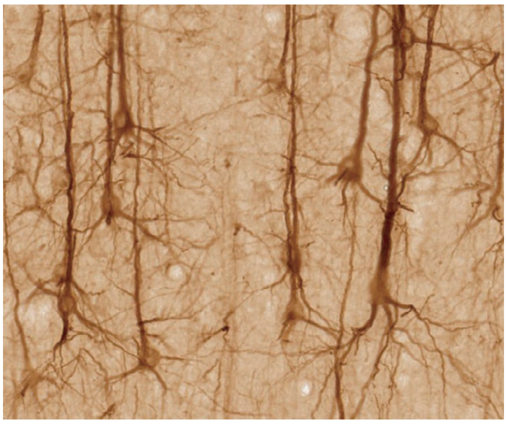

FIGURE 164 Photograph of stained pyramidal neurons in the cerebral cortex of the human cortex, showing their interconnections (© Medlat/Wikimedia).

Memory is the collection of records of perceptions. The production of such records is the essential aspect of observation. Records can be stored in human memory, i.e., in the brain, or in machine memory, as in computers, or in object memory, such as notes on paper. Without memory, there is no science, no life – since life is based on the records inside the DNA – and especially, no fun, as proven by the sad life of those who lose their memory. Ref. 214

Many animals and people have a memory, because a memory helps to move in a way that maximises reproduction and survival. Memory is found in all mammals, but also in insects and snails. The well-known sea snail *Aplysia californica* has memory – is shows conditioning, like Pawlow's dogs – even though it has only 20 000 neurons. Experiments confirm that individual memory is stored in the strength of neuron connections, the *synapses*. This statement was made already in 1949 by the Canadian psychologist Donald Hebb. In that year Hebb specified the physical embodiment of the observations of the psychologists Sigmund Freud and William James from the 1890s, who had already deduced that memory is about the strengthening and weakening of connections inside the brain. In short, observations and learning, everything we call *memories*, are recorded in the synapses.*

Obviously every record is an object. But under which conditions does an object qualify as a record? A signature can be the record of the agreement on a commercial transaction. A single small dot of ink is not a record, because it could have appeared by mistake, for example by an accidental blot. In contrast, it is improbable that ink should fall on

* The brain has various modes of learning that depends on its hardware. In a traumatic event, the brain learns within a few seconds to avoid similar situations for the rest of its life. In contrast, learning at school can take many months for a simple idea. It fact everybody can influence the ease and speed of learning; by mentally attaching images, voices, emotions, fantasies or memories to a topic or situation, one can speed up learning and reduce learning effort considerably.

Ref. 221 Research has shown that in the amygdala, where emotions and memory are combined, the enzyme calcineurin and the gene regulator Zif268 are important for traumatic memory: low calcineurin levels lead to increased expression of the gene regulator and to longer-lasting traumatic memory, high levels reduce the traumatic effect.

paper exactly in the shape of a signature. (The simple signatures of physicians are obviously exceptions.) Simply speaking, a *record* is any object, which, in order to be copied, has to be forged. More precisely, a record is an object or a situation that cannot arise nor disappear by mistake or by chance. Our personal memories, be they images or voices, have the same property; we can usually trust them, because they are so detailed that they cannot have arisen by chance or by uncontrolled processes in our brain.

Can we estimate the probability for a record to appear or disappear by chance? Yes, we can. Every record is made of a characteristic number N of small entities, for example the number of the possible ink dots on paper, the number of iron crystals in a cassette tape, the electrons in a bit of computer memory, the silver iodide grains in a photographic negative, etc. The chance disturbances in any memory are due to internal fluctuations, also called *noise*. Noise makes the record unreadable; it can be dirt on a signature, thermal magnetization changes in iron crystals, electromagnetic noise inside a solid state memory, etc. Noise is found in all classifiers, since it is inherent in all interactions and thus in all information processing.

It is a general property that internal fluctuations due to noise decrease when the size, i.e., the number of components of the record is increased. In fact, the probability p_{mis} for a misreading or miswriting of a record changes as

Challenge 248 ny

$$p_{mis} \sim 1/\sqrt{N} , \qquad (93)$$

where N is the number of particles or subsystems used for storing it. This relation appears because, for large numbers, the so-called *normal distribution* is a good approximation of almost any process. In particular, the width of the normal distribution, which determines the probability of record errors, grows less rapidly than its integral when the number of entities is increased; for large numbers, such statements become more and more precise.

We conclude that any good record must be made from a *large* number of entities. The larger the number, the less sensitive the memory is to fluctuations. Now, a system of large size with small fluctuations is called a *(physical) bath*. Only baths make memories possible. In other words, every record contains a bath. We conclude that any *observation* of a system is the interaction of that system with a bath. This connection will be used several times in the following, in particular in quantum theory. When a record is produced by a machine, the 'observation' is usually called a *(generalized) measurement*. Are you able to specify the bath in the case of a person looking at a landscape?

Challenge 249 s

From the preceding discussion we can deduce a powerful conclusion: since we have such a good memory at our disposition, we can deduce that we are made of many small parts. And since records exist, the world must also be made of a large number of small parts. No microscope of any kind is needed to confirm the existence of molecules or similar small entities; such a tool is only needed to determine the *sizes* of these particles. Their existence can be deduced simply from the observation that we have memory. (Of course, another argument proving that matter is made of small parts is the ubiquity of noise.)

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A second conclusion was popularized in the late 1920s by Leo Szilard. Writing a memory does not necessarily produce entropy; it is possible to store information into a memory without increasing entropy. However, entropy is produced in every case that the

memory is *erased*. It turns out that the (minimum) entropy created by erasing one bit
Challenge 250 e is given by

$$S_{\text{per erased bit}} = k \ln 2 \,, \tag{94}$$

and the number $\ln 2 \approx 0.69$ is the natural logarithm of 2. Erasing thus on the one hand reduces the disorder of the data – the local entropy–, but on the other hand increases the total entropy. As is well known, energy is needed to reduce the entropy of a local system. In short, any system that erases memory requires energy. For example, a logical AND gate effectively erases one bit per operation. Logical thinking thus requires energy.

It is also known that *dreaming* is connected with the erasing and reorganization of information. Could that be the reason that, when we are very tired, without any energy
Challenge 251 s left, we do not dream as much as usual? In dreams, the brain reorganizes the experiences made in the past. Dreams tell us what keeps our unconscious busy. Every person must decide by herself what to do with dreams that we recall. In short, dreams have no meaning – we give them meaning. In any case, dreams are one of the brain's ways to use memory efficiently.

Entropy is thus necessarily created when we forget. This is evident when we remind
Ref. 222 ourselves that forgetting is similar to the deterioration of an ancient manuscript. Entropy increases when the manuscript is not readable any more, since the process is irreversible and dissipative.* Another way to see this is to recognize that to clear a memory, e.g. a magnetic tape, we have to put energy into it, and thus increase its entropy. Conversely, *writing* into a memory can often *reduce* entropy; we remember that signals, the entities that write memories, carry negative entropy. For example, the writing of magnetic tapes usually reduces their entropy.

The capacity of the brain

> Computers are boring. They can give only answers.
> (Wrongly) attributed to Pablo Picasso

The human brain is built in such a way that its fluctuations cannot destroy its contents. The brain is well protected by the skull for exactly this reason. In addition, the brain literally grows connections, called *synapses*, between its various *neurons*, which are the cells doing the signal processing. The neuron is the basic processing element of the brain, performing the basic classification. It can only do two things: to fire and not to fire. (It is possible that the time at which a neuron fires also carries information; this question is
Ref. 225, Ref. 226 not yet settled.) The neuron fires depending on its input, which comes via the synapses from hundreds of other neurons. A neuron is thus an element that can distinguish the

Ref. 223 * As Wojciech Zurek clearly explains, the entropy created inside the memory is the main reason that even Maxwell's demon cannot reduce the entropy of two volumes of gases by opening a door between them in such a way that fast molecules accumulate on one side and slow molecules accumulate on the other. (Maxwell had introduced the 'demon' in 1871, to clarify the limits posed by nature to the gods.) This is just another way to rephrase the old result of Leo Szilard, who showed that the measurements by the demon
Ref. 224 create more entropy than they can save. And every measurement apparatus contains a memory.

To play being Maxwell's demon, look for one of the many computer game implementations around the internet.

inputs it receives into two cases: those leading to firing and those that do not. Neurons are thus classifiers of the simplest type, able only to distinguish between two situations.

Every time we store something in our long term memory, such as a phone number, the connection strength of existing synapses is changed or new synapses are grown. The connections between the neurons are much stronger than the fluctuations in the brain. Only strong disturbances, such as a blocked blood vessel or a brain lesion, can destroy neurons and lead to loss of memory.

As a whole, the brain provides an extremely efficient memory. Despite intense efforts, engineers have not yet been able to build a memory with the capacity of the brain in the same volume. Let us estimated this memory capacity. By multiplying the number of neurons, about 10^{11},* by the average number of synapses per neuron, about 100, and also by the estimated average number of bits stored in every synapse, about 10**, we arrive at a conservative estimate for the storage capacity of the brain of about

$$M_{\text{rewritable}} \approx 10^{14}\,\text{bit} \approx 10^{4}\,\text{GB}\,. \qquad (95)$$

(One *byte*, abbreviated B, is the usual name for eight bits of information.) Note that evolution has managed to put as many neurons in the brain as there are stars in the galaxy, and that if we add all the dendrite lengths, we get a total length of about 10^{11} m, which corresponds to the distance to from the Earth to the Sun. Our brain truly is *astronomically* complex.

However, this standard estimate of 10^{14} bits is not really correct! It assumes that the only component storing information in the brain is the synapse strength. Therefore it only measures the *erasable* storage capacity of the brain. In fact, information is also stored in the structure of the brain, i.e., in the exact configuration in which every cell is connected to other cells. Most of this structure is fixed at the age of about two years, but it continues to develop at a lower level for the rest of human life. Assuming that for each of the N cells with n connections there are $f\,n$ connection possibilities, this *write once*
Challenge 252 e capacity of the brain can be estimated as roughly $N\sqrt{fn}\,fn\log fn$ bits. For $N = 10^{11}$, $n = 10^2$, $f = 6$, this gives

$$M_{\text{writeonce}} \approx 10^{16}\,\text{bit} \approx 10^{6}\,\text{GB}\,. \qquad (96)$$

Ref. 227 Structural brain changes are measurable. Recent measurements confirmed that bilingual persons, especially early bilinguals, have a higher density of grey mass in the small parietal cortex on the left hemisphere of the brain. This is a region mainly concerned with language processing. The brain thus makes also use of structural changes for optimized storage and processing. Structure changes are also known for other populations, such as autistics, homophiles and hyperactive children. Intense and prolonged experiences during pregnancy or childhood seem to induce such structural developments.

Sometimes it is claimed that people use only between 5 % or 10 % of their brain capac-

* The number of neurons seems to be constant, and fixed at birth. The growth of interconnections is highest between age one and three, when it is said to reach up to 10^7 new connections per second.

** This is an average. Some types of synapses in the brain, in the hippocampus, are known to store only one bit.

ity. This myth, which goes back to the nineteenth century, would imply that it is possible to measure the actual data stored in the brain and compare it with its available maximum. Alternatively, the myth implies that the processing capacity can be measured and compared with an available maximum capacity. The myth also implies that nature would develop and maintain an organ with 90 % overcapacity, wasting all the energy and material to build, repair and maintain it. The myth is wrong. At present, the storage capacity and the processing capacity of a brain cannot be measured, but only estimated.

The large storage capacity of the brain also shows that human memory is filled by the environment and is not inborn: one human ovule plus one sperm have a mass of about 1 mg, which corresponds to about $3 \cdot 10^{16}$ atoms. Obviously, fluctuations make it impossible to store 10^{16} bits in these systems. In fact, nature stores only about $6 \cdot 10^9$ DNA base pairs or $12 \cdot 10^9$ bits in the genes of a fecundated ovule, using $3 \cdot 10^6$ atoms per bit. In contrast, a typical brain has a mass of 1.5 to 2 kg and contains about 5 to $7 \cdot 10^{25}$ atoms, which makes it as efficient a memory as an ovule. The difference between the number of bits in human DNA and those in the brain nicely shows that almost all information stored in the brain is taken from the environment; it cannot be of genetic origin, even allowing for smart decompression of stored information.

In total, all the tricks used by nature result in the most powerful classifier yet known.* Are there any limits to the brain's capacity to memorize and to classify? With the tools that humans have developed to expand the possibilities of the brain, such as paper, writing and printing to support memory, and the numerous tools available to simplify and to abbreviate classifications explored by mathematicians, brain classification is only limited by the time spent practising it. Without tools, there are strict limits, of course. The two-millimetre thick cerebral cortex of humans has a surface of about four sheets of A4 paper, a chimpanzee's yields one sheet and a monkey's is the size of a postcard. It is estimated that the total intellectually accessible memory is of the order of

Ref. 228

$$M_{\text{intellectual}} \approx 1\ \text{GB}\ , \tag{97}$$

though with a large experimental error.

The brain is also unparalleled in its processing capacity. This is most clearly demonstrated by the most important consequence deriving from memory and classification: thought and language. Indeed, the many types of thinking or language we use, such as comparing, distinguishing, remembering, recognizing, connecting, describing, deducing, explaining, imagining, etc., all describe different ways to classify memories or perceptions. In the end, every type of thinking or talking directly or indirectly classifies observations. But how far are computers from achieving this! The first attempt, in 1966, was a programming joke by Joseph Weizenbaum: the famous chatterbot program Eliza (try it at www.manifestation.com/neurotoys/eliza.php3) is a parody of a psychoanalyst. Even today, over 40 years later, conversation with a computer program, such as Friendbot (found at www.friendbot.co.uk), is still a disappointing experience. The huge capacity of the brain is the main reason for this disappointment.

Incidentally, even though the brains of sperm whales and of elephants can be five to

* Also the power consumption of the brain is important: even though it contains only about 2 % of the body's mass, it uses 25 % of the energy taken in by food.

six times as heavy as those of humans, the number of neurons and connections, and thus the capacity, is lower than for humans. Snails, ants, small fish have neuron numbers of the order of 10 000; the well-studied nematode *Caenorhabditis elegans* has only 302, though other animals have even fewer.

Curiosities about the brain

Teachers should all be brain experts. The brain learns best when it has an *aim*. Without an aim, both the lecture preparation and the lecture performance will lose most of its possible effects. How many teachers state the aim of their class at its beginning?

The brain also leans best when it is *motivated*. Different students need different motivations: potential applications, curiosity, competition, activation of already acquired knowledge, impressing the opposite sex, or exploring the unknown. And students need motivations on different levels of difficulty. Which teacher provides this mix?

Finally, brains in students have different ways to *create concepts*: using words, sounds, images, emotions, body sensations, etc. Which teacher addresses them all in his lessons?

* *

The brain plays strange games on the people that carry it. Modern research has shown that school pupils can be distinguished into five separate groups. Ref. 229

1. Smart students
2. Uninterested students
3. Students that overestimate themselves (often, but not always, boys)
4. Students that underestimate themselves (often, but not always, girls)
5. Struggling/weak students

This has to be kept in mind when teaching classes. To which group do/did you belong?

* *

Many cognitive activities of the brain are located in specific regions of the *cerebral cortex*, also called *grey matter* (see Figure 161). It is known that all grey matter is built of a large number of parallel, but largely independent structures, the so-called *neocortical columns*; they are similar to microprocessors. Each neocortical column has input and outputs, but works independently of the others; it is about 2 mm in height, 0.5 mm in diameter, and contains about 10 00 neurons of various types. The human cortex contains several millions of these columns, arranged in six layers. At present, researchers are able to simulate *one* neocortical column with *one* supercomputer. For more details, see the bluebrain.epfl.ch website. In short, your brain corresponds to several million supercomputers. Take good care of it.

* *

A beautiful atlas of the brain can be found at bigbrain.loris.ca. On this website, researchers from across the world collect the best images of the brain that modern research provides. Ref. 230

* *

The brain has many interesting sides. The technique of *neurofeedback* is an example. A

few electrodes are attached to the skin of the head, and a feedback loop is created with help of a visualization on a screen. Such a visualisation helps to put oneself into high-theta state – corresponding to deep relaxation –, or into the SMR state – corresponding to rest and concentration –, or into alpha-dominated states – corresponding to relaxation with closed eyes. Learning to switch rapidly between these states is helping athletes, surgeons, dancers, musicians, singers and children with attention deficit syndrome. After a few sessions, the effects stay for over a year. For attention deficit syndrome, the results Ref. 231 are as good as with medication.

* *

One interesting side of the human brain is the wide range of *passions* it produces. For example, there are people whose passion drives them to dedicate all their life to singing. There are people whose life-long passion is to invent languages; John Ronald Tolkien is the most famous example. There are other people whose passion is to help murderers to find peace of mind. Some people dedicate their life to raising handicapped children unwanted by their parents. Other people dedicate their life to implementing rapid solutions for infrastructure problems – water, gas and electricity supplies – in cities under war. The examples one can find are fascinating.

* *

Many functions in the brain are not performed by the programmable part of the brain, the cortex, but by specialized hardware. The list of known specialized hardware parts of the brain is still growing, as discoveries are still being made. Researchers have discovered Ref. 232 dedicated neurons that control the walking process in each leg, dedicated neurons – the so-called *mirror neurons* – that re-enact what people we see are feeling or acting, and dedicated neurons from the eye to the brain that control the day–night cycle. These recent discoveries complement the older ones that there is specialized hardware for every sense in the neural system, from touch to smell to proprioception. In short, many basic functions of the neural system are wired in, and many advanced functions are as well. The full list of wired-in systems is not known yet. For example, only future research will help us to understand how much of our subconscious is due to hardware, and how much is due to the software in the cortex.

* *

Cats are smart animals, and everybody who interacts with them knows how elaborate their behaviour and the spectrum of their activities is. All this is organized by a brain of the size of a walnut, with about 300 million neurons.

Interestingly, every human has roughly the same number of neurons that are found in a cat's brain in a place outside the brain: the belly. This group of neurons is called the enteric nervous system. This large collection of neurons, over 100 millions of them, controls the behaviour of the gut cells – in particular, the first layer of gut cells that comes in contact with food – and controls the production of many enzymes and neurotransmitters, which in turn influence our mood. 95 % of the serotonin produced in the body are produced by the intestine. It may well turn out that treating depression requires treating Vol. V, page 52 the intestine first. The enteric nervous system also determines whether vomiting is necessary or not, it triggers constipation and diarrhea, influences stress levels, regulates

our immune system and controls numerous other processes. In short, the enteric nervous system is the anatomical basis for our 'gut feelings' and probably for our general well-being. Research on these topics is still ongoing.

Ref. 233

* *

We learn better if we *recall* what we learned. Experiments show that remembering strengthens synapses, and thus strengthens our memory. We learn better if we know the *reasons* for the things we are learning. Experiments show that causality strengthens synapses.

* *

We learn while sleeping. The brain stores most things we experience during the day in a region called the *hippocampus*. During deep sleep, i.e., in the sleep time without dreams, the brain selects which of those experiences need to be stored in its long-time memory, the *neocortex*. The selection is based on the emotions attached to the memory, especially excitement, fear or anger. But also the expectation of a reward – such as a present or the possibility to make good impression when asked about the topic – is extremely effective in transferring content into the neocortex, as research by Jan Born has shown. If this rule is followed, sleeping just after learning, and in particular, deep sleep, is the best way to study. The most effective way to learn a language, to learn a new topic, or to memorize a presentation is to sleep just after study or training.

Deep sleep helps learning. Deep sleep can be promoted in many ways. Effort, sport, and even electric stimulation increases deep sleep. The pharmaceutic industry is now trying to develop sleeping pills that increase deep sleep. Alcohol, most sleeping pills, television, the internet and traumatic events decrease deep sleep. Jan Born states that most probably, sleep exists in order to enable us to learn; no other explanation for the loss of consciousness during deep sleep is convincing.

How do we sleep? When we are awake, all sense input is sent to the *thalamus*, which filters it and sends it to the neocortex. During sleep the neocortex effectively switches off large parts of the thalamus, so that almost no sense input arrives to the neocortex. Modelling these processes even allows understanding how sleep starts and to reproduce the brain waves seen during the beginning of sleep.

Ref. 234

* *

Many modern research results on animal and human brains can be found at the Brain Map website, available at www.brain-map.org.

* *

Brains and computers differ markedly in the way they work. Brains are analog, computers are digital. How exactly do computers work? The general answer is: computers are a smart and organized collection of electrical switches. To make matters as easy as possible, the calculation engine inside a computer – the so-called *central processing unit*, the heart of the computer – calculates using binary numbers. The 'on' and 'off' states of a switch are associated to the digits '1' and '0'. Can you devise a simple collection of switches that allows adding two binary numbers of one digit? Of many digits? And to multiply two numbers? Try it – it is an interesting exercise.

Computers are called *digital* because they are based on switches. Indeed, all integrated circuits inside a pocket calculator or inside a laptop are just collections of electrical switches; modern specimen can contain several millions of them, each switch with a specific function.

* *

During pregnancy, the brain of the embryo grows at a rate of 250 000 neurons per minute. The rate shows how fascinating a process life is.

* *

The signal communication between the brain and the arms differs from the signal communication between the brain and the legs. When the brain sends a command for some arm or leg movement to the spine, the spine then in turns sends its to the arms or to the legs. For the arms (and hands) – but *not* for the legs – the spine sends a copy of the command it is sending there back to the brain. This feedback seems to allow the brain to specify its next motion command more precisely. Thus the body and the brain are hard-wired for the fine motor skills that help us to use our fingers and hands as precisely as possible. The importance of the fine motor skills was already known to the ancient Greeks; Anaxagoras said that humans are the most clever living beings because they have hands.

* *

Epilepsy is a group of brain disorders that affects a large percentage of the human population. Epilepsy is an electric malfunction of the brain. It leads to regular electrical oscillations inside the brain, during which the person loses awareness or even gets fits. Epilepsy is also one of the reasons for autistic behaviour. Epilepsy can be triggered by genetic defects, by injuries and by other causes. Research in epilepsy is a vast field.

Many genetic types of epilepsy are due to mutations in genes that code ion channels. When ion channels do not work properly, the concentration of cations such as sodium does not behave properly, leading to the electric malfunctions. Research into the origin of epilepsy has shown that some genetic mutations are not inherited from the parents, but are *de novo*: they appear only in the child.

* *

Researchers have tentative linked gene defects to the propensity to forget things in everyday life. However, one can question whether an error in the DRD2 gene is really the cause for forgetting where the car keys are.

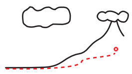
* *

Many switches has three states; one could call them '-1', '0', '1'. Thus, building computers based on 'trits' instead of 'bits' is a realistic option. Why are there no 27-trit computers? Challenge 253 s

* *

The neurotransmitters that influence moods are still a topic of intense research. Such research has shown that a specific peptide called *hypocretin* or also *orexin* leads to high alertness, to increases appetite and above all to good mood. Whether this really is the

‘happiness hormone’, as is sometimes claimed, still has to be tested.



Chapter 8

THOUGHT AND LANGUAGE

> Reserve your right to think, for even to think wrongly is better than not to think at all.
>
> Hypatia of Alexandria

Language possibly is the most wonderful gift of human nature. We have all learned it from somebody who cared about us. Nevertheless, the origins of language are hidden in the distant past of humanity. But we must explore language, because we have repeatedly stated that physics is talking about motion. Physics is a precise language specialized for the description of motion. We will find out in our walk that this is not a restriction, because everything in the world moves. But our quest for precision demands that we explore the meaning, use and limits of language.

What is language?

> Ein Satz kann nur sagen, *wie* ein Ding ist, nicht *was* es ist.*
>
> Ludwig Wittgenstein, *Tractatus*, 3.221

Using the ability to produce sounds and to put ink on paper, people attach certain *symbols*,** also called *words* or *terms* in this context, to the many partitions they specify with the help of their thinking. Such a categorization is then said to define a *concept* or *notion*, and is set in *italic typeface* in this text. *A standard set of concepts forms a language.**** In
Ref. 235 other words, we have:

▷ A *(human) language* is a standard way of symbolic interaction between people.

* 'A proposition can only say *how* a thing is, not *what* it is.'

** A symbol is a type of *sign*, i.e., an entity associated by some convention to the object it refers. Following Charles Peirce (b. 1839 Cambridge, d. 1914 Milford) – see www.peirce.org – the most original philosopher born in the United States, a symbol differs from an *icon* (or *image*) and from an *index*, which are also attached to objects by convention, in that it does not resemble the object, as does an icon, and in that it has no contact with the object, as is the case for an index.

*** The recognition that language is based on a partition of ideas, using the various differences between them to distinguish them from each other, goes back to Ferdinand de Saussure (b. 1857 Geneva, d. 1913 Vufflens), who is regarded as the founder of linguistics. His textbook *Cours de linguistique générale*, Editions Payot, 1985, has been the reference work of the field for over half a century. Note that Saussure, in contrast to Peirce, prefers the term 'sign' to 'symbol', and that his definition of the term 'sign' includes also the object to which it refers.

TABLE 20 Language basics.

Aspect	Value
Human phonemes	*c.* 70
English phonemes	44
German phonemes	40
Italian phonemes	30
Words of the English language (more than all other languages, with the possible exception of German)	*c.* 350 000
Number of languages on Earth in the year 2000	*c.* 6000

There are human languages based on facial expressions, on gestures, on spoken words, on whistles, on written words, and more. The use of *spoken* language is considerably younger than the human species; it seems that it appeared only about two hundred thousand years ago. Written language is even younger, namely only about six thousand years old. But the set of concepts used, the *vocabulary*, is still expanding. For humans, the understanding of language begins soon after birth (perhaps even before), the active use begins at around a year of age, the ability to read can start as early as two, and personal vocabulary continues to grow as long as curiosity is alive.

Physics being a lazy way to chat about motion, it needs language as an essential tool. Of the many aspects of language, from literature to poetry, from jokes to military orders, from expressions of encouragement, dreams, love and emotions, physics uses only a small and rather special segment. This segment is defined by the inherent restriction to talk about motion. Since motion is an observation, i.e., an interaction with the environment that several people experience in the same way, this choice puts a number of restrictions on the contents – the vocabulary – and on the form – the grammar – of such discussions.

For example, from the definition that observations are shared by others, we get the requirement that the statements describing them must be translatable into all languages. But when can a statement be translated? On this question two extreme points of view are possible: the first maintains that *all* statements can be translated, since it follows from the properties of human languages that each of them can express every possible statement. In this view, we can say:

▷ Only sign systems that allow one to express the complete spectrum of human messages form a *human language*.

This definition of language distinguishes human spoken and sign language from animal languages, such as the signs used by apes, birds or honey bees, and also from computer languages, such as Pascal or C. With this meaning of language, all statements can be translated by definition.

It is more challenging for a discussion to follow the opposing view, namely that precise translation is possible only for those statements which use terms, word types and grammatical structures found in *all* languages. Linguistic research has invested consid-

erable effort in the distillation of phonological, grammatical and semantic *universals*, as they are called, from the 6000 or so languages thought to exist today.*

The investigations into the *phonological* aspect, which showed for example that every language has at least two consonants and two vowels, does not provide any material for the discussion of translation.** Studying the *grammatical* (or *syntactic*) aspect, one finds that all languages use smallest elements, called 'words', which they group into sentences. They all have pronouns for the first and second person, 'I' and 'you', and always contain nouns and verbs. All languages use *subjects* and *predicates* or, as one usually says, the three entities *subject*, *verb* and *object*, though not always in this order. Just check the Challenge 254 e languages you know.

On the *semantic* aspect, the long list of lexical universals, i.e., words that appear in all languages, such as 'mother' or 'Sun', has recently been given a structure. The linguist Anna Wierzbicka performed a search for the building blocks from which all concepts can be built. She looked for the definition of every concept with the help of simpler ones, and continued doing so until a fundamental level was reached that cannot be further reduced. The set of concepts that are left over are the primitives. By repeating this exercise in many languages, Wierzbicka found that the list is the same in all cases. She thus had discovered *universal semantic primitives*. In November 1992, the list contained the terms given in Table 21.

Following the life-long research of Anna Wierzbicka and her research school, all these concepts exist in all languages of the world studied so far.*** They have defined the meaning of each primitive in detail, performed consistency checks and eliminated alternative approaches. They have checked this list in languages from all language groups, in languages from all continents, thus showing that the result is valid everywhere. In every Ref. 237 language all other concepts can be defined with the help of the semantic primitives.

Simply stated, learning to speak means learning these basic terms, learning how to combine them and learning the names of these composites. The definition of language given above, namely as a means of communication that allows one to express everything one wants to say, can thus be refined:

▷ Only a set of concepts that includes the universal semantic primitives forms

* A professional database by the linguist Merritt Ruhlen with 5700 languages and many details on each language can be found at ehl.santafe.edu/intro1.htm. A long but unprofessional list with 6 900 languages (and with 39 000 language and dialect names) can be found on the website www.ethnologue.com. Beware, it is edited by a fringe religious group that aims to increase the number of languages as much as possible.

It is estimated that 15 000 ± 5 000 languages have existed in the past.

Nevertheless, in today's world, and surely in the sciences, it is often sufficient to know one's own language plus English. Since English is the language with the largest number of words, learning it well is a greater Ref. 236 challenge than learning most other languages.

** Phonological studies also explore topics such as the observation that in many languages the word for 'little' contains an 'i' (or high pitched 'e') sound: petit, piccolo, klein, tiny, pequeño, chiisai; exceptions are: small, parvus.

*** It is easy to imagine that this research steps on the toes of many people. A list that maintains that we only have about thirty basic concepts in our heads is taken to be offensive by many small minds. In addition, a list that maintains that 'true', 'good', 'creation', 'life', 'mother' or 'god' are composite will elicit violent reactions, despite the correctness of the statements. Indeed, some of these terms were added in the 1996 list, which is somewhat longer.

TABLE 21 The semantic primitives, following Anna Wierzbicka.

I, you, someone, something, people	[substantives]
this, the same, one, two, all, much/many	[determiners and quantifiers]
know, want, think, feel, say	[mental predicates]
do, happen	[agent, patient]
good, bad	[evaluative]
big, small	[descriptors]
very	[intensifier]
can, if (would)	[modality, irrealis]
because	[causation]
no (not)	[negation]
when, where, after (before), under (above)	[time and place]
kind of, part of	[taxonomy, partonomy]
like	[hedge/prototype]

a *human language*.

For physicists – who aim to talk in as few words as possible – the list of semantic primitives has three facets. First, the approach is similar to physics' own aim: the idea of primitives gives a structured summary of everything that can be said, just as the atomic elements structure all objects that can be observed. Second, the list of primitives can be divided into two groups: one group contains all terms describing motion (do, happen, when, where, feel, small, etc. – probably a term from the semantic field around light or colour should be added) and the other group contains all terms necessary to talk about abstract sets and relations (this, all, kind of, no, if, etc.). Even for linguistics, aspects of motion and logical concepts are the basic entities of human experience and human thinking. To bring the issue to a point, the semantic primitives contain the basic elements of physics and the basic elements of mathematics. All humans are thus both physicists and mathematicians. The third point is that the list of primitives is too long. The division of the list into two groups directly suggests shorter lists; we just have to ask physicists and mathematicians for concise summaries of their respective fields. To appreciate this aim, try to define what 'if' means, or what an 'opposite' is – and explore your own ways of reducing the list.

Challenge 255 d

Reducing the list of primitives is also one of our aims in this adventure. We will explore the mathematical group of primitives in this chapter. The physical group will occupy us in the rest of our adventure. However, a shorter list of primitives is not sufficient. Our goal is to arrive at a list consisting of only *one* basic concept. Reaching this goal is not simple, though. First, we need to check whether the set of classical physical concepts that we have discovered so far is *complete*. Can classical physical concepts describe *all* observations? The volume on quantum physics of our adventure is devoted to this question. The second task is to reduce the list. This task is not straightforward; we have already discovered that physics is based on a circular definition: in Galilean physics, space and time are defined using matter, and matter is defined using space and time. We will need quite some effort

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Vol. I, page 406

to overcome this obstacle. The final part of this text tells the precise story. After numerous adventures we will indeed discover a basic concept on which all other concepts are based. Vol. VI, page 140

We can summarize all the above-mentioned results of linguistics in the following way. By constructing a statement made only of subject, verb and object, consisting only of nouns and verbs, using only concepts built from the semantic primitives, we are sure that it can be translated into all languages. This explains why physics textbooks are often so boring: the authors are often too afraid to depart from this basic scheme. On the other hand, research has shown that such straightforward statements are not restrictive: with them one can say everything that can be said.

> Jedes Wort ist ein Vorurteil.
>
> Friedrich Nietzsche*

> Every word was once a poem.
>
> Ralph Waldo Emerson**

What is a concept?

> Concepts are merely the results, rendered permanent by language, of a previous process of comparison.
>
> William Hamilton

There is a group of people that has taken the strict view on translation and on precision to the extreme. They build all concepts from an even smaller set of primitives, namely only two: 'set' and 'relation', and explore the various possible combinations of these two concepts, studying their various classifications. Step by step, this radical group, commonly called *mathematicians*, came to define with full precision concepts such as numbers, points, curves, equations, symmetry groups and more. The construction of these concepts is summarized partly in the following and partly in the next volume of this adventure. Vol. IV, page 220

What properties must a useful concept have? For example, what is 'passion' and what is a 'cotton bud'? Obviously, a useful concept implies a list of its parts, its aspects and their internal relations, as well as their relation to the exterior world. Thinkers in various fields, from philosophy to politics, agree that the definition is:

▷ A *concept* has
1. explicit and fixed content,
2. explicit and fixed limits,
3. explicit and fixed domain of application.

The inability to state these properties or to keep them fixed is often the easiest way to distinguish *crackpots* from more reliable thinkers. Unclearly defined terms, which thus do not qualify as concepts, regularly appear in myths, e.g. 'dragon' or 'sphinx', or in ideologies, e.g. 'worker' or 'soul'. Even physics is not immune. For example, we will discover later that neither 'universe' nor 'creation' are concepts. Are you able to argue the case? Challenge 256 s

* 'Every word is a prejudice.' Friedrich Nietzsche (b. 1844 Röcken, d. 1900 Weimar) was an influential philologist and philosopher.
** Ralph Waldo Emerson (b. 1803 Boston, d. 1882 Concord) was an influential essayist and philosopher.

But the three defining properties of any concept are interesting in their own right. Explicit content means that concepts are built one onto another. In particular, the most fundamental concepts appear to be those that have no parts and no external relations, but only internal ones. Can you think of one? Only the last part of this walk will uncover the final word on the topic.

Challenge 257 s

The requirements of explicit limits and explicit contents also imply that all concepts describing nature are *sets* or *relations* or both – since sets and relations obey the requirements for concepts.* Since mathematics is based on the concepts of 'set' and of 'relation', we follow directly that mathematics can provide the *form* for any concept, especially whenever high precision is required, as in the study of motion. Obviously, the *content* of the description is only provided by the study of nature itself; only then do concepts become useful.

Indeed, despite the involved precision, in fact precisely because of it, no mathematical concept talks about nature or about observations.** Therefore the study of motion needs other, more useful concepts.

Physics is the precise description of motion. In physics, the search for sufficiently precise concepts can be seen as the single theme structuring the long history of the field. Regularly, new concepts have been proposed, explored in all their properties, and tested. Finally, concepts are rejected or adopted, in the same way that children reject or adopt a new toy. Children do this unconsciously, scientists do it consciously, using language.*** For this reason, concepts are universally intelligible.

Note that the concept '*concept*' itself is not definable independently of experience; a concept is something that helps us to act and react to the world in which we live. Moreover, concepts do not live in a world separate from the physical one: every concept requires memory from its user, since the user has to remember the way in which it was formed; therefore every concept needs a material support for its use and application. Thus all thinking and all science is fundamentally based on experience.

In conclusion, all concepts are based on the idea that nature is made of related parts. This idea leads to complementing couples such as 'noun–verb' in linguistics, 'set–relation' or 'definition–theorem' in mathematics, and 'aspect of nature–pattern of nature'

* We see that every physical concept is an example of a (mathematical) *category*, i.e., a combination of objects and mappings/relations. For more details about categories, with a precise definition of the term, see page 266.

** Insofar as one can say that mathematics is based on the concepts of 'set' and 'relation', which are based on experience, one can say that mathematics explores a section of reality, and that its concepts are *derived* from experience. This and similar views of mathematics are called *platonism*. More concretely, platonism is the view that the concepts of mathematics exist *independently* of people, and that they are discovered, and not created, by mathematicians.

In short, since mathematics makes use of the brain, which is a physical system, actually *mathematics is applied physics*.

However, we will discover that the concept of 'set' does *not* apply to nature; this changes the discussion in completely.

Vol. VI, page 101

*** Concepts formed unconsciously in our early youth are the most difficult to define precisely, i.e., with language. Some who were unable to define them, such as the influential philosopher Immanuel Kant (b. 1724 Königsberg, d. 1804 Königsberg), used to call them 'a priori' concepts (such as 'space' and 'time') to contrast them with the more clearly defined 'a posteriori' concepts. Today, this distinction has been shown to be unfounded both by the study of child psychology (see the footnote on page 238) and by physics itself, so that these qualifiers are therefore not used in our walk.

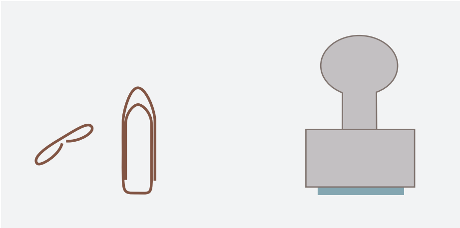

FIGURE 165 Devices for the definition of sets (left) and of relations (right).

in physics. These couples constantly guide human thinking, from childhood onwards, as developmental psychology can testify.

What are sets? What are relations?

> Alles, was wir sehen, könnte auch anders sein. Alles, was wir überhaupt beschreiben können, könnte auch anders sein. Es gibt keine Ordnung der Dinge a priori.*
>
> Ludwig Wittgenstein, *Tractatus*, 5.634

Defining sets and defining relations are the two fundamental acts of our thinking. This can be seen most clearly in any book about mathematics; such a book is usually divided into paragraphs labelled 'definition', 'theorem', 'lemma' and 'corollary'. The first type of paragraph defines concepts, i.e., defines sets, and the other three types of paragraphs express relations, i.e., connections between these sets. *Mathematics* is thus the exploration of the possible symbolic concepts and their relations. Mathematics is the science of symbolic necessities.

Sets and relations are tools of classification; that is why they are also the tools of any bureaucrat. (See Figure 165.) This class of humans is characterized by heavy use of paper clips, files, metal closets, archives – which all define various types of sets – and by the extensive use of numbers, such as reference numbers, customer numbers, passport numbers, account numbers, law article numbers – which define various types of relations between the items, i.e., between the elements of the sets.

Both the concepts of set and of relation express, in different ways, the fact that nature can be *described*, i.e., that it can be *classified* into parts that form a whole. The act of grouping together aspects of experience, i.e., the act of classifying them, is expressed in formal language by saying that a set is defined. In other words, a *set* is a collection of *elements* of our thinking. Every set distinguishes the elements from each other and from the set itself. This definition of 'set' is called the *naive* definition. For physics, the definition is sufficient, but you won't find many who will admit this. In fact, mathematicians have refined the definition of the concept 'set' several times, because the naive definition does not work well for infinite sets. A famous example is the story about sets which do not contain themselves. Obviously, any set is of two sorts: either it contains itself or it does not. If we take the set of all sets that do *not* contain themselves, to which sort does it belong?

Challenge 258 s

* 'Everything we see could also be otherwise. Everything we describe at all could also be otherwise. There is no order of things a priori.'

TABLE 22 The defining properties of a set – the ZFC axioms.

The axioms of Zermelo–Fraenkel–C set theory
– Two sets are equal if and only if they have the same elements. (Axiom of extensionality)
– The empty set is a set. (Axiom of the null set)
– If x and y are sets, then the unordered pair $\{x, y\}$ is a set. (Axiom of unordered pairs)
– If x is a set of sets, the union of all its members is a set. (Union or sum set axiom)
– The entity $\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\{\{\emptyset\}\}\}, ...\}$ is a set [a] – in other words, infinite collections, such as the natural numbers, are sets. (Axiom of infinity)
– An entity defined by all elements having a given property is a set, provided this property is reasonable; some important technicalities defining 'reasonable' are necessary. (Axiom of separation)
– If the domain of a function is a set, so is its range. (Axiom of replacement)
– The entity y of all subsets of x is also a set, called the power set. (Axiom of the power set)
– A set is not an element of itself – plus some technicalities. (Axiom of regularity)
– The product of a family of non-empty sets is non-empty. Equivalently, picking elements from a list of sets allows one to construct a new set – plus technicalities. (Axiom of choice)

a. The more common formulation (though equivalent to the above) is: The entity $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}, ...\}$ is a set.

To avoid problems with the concept of 'set', mathematics requires a precise definition. The first such definition was given by the mathematician Ernst Zermelo (b. 1871 Berlin, d. 1951 Freiburg i.B.) and the mathematician Adolf/Abraham Fraenkel (b. 1891 München, d. 1965 Jerusalem). Later, the so-called *axiom of choice* was added, in order to make it possible to manipulate a wider class of infinite sets. The result of these efforts is called the *ZFC* definition.* From this basic definition we can construct all mathematical concepts used in physics. From a practical point of view, it is sufficient to keep in mind that for the whole of physics, the naive definition of a set is equivalent to the precise ZFC definition, actually even to the simpler ZF definition. Subtleties appear only for some special types of infinite sets, but these are not used in physics. In short, from the basic, naive set definition we can construct all concepts used in physics.

Ref. 239 The naive set definition is far from boring. To satisfy two people when dividing a cake, we follow the rule: I cut, you choose. The method has two properties: it is *just*, as everybody thinks that they have the share that they deserve, and it is *fully satisfying*, as everybody has the feeling that they have at least as much as the other. What rule is needed for three people? And for four? Challenge 260 d

* A global overview of axiomatic set theory is given by Paul J. Cohen & Reuben Hersch, *Non-Cantorian set theory*, Scientific American **217**, pp. 104–116, 1967. Those were the times when Scientific American was a quality magazine.

For a good introduction to the axiom of choice, see the www.math.vanderbilt.edu/~schectex/ccc/choice.html website.

Ref. 238 Other types of entities, more general than standard sets, obeying other properties, can also be defined, and are also subject of (comparatively little) mathematical research. To find an example, see the section on cardinals later on. Such more general entities are called classes whenever they contain at least one set. Can you give an example? In the final part of our mountain ascent we will meet physical concepts that are described neither by sets nor by classes, containing no set at all. That is where the real fun starts. Page 265 Challenge 259 s

Apart from defining sets, every child and every brain creates links between the different aspects of experience. For example, when it hears a voice, it automatically makes the connection that a human is present. In formal language, connections of this type are called *relations*. Relations connect and differentiate elements along other lines than sets: the two form a complementing couple. Defining a set unifies many objects and at the same time divides them into two: those belonging to the set and those that do not; defining a (binary) relation unifies elements two by two and divides them into many, namely into the many couples it defines.

Sets and relations are closely interrelated concepts. Indeed, one can define (mathematical) relations with the help of sets. A *(binary) relation* between two sets X and Y is a subset of the product set, where the *product set* or *Cartesian product* $X \times Y$ is the set of all ordered pairs (x, y) with $x \in X$ and $y \in Y$. An *ordered pair* (x, y) can easily be defined with the help of sets. Can you find out how? For example, in the case of the relation 'is wife of', the set X is the set of all women and the set Y that of all men; the relation is given by the list all the appropriate ordered pairs, which is much smaller than the product set, i.e., the set of all possible woman–man combinations.

Challenge 261 s

It should be noted that the definition of relation just given is not really complete, since every construction of the concept 'set' already contains certain relations, such as the relation 'is element of.' It does not seem to be possible to reduce either one of the concepts 'set' or 'relation' completely to the other one. This situation is reflected in the physical cases of sets and relations, such as space (as a set of points) and distance, which also seem impossible to separate completely from each other. In other words, even though mathematics does not pertain to nature, its two basic concepts, sets and relations, are taken from nature. In addition, the two concepts, like those of space-time and particles, are each defined with the other.

Infinity – and its properties

Mathematicians soon discovered that the concept of 'set' is only useful if one can also call collections such as $\{0, 1, 2, 3...\}$, i.e., of the number 0 and all its successors, a 'set.' To achieve this, one property in the Zermelo–Fraenkel list defining the term 'set' explicitly specifies that this infinite collection can be called a set. (In fact, also the axiom of replacement states that sets may be infinite.) Infinity is thus put into mathematics and added to the tools of our thinking right at the very beginning, in the definition of the term 'set.' When describing nature, with or without mathematics, we should never forget this fact. A few additional points about infinity should be of general knowledge to any expert on motion.

A set is infinite if there is a function from it into itself that is *injective* (i.e., different elements map to different results) but not *onto* (i.e., some elements do not appear as images of the map); e.g. the map $n \mapsto 2n$ shows that the set of integers is infinite. Infinity also can be checked in another way: a set is infinite if it remains so also after removing one element, even repeatedly. We just need to remember that the empty set is *finite*.

Only *sets* can be infinite. And sets have parts, namely their elements. When a thing or a concept is called 'infinite' one can *always* ask and specify what its parts are: for space the parts are the points, for time the instants, for the set of integers the integers, etc. An

indivisible or a finitely divisible entity cannot be called infinite.*

There are *many types* of infinities, all of different sizes.** This important result was discovered by the mathematician Georg Cantor (b. 1845 Saint Petersburg, d. 1918 Halle an der Saale). He showed that from the countable set of natural numbers one can construct other infinite sets which are not countable. He did this by showing that the *power set* $P(\omega)$, namely the set of all subsets, of a countably infinite set is infinite, but *not* countably infinite. Sloppily speaking, the power set is 'more infinite' than the original set. The real numbers $\mathbb{R}$, to be defined shortly, are an example of an uncountably infinite set; there are many more of them than there are natural numbers. (Can you show Challenge 262 s this?) However, *any* type of infinite set contains at least one subset which is countably infinite.

Even for an infinite set one can define size as the number of its elements. Cantor called this concept of size the *cardinality* of a set. The cardinality of a finite set is simply given by the number of its elements. The cardinality of a power set is 2 exponentiated by the cardinality of the set. The cardinality of the set of integers is called $\aleph_0$, pronounced 'aleph zero', Vol. I, page 413 after the first letter of the Hebrew alphabet. The smallest *uncountable* cardinal is called $\aleph_1$. The next cardinal is called $\aleph_2$ etc. A whole branch of mathematics is concerned with the manipulation of these infinite 'numbers'; addition, multiplication, exponentiation are easily defined. For some of them, even logarithms and other functions make sense.***

The cardinals defined in this way, including $\aleph_n$, $\aleph_\omega$, $\aleph_{\aleph_\aleph}$ are called accessible, because since Cantor, people have defined even larger types of infinities, called *inaccessible*. These numbers (inaccessible cardinals, measurable cardinals, supercompact cardinals, etc.) need additional set axioms, extending the ZFC system. Like the ordinals and the cardinals, they form examples of what are called *transfinite* numbers.

The real numbers have the cardinality of the power set of the integers, namely $2^{\aleph_0}$. Challenge 263 s Can you show this? The result leads to the famous question: Is $\aleph_1 = 2^{\aleph_0}$ or not? The statement that this be so is called the *continuum hypothesis* and was unproven for several generations. The surprising answer came in 1963: the usual definition of the concept of Ref. 240 set is not specific enough to fix the answer. By specifying the concept of set in more detail, with additional axioms – remember that axioms are defining properties – you can make the continuum hypothesis come out either right or wrong, as you prefer.

Another result of research into transfinites is important: for every definition of a type of infinite cardinal, it seems to be possible to find a larger one. In everyday life, the idea of infinity is often used to stop discussions about size: 'My big brother is stronger than yours.' 'But mine is infinitely stronger than yours!' Mathematics has shown that questions on size do continue afterwards: 'The strength of my brother is the power set of that of Ref. 241 yours!' Rucker reports that mathematicians conjecture that there is no possible nor any conceivable end to these discussions.

* Therefore, most gods, being concepts and thus sets, are either finite or, in the case where they are infinite, they are divisible. It seems that only polytheistic and pantheistic world views are not disturbed by this conclusion.

** In fact, there is such a huge number of types of infinities that none of these infinities itself actually describes this number. Technically speaking, there are as many infinities as there are ordinals.

*** Many results are summarized in the excellent and delightful paperback by RUDY RUCKER, *Infinity and the Mind – the Science and Philosophy of the Infinite*, Bantam, Toronto, 1983.

For physicists, a simple question appears directly. Do infinite quantities exist in nature? Or better, is it necessary to use infinite quantities to describe nature? You might want to clarify your own opinion on the issue. It will be settled during the rest of our adventure.

Challenge 264 e

Functions and structures

Which relations are useful to describe patterns in nature? A typical example is 'larger stones are heavier'. Such a relation is of a specific type: it relates one specific value of an observable 'volume' to one specific value of the observable 'weight'. Such a one-to-one relation is called a *(mathematical) function* or *mapping*. Functions are the most specific types of relations; thus they convey a maximum of information. In the same way as numbers are used for observables, functions allow easy and precise communication of relations between observations. All physical rules and 'laws' are therefore expressed with the help of functions and, since physical 'laws' are about measurements, functions of numbers are their main building blocks.

A *function* f, or *mapping*, is a thus binary relation, i.e., a set $f = \{(x, y)\}$ of ordered pairs, where for every value of the first element x, called the *argument*, there is only *one* pair (x, y). The second element y is called the *value* of the function at the argument x. The set X of all arguments x is called the *domain of definition* and the set Y of all second arguments y is called the *range* of the function. Instead of $f = \{(x, y)\}$ one writes

$$f : X \to Y \quad \text{and} \quad f : x \mapsto y \quad \text{or} \quad y = f(x) \,, \tag{98}$$

where the type of arrow – with initial bar or not – shows whether we are speaking about sets or about elements.

We note that it is also possible to use the couple 'set' and 'mapping' to define all mathematical concepts; in this case a relation is defined with the help of mappings. A modern school of mathematical thought formalized this approach by the use of (mathematical) *categories*, a concept that includes both sets and mappings on an equal footing in its definition.*

To think and talk more clearly about nature, we need to define more specialized concepts than sets, relations and functions, because these basic terms are too general. The most important concepts derived from them are operations, algebraic structures and numbers.

A *(binary) operation* is a function that maps the Cartesian product of two copies of a set X into itself. In other words, an operation w takes an ordered couple of arguments $x \in X$ and assigns to it a value $y \in X$:

$$w : X \times X \to X \quad \text{and} \quad w : (x, x) \mapsto y \,. \tag{99}$$

* A *category* is defined as a collection of objects and a collection of 'morphisms', or mappings. Morphisms can be composed; the composition is associative and there is an identity morphism. More details can be found in the literature. Ref. 242

Note that every category contains a set; since it is unclear whether nature contains sets, as we will discuss below,, it is questionable whether categories will be useful in the unification of physics, despite their intense and abstract charm. Page 304

Challenge 265 s
Is division of numbers an operation in the sense just defined?

Now we are ready to define the first of three basic concepts of mathematics. An *algebraic structure*, also called an *algebraic system*, is (in the most restricted sense) a set together with certain operations. The most important algebraic structures appearing in physics are groups, vector spaces, and algebras.

Vol. V, page 350

In addition to algebraic structures, mathematics is based on *order structures* and on *topological structures*. Order structures are building blocks of numbers and necessary to define comparisons of any sort. Topological structures are built, via subsets, on the concept of neighbourhood. They are necessary to define continuity, limits, dimensionality, topological spaces and manifolds.

Vol. V, page 357

Obviously, most mathematical structures are combinations of various examples of these three basic structure types. For example, the *system* of real numbers is given by the *set* of real numbers with the *operations* of addition and multiplication, the *order relation* 'is larger than' and a *continuity* property. They are thus built by combining an algebraic structure, an order structure and a topological structure. Let us delve a bit into the details.

Ref. 243

Numbers

> Which numbers are multiplied by six when their last digit is taken away and transferred to the front?

Challenge 266 s

Numbers are the oldest mathematical concept and are found in all cultures. The notion of number, in Greek ἀριθμός, has been changed several times. Each time the aim was to include wider classes of objects, but always retaining the general idea that numbers are entities that can be added, subtracted, multiplied and divided.

The modern way to write numbers, as e.g. in $12\,345\,679 \cdot 54 = 666\,666\,666$, is essential for science.* It can be argued that the lack of a good system for writing down and for calculating with numbers delayed the progress of science by several centuries. (By the way, the same can be said for the affordable mass reproduction of written texts.)

The simplest numbers, $0, 1, 2, 3, 4, ...$, are usually seen as being taken directly from experience. However, they can also be constructed from the notions of 'relation' and 'set'. One of the many possible ways to do this (can you find another?) is by identifying a natural number with the set of its predecessors. With the relation 'successor of', abbreviated S, this definition can be written as

Challenge 267 s

$$0 := \varnothing \quad , \quad 1 := S\,0 = \{0\} = \{\varnothing\} \quad ,$$
$$2 := S\,1 = \{0, 1\} = \{\varnothing, \{\varnothing\}\} \quad \text{and} \quad n+1 := S\,n = \{0, ..., n\} \; . \qquad (100)$$

This set, together with the binary operations 'addition' and 'multiplication,' constitutes the algebraic system $N = (N, +, \cdot, 1)$ of the *natural numbers*. For all number systems the algebraic system and the set are often sloppily designated by the same symbol. The algebraic system N is what mathematician call a semi-ring. (Some authors prefer not to

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* However, there is no need for written numbers for doing mathematics, as shown in the interesting book by Marcia Ascher, *Ethnomathematics – A Multicultural View of Mathematical Ideas*, Brooks/Cole, 1991.

count the number zero as a natural number.) Natural numbers are fairly useful.

TABLE 23 Some large numbers.

Number	Example in nature
Around us	
1	number of angels that can be in one place at the same time, following Thomas Aquinas Ref. 245
8	number of times a newspaper can be folded in alternate perpendicular directions
12	largest number of times a paper strip has been folded in the same direction Ref. 246
20	number of digits in precision measurements that will probably never be achieved
21, 34, 55, 89	petals of common types of daisy and sunflower Ref. 247
57	faces of a diamond with brilliant cut
2000 to 6000	stars visible in the night sky
15 000	average number of objects in a European household
10^5	leaves of a tree (10 m beech)
6 to $7 \cdot 10^9$	humans in the year 2000
10^{17}	ants in the world
c. 10^{20}	number of snowflakes falling on the Earth per year
c. 10^{24}	grains of sand in the Sahara desert
10^{22}	stars in the universe
10^{25}	cells on Earth
$1.1 \cdot 10^{50}$	atoms making up the Earth ($6370^3\,\mathrm{km}^3 \cdot 4 \cdot 3.14/3 \cdot 5500\,\mathrm{kg/m^3} \cdot 30\,\mathrm{mol/kg} \cdot 6 \cdot 10^{23}\,/\mathrm{mol}$)
10^{81}	atoms in the visible universe
10^{90}	photons in the visible universe
10^{169}	number of atoms fitting in the visible universe
10^{244}	number of space-time points inside the visible universe
Information	
51	record number of languages spoken by one person
c. 5000	words spoken on an average day by a man
c. 7000	words spoken on an average day by a woman
c. 2 000 000	number of scientists on Earth around the year 2000
$3 \cdot 10^8$	words spoken during a lifetime (2/3 time awake, 30 words per minute)
10^9	words heard and read during a lifetime
$4 \cdot 10^9$	pulses exchanged between both brain halves every second
10^{17}	image pixels seen in a lifetime ($3 \cdot 10^9\,\mathrm{s} \cdot (1/15\,\mathrm{ms}) \cdot 2/3$ (awake) $\cdot 10^6$ (nerves to the brain) Ref. 248
10^{19}	bits of information processed in a lifetime (the above times 32)
c. $5 \cdot 10^{12}$	printed words available in (different) books around the world (*c.* $100 \cdot 10^6$ books consisting of 50 000 words)

Number	Example in nature
$2^{10} \cdot 3^7 \cdot 8! \cdot 12!$	
$= 4.3 \cdot 10^{19}$	possible positions of the $3 \times 3 \times 3$ Rubik's Cube Ref. 249
$5.8 \cdot 10^{78}$	possible positions of the $4 \times 4 \times 4$ Rubik-like cube
$5.6 \cdot 10^{117}$	possible positions of the $5 \times 5 \times 5$ Rubik-like cube
c. 10^{200}	possible games of chess
c. 10^{800}	possible games of go
c. 10^{10^7}	possible states in a personal computer
Parts of us	
600	numbers of muscles in the human body, of which about half are in the face
150 000 ± 50 000	hairs on a healthy head
900 000	neurons in the brain of a grasshopper
$126 \cdot 10^6$	light sensitive cells per retina (120 million rods and 6 million cones)
10^{10} to 10^{11}	neurons in the human brain
$500 \cdot 10^6$	blinks of the eye during a lifetime (about once every four seconds when awake)
$300 \cdot 10^6$	breaths taken during human life
$3 \cdot 10^9$	heart beats during a human life
$3 \cdot 10^9$	letters (base pairs) in haploid human DNA
$10^{15\pm1}$	cells in the human body
$10^{16\pm1}$	bacteria carried in the human body

The system of *integers* $Z = (..., -2, -1, 0, 1, 2, ..., +, \cdot, 0, 1)$ is the minimal ring that is an extension of the natural numbers. The system of *rational numbers* $Q = (Q, +, \cdot, 0, 1)$ is the minimal field that is an extension of the ring of the integers. (The terms 'ring' and 'field' are defined in all details in the next volume.) Vol. IV, page 220 The system of *real numbers* $R = (R, +, \cdot, 0, 1, >)$ is the minimal extension of the rationals that is continuous and totally ordered. (For the definition of continuity, see volume IV, page 221, and volume V, page 359.) Equivalently, the reals are the minimal extension of the rationals forming a complete, totally strictly-Archimedean ordered field. This is the historical construction – or definition – of the integer, rational and real numbers from the natural numbers. However, it is not the only one construction possible. The most beautiful definition of all these types of numbers is the one discovered in 1969 by John Conway, and popularized by him, Donald Knuth and Martin Kruskal. Ref. 250

▷ A *number* is a sequence of bits.

The two bits are usually called 'up' and 'down'. Examples of numbers and the way to write them are given in Figure 166.

The empty sequence is the number zero. A finite sequence of n ups is the integer number n, and a finite sequence of n downs is the integer $-n$. Finite sequences of mixed ups and downs give the *dyadic rational numbers*. Examples are 1, 2, 3, -7, $19/4$, $37/256$, etc. They all have denominators with a power of 2. The other *rational numbers* are those that end in an infinitely repeating string of ups and downs, such as the *reals*, the *infinitesi-*

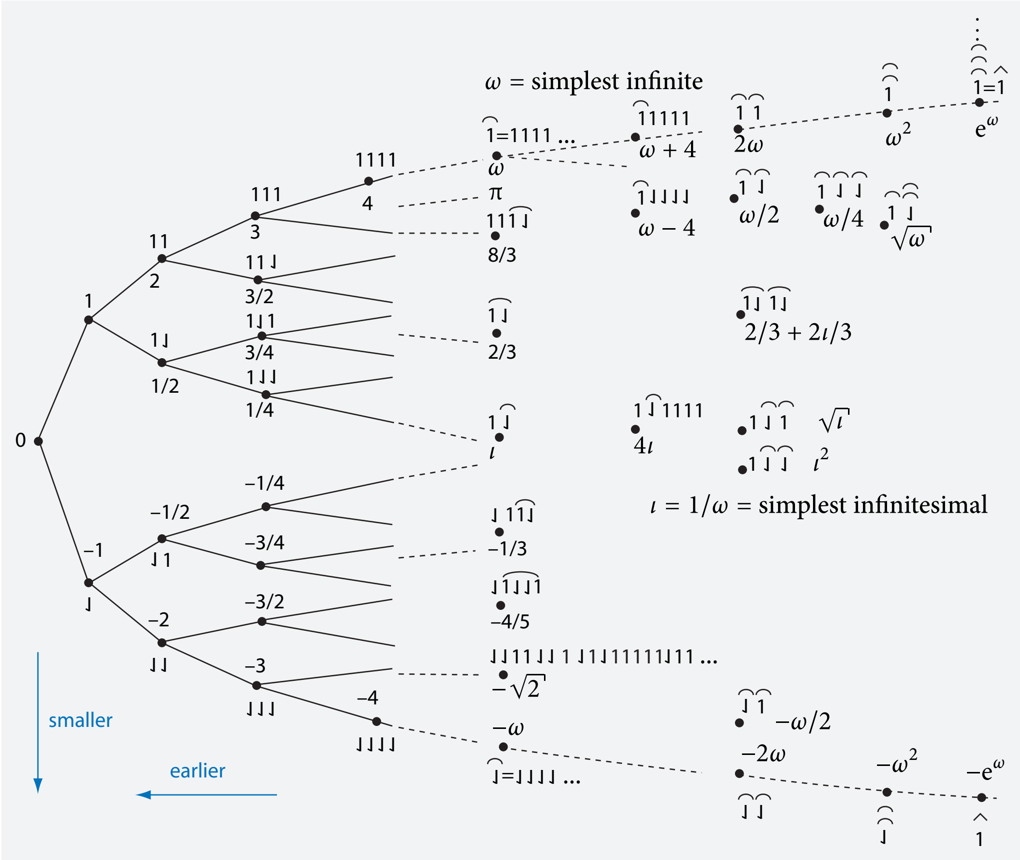


FIGURE 166 The surreal numbers in conventional and in bit notation.

mals and simple infinite numbers. Longer countably infinite series give even more crazy numbers.

The complete class of numbers that is defined by a sequence of bits is called the class of *surreal* numbers.*

There is a second way to write surreal numbers. The first is the just mentioned sequence of bits. But in order to define addition and multiplication, another notation is usually used, deduced from Figure 166. A surreal α is defined as the earliest number of all those between two series of earlier surreals, the left and the right series:

$$\alpha = \{a, b, c, ...|A, B, C, ...\} \quad \text{with} \quad a, b, c, < \alpha < A, B, C\ . \qquad (101)$$

* The surreal numbers do *not* form a set since they contain all *ordinal numbers*, which themselves do not form a set, even though they of course *contain* sets. In short, ordinals and surreals are classes which are *larger* than sets.

For example, we have

$$\{0|\} = 1\ ,\ \{0,1\,|\} = 2\ ,\ \{|0\} = -1\ ,\ \{|-1,0\} = -2\ ,\ \{0\,|1\} = 1/2\ ,$$
$$\{0\,|1/2,1/4\} = 1\ ,\ \{0,1,3/2,25/16\,|\,41/16,13/8,7/4,2\} = 1+37/64\ , \qquad (102)$$

showing that the finite surreals are the *dyadic numbers* $m/2^n$ (n and m being integers). Given two surreals $\alpha = \{..., a, ...|..., A, ...\}$ with $a < \alpha < A$ and $\beta = \{..., b, ...|..., B, ...\}$ with $b < \beta < B$, addition is defined recursively, using earlier, already defined numbers, as

$$\alpha + \beta = \{..., a+\beta, ..., \alpha+b, ...|..., A+\beta, ..., \alpha+B, ...\}\ . \qquad (103)$$

Challenge 268 s

This definition is used simply because it gives the same results as usual addition for integers and reals. Can you confirm this? By the way, addition is not always commutative. Are you able to find the exceptions, and to find the definition for subtraction? Multiplication is also defined recursively, namely by the expression

$$\alpha\beta = \{..., a\beta+\alpha b-ab, ..., A\beta+\alpha B-AB, ...|$$
$$..., a\beta+\alpha B-aB, ..., A\beta+\alpha b-Ab, ...\}\ . \qquad (104)$$

Ref. 250

These definitions allow one to write $\iota = 1/\omega$, and to talk about numbers such as $\sqrt{\omega}$, the square root of infinity, about $\omega+4$, $\omega-1$, 2ω, e^{ω} and about other strange numbers shown in Figure 166. However, the surreal numbers are not commonly used. More common is one of their subsets.

Challenge 269 s

The *real numbers* are those surreals whose decimal expansion is not larger than infinity and in addition, equate numbers such as 0.999999... and 1.000000..., as well as all similar cases. In other words, the surreals distinguish the number 0.999999... from the number 1, whereas the reals do not. Indeed, between these two surreal numbers there are infinitely many other surreals. Can you name a few?

Vol. IV, page 220

Reals are more useful for describing nature than surreals, first because they form a *set* – which the surreals do not – and secondly because they allow the definition of *integration*. Other numbers defined with the help of reals, e.g. the complex numbers $\mathbb{C}$, the quaternions $\mathbb{H}$and a few more elaborate number systems, are presented in the next volume.

To conclude, in physics it is usual to call *numbers* the elements of any set that is a semi-ring (e.g. $\mathbb{N}$), a ring (e.g. $\mathbb{Z}$) or a field ($\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$ or $\mathbb{H}$). Since numbers allow one to compare magnitudes and thus to *measure*, these numbers play a central role in the description of observations.

Ref. 251

Challenge 270 s

“A series of equal balls is packed in such a way that the area of needed wrapping paper is minimal. For small numbers of balls the linear package, with all balls in one row, is the most efficient. For which number of balls is the linear package no longer a minimum?”

Is mathematics a language?

> Die Sätze der Mathematik sind Gleichungen, also Scheinsätze. Der Satz der Mathematik drückt keinen Gedanken aus.*
>
> Ludwig Wittgenstein, *Tractatus*, 6.2, 6.21

Surely, mathematics is a *vocabulary* that helps us to talk with precision. Mathematics can be seen as the exploration of *all* possible concepts that can be constructed from the two fundamental bricks 'set' and 'relation' (or some alternative, but equivalent pair).

▷ *Mathematics* is the science of symbolic necessities.

Rephrased again, mathematics is the exploration of all possible types of classifications. Or, less humorously: mathematics is the exploration of tautologies. These aspects explain the usefulness of mathematics in all situations where complex, yet precise classifications of observations are necessary, such as in physics.

However, mathematics cannot express everything that humans want to communicate, such as wishes, ideas or feelings. Just try to express the fun of swimming using mathematics. Indeed, mathematics is the science of symbolic *necessities*; thus mathematics is not a language, nor does it contain one. Mathematical concepts, being based on *abstract* sets and relations, do not pertain to nature. Despite its beauty, mathematics does not allow us to talk about nature or the observation of motion. Mathematics does not tell *what* to say about nature; it does tell us *how* to say it.

In his famous 1900 lecture in Paris, the famous mathematician David Hilbert** gave a list of 23 great challenges facing mathematics. The sixth of Hilbert's problems was to find a mathematical treatment of the axioms of physics. Our adventure so far has shown that physics started with a *circular definition* that has not yet been eliminated after 2500 years of investigations: space-time is defined with the help of objects and objects are defined with the help of space and time. Being based on a circular definition, physics is thus *not* modelled after mathematics, even if many physicists and mathematicians, including Hilbert, would like it to be so. Physicists must live with logical problems and must walk on unsure ground in order to achieve progress. In fact, they have done so for 2500 years. If physics were an axiomatic system, it would not contain circular definitions; on the other hand, it would also cease to be a language and would cease to describe nature. We will return to this issue in detail in the last part of our adventure.

Vol. I, page 406
Ref. 252
Vol. VI, page 104

* 'The propositions of mathematics are equations, and therefore pseudo-propositions. A proposition of mathematics does not express a thought.'

** David Hilbert (b. 1862 Königsberg, d. 1943 Göttingen) was professor of mathematics in Göttingen and the greatest mathematician of his time. He was a central figure to many parts of mathematics, and also played an important role both in the birth of general relativity and of quantum theory. His textbooks are still in print. His famous personal credo was: 'Wir müssen wissen, wir werden wissen.' (We must know, we will know.) His famous Paris lecture is published e.g. in *Die Hilbertschen Probleme*, Akademische Verlagsgesellschaft Geest & Portig, 1983. The lecture galvanized all of mathematics. (Despite efforts and promises of similar fame, *nobody* in the world had a similar overview of mathematics that allowed him or her to repeat the feat in the year 2000.) In his last decade he suffered the persecution of the Nazi regime; the persecution eliminated Göttingen from the list of important science universities, without recovering its place up to this day.

Why use mathematics?

> Die Forderung der Möglichkeit der einfachen Zeichen ist die Forderung der Bestimmtheit des Sinnes.*
>
> Ludwig Wittgenstein, *Tractatus*, 3.23

Numbers, as well as most other mathematical concepts, were developed precisely with the aim of describing nature.

▷ Numbers and mathematical concepts were developed right from the start to provide as succinct a description as possible.

This property is one consequence of mathematics being the science of *symbolic* necessities. Mathematical concepts are *tools* that help our thinking. This is the reason that mathematics is used in physics.

Several well-known physicists have repeatedly asked why mathematics is so important. Ref. 253 For example, Niels Bohr is quoted as having said: 'We do not know why the language of mathematics has been so effective in formulating those laws in their most succinct form.' Eugene Wigner wrote an often cited paper entitled *The unreasonable effectiveness of mathematics*. Ref. 254 At the start of science, many centuries earlier, Pythagoras and his contemporaries were so overwhelmed by the usefulness of numbers in describing nature that Pythagoras was able to organize a sect based on this connection. The members of the inner circle of this sect were called 'learned people,' in Greek '*mathematicians*', from the Greek μάθημα 'teaching'. This sect title then became the name of the modern profession. But asking about the effectiveness of mathematics is akin to asking about the effectiveness of carpenter tools.

Perhaps we are being too dismissive. Perhaps the mentioned thinkers mainly wanted to express their feeling of wonder when experiencing that language works, that thinking and our brain works, and that life and nature are so beautiful. This would put the title question nearer to the well-known statement by Albert Einstein: 'The most incomprehensible fact about the universe is that it is comprehensible.' Comprehension is another word for description, i.e., for classification. Obviously, any separable system is comprehensible, and there is nothing strange about it. But is the universe separable? As long as is it described as being made of particles and vacuum, this is the case.

The basic assumption we made at our start was the separability of nature. This is the central idea that Pythagoras' sect expressed in their core belief

▷ Everything in nature is numbers.

But Pythagoras and his sect were wrong. Like for so many beliefs, observation will show the opposite. We will discover this in the last part of our adventure. The assumption that observations in nature can be separated is an approximation. In short:

▷ Counting is an approximation.

* 'The requirement that simple signs be possible is the requirement that sense be determinate.'

The foundations of physics we used so far are built on sand. A better foundation is needed. The quoted 'incomprehensibility' of nature then becomes amazement at the precision of this approximation. This experience is the high point of our adventure.

> Die Physik ist für Physiker viel zu schwer.*
> David Hilbert

Curiosities and fun challenges about mathematics

Challenge 271 s What is the largest number that can be written with four digits of 2 and no other sign? And with four 4s?

* *

Pythagorean triplets are integers that obey $a^2 + b^2 = c^2$. Give at least ten examples. Then show the following three properties: at least one number in a triplet is a multiple of 3; at least one number in a triplet is a multiple of 4; at least one number in a triplet is a multiple of 5. Challenge 272 e

* *

Challenge 273 e How many zeroes are there at the end of 1000! ?

* *

A mother is 21 years older than her child, and in 6 years the child will be 5 times younger than the mother. Where is the father? This is the *young mother puzzle*. Challenge 274 s

* *

The number $1/n$, when written in decimal notation, has a periodic sequence of digits. The period is at most $n-1$ digits long, as for $1/7 = 0.142857\,142857\,1428....$ Which other numbers $1/n$ have periods of length $n - 1$? Challenge 275 d

* *

Felix Klein was a famous professor of mathematics at Göttingen University. There were two types of mathematicians in his department: those who did research on whatever they wanted and those for which Klein provided the topic of research. To which type did Klein belong? Challenge 276 s

Obviously, this is a variation of another famous puzzle. A barber shaves all those people who do not shave themselves. Does the barber shave himself? Challenge 277 s

* *

Everybody knows what a *magic square* is: a square array of numbers, in the simplest case from 1 to 9, that are distributed in such a way that the sum of all rows, columns (and possibly all diagonals) give the same sum. Can you write down the simplest $3 \times 3 \times 3$ *magic cube*? Challenge 278 s

* *

* 'Physics is much too difficult for physicists.'

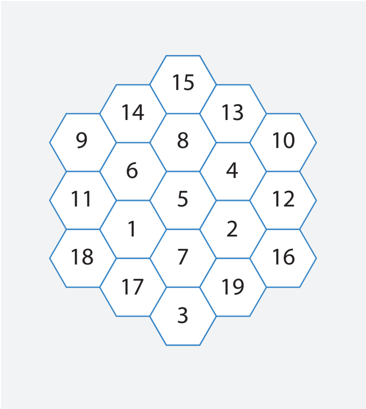


FIGURE 167 The only magic hexagon starting with the number 1 (up to reflections and rotations).

The digits 0 to 9 are found on keyboards in two different ways. Calculators and keyboards have the 7 at the top left, whereas telephones and automatic teller machines have the digit 1 at the top left. The two standards, respectively by the International Standards Organization (ISO) and by the International Telecommunication Union (ITU, formerly CCITT), evolved separately and have never managed to merge. Ref. 255

* *

Leonhard Euler in his notebooks sometimes wrote down equations like

$$1 + 2^2 + 2^4 + 2^6 + 2^8 + ... = -\frac{1}{3} . \tag{105}$$

Challenge 279 d Can this make sense?

* *

In the history of recreational mathematics, several people have independently found the well-known magic hexagon shown in Figure 167. The first discoverer was, in 1887, Ernst von Hasselberg. The hexagon is called magic because all lines add up to the same number, 38. Hasselberg also proved the almost incredible result that no other magic hexagon exists. Can you confirm this? Challenge 280 d

* *

In many flowers, numbers from the *Fibonacci series* 1, 1, 2, 3, 5, 8, 13, 21 etc., appear. Figure 174 gives a few examples. It is often suggested that this is a result of some deep sense of beauty in nature. This is *not* the case, as Figure 168 shows. Mark a spot on a surface, and put washers around it in by hand in a spiral manner; you will find the same spirals that you find in many flowers, and thus, at their border, the same Fibonacci numbers. This argument by Donald Simanek shows that there is nothing deep, complicated or even mysterious in the appearance of Fibonacci numbers in plants. For an opposite point of view, see reference Ref. 247.

Page 228
Ref. 247

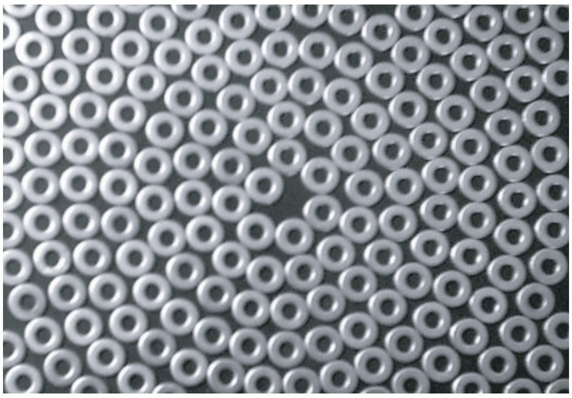


FIGURE 168 Fibonacci numbers and spirals from washers (© Donald Simanek).

* *

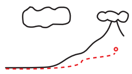
Prime numbers are a favourite playground for mathematicians. A famous result on all prime numbers p_i states

$$\prod_{i=1}^{\infty}\left(1-\frac{1}{p_i^2}\right)=\frac{6}{\pi^2} \qquad (106)$$

Challenge 281 s Can you imagine how this result is proven?

* *

Digits owe their name to the latin word 'digitum' or finger. In times when writing on paper was expensive, it was already possible to count up to 9999 using the two hands, with a system developed by Beda Venerabilis and popularized, for example, by Luca Pacioli.
Challenge 282 e Can you develop a similar system?



CHAPTER 9

CONCEPTS, LIES AND PATTERNS OF NATURE

> Die Grenzen meiner Sprache bedeuten die Grenzen meiner Welt.*
>
> Ludwig Wittgenstein, *Tractatus*, 5.6

> Der Satz ist ein Bild der Wirklichkeit. Der Satz ist ein Modell der Wirklichkeit, so wie wir sie uns denken.**
>
> Ludwig Wittgenstein, *Tractatus*, 4.01

Ref. 256

In contrast to mathematics, physics does aim at being a language. But it is ambitious: it aims to express *everything*, with *complete* precision, and, in particular, all examples and possibilities of change.*** Like any language, physics consists of concepts and sentences. In order to be able to express everything, it must aim to use few words for a lot of facts.**** Physicists are essentially *lazy* people: they try to minimize the effort in everything they do. The concepts in use today have been optimized by the combined effort of many people to be as practical, i.e., as powerful as possible. A concept is called *powerful* when it allows one to express in a compact way a large amount of information, meaning that it can rapidly convey a large number of details about observations.

General statements about many examples of motion are called *rules* or *patterns*. In the past, it was often said that 'laws govern nature', using an old and inappropriate ideology. A physical 'law' is only a way of saying as much as possible with as few words as possible.

* 'The limits of my language are the limits of my world.'

** 'A proposition is a picture of reality. A proposition is a model of reality as we imagine it.'

*** All observations are about change. The various types of change are studied by the various sciences; they are usually grouped in the three categories of *human sciences*, *formal sciences* and *natural sciences*. Among the latter, the oldest are astronomy and metallurgy. Then, with the increase of curiosity in early antiquity, came the natural science concerned with the topic of motion: *physics*. In the course of our walk it will become clear that the unusual definition of physics as the study of change indeed covers the whole set of topics studied in physics. In particular it includes the more common definition of physics as the study of matter, its properties, its components and their interactions.

**** A particular, specific observation, i.e., a specific example of input shared by others, is called a *fact*, or in other contexts, an *event*. A striking and regularly observed fact is called a *phenomenon*, and a general observation made in many different situations is called a *(physical) principle*. (Often, when a concept is introduced that is used with other meaning in other fields, in this walk it is preceded by the qualifier 'physical' or 'mathematical' in parentheses.) Actions performed towards the aim of collecting observations are called *experiments*. The concept of experiment became established in the sixteenth century; in the evolution of a child, it can best be compared to that activity that has the same aim of collecting experiences: *play*.

When saying 'laws govern nature' we actually mean to say 'being lazy, we describe observations with patterns'. Laws are the epitome of laziness. Formulating laws is pure sloth. In fact, the correct expression is

▷ Patterns describe nature.

Physicists have written about the laziness necessary to find patterns in much detail. In order to become a master of laziness, we need to distinguish lazy patterns from those which are not, such as lies, beliefs, statements that are not about observations, and statements that are not about motion. We do this below.

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The quest for laziness is the origin, among others, of the use of numbers in physics. Observables are often best described with the help of numbers, because numbers allow easy and precise communication and classification. Length, velocity, angles, temperature, voltage or field strength are of this type. The notion of 'number', used in every measurement, is constructed, often unconsciously, from the notions of 'set' and 'relation', as shown above. Apart from the notion of number, other concepts are regularly defined to allow fast and compact communication of the 'laws' of nature; all are 'abbreviation tools.' In this sense, the statement 'the level of the Kac–Moody algebra of the Lagrangian of the heterotic superstring model is equal to one' contains precise information, explainable to everybody; however, it would take dozens of pages to express it using only the terms 'set' and 'relation.' In short, the *precision* common in physics results from its *quest for laziness*.

> “Es ist besser, daß die Leute nicht wissen, wie Gesetze und Wurst zustande kommen. Sonst könnten sie nachts nicht ruhig schlafen.*”
>
> Bismarck, Otto von

Are physical concepts discovered or created?

> “Das logische Bild der Tatsachen ist der Gedanke.**”
>
> Ludwig Wittgenstein, *Tractatus*, 3

The title question is often rephrased as: are physical concepts free of beliefs, taste or personal choices? The question has been discussed so much that it even appears in Hollywood films. We give a short summary that can help you to distinguish honest from dishonest teachers.

Creation of concepts, in contrast to their discovery, would imply free choice between many alternative possibilities. The chosen alternative for the definition of concepts would then be due to the beliefs or tastes used. In physics (in obvious contrast to other, more ideological fields of enquiry), we know that different physical descriptions of observations are either equivalent or, in the opposite case, imprecise or even wrong. A physical description of observations is thus essentially unique: any choices of concepts are only apparent. There is no real freedom in the definition of physical concepts. In this property, physics is in strong contrast to artistic activity.

* 'It is better that people do not know how laws and sausages are made. Otherwise they would not sleep well at night.' Otto von Bismarck (b. 1815 Schönhausen, d. 1898 Friedrichsruh) was Prussian Chancellor.

** 'A logical picture of facts is a thought.'

If two different physical concepts can be used to describe the same aspect of observations, they must be equivalent, even if the relation that leads to the equivalence is not immediately clear. In fact, the requirement that people with different standpoints and observing the same event deduce equivalent descriptions lies at the very basis of physics. It expresses the requirement that observations are observer-independent. In short, the strong requirement of viewpoint independence makes the free choice of concepts a logical impossibility.

The conclusion that concepts describing observations are discovered rather than created is also reached independently in the field of linguistics by the above-mentioned research on semantic primitives,* in the field of psychology by the observations on the formation of the concepts in the development of young children, and in the field of ethology by the observations of animal development, especially in the case of mammals. In all three fields detailed observations have been made of how the interactions between an individual and its environment lead to concepts, of which the most basic ones, such as space, time, object or interaction, are common across the sexes, cultures, races and across many animal species populating the world. Curiosity and the way that nature works leads to the same concepts for all people and even the animals; the world offers only one possibility, without room for imagination. Imagining that physical concepts can be created at your leisure is a belief – or a useful exercise, but never successful.

Physical concepts are classifications of observations. The activity of classification itself follows the patterns of nature; it is a mechanical process that machines can also perform. This means that any distinction, i.e., any statement that A is different from B, is a theory-free statement. No belief system is necessary to distinguish different entities in nature. Cats and pigs can also do so. Physicists *can* be replaced by animals, even by machines. Our mountain ascent will repeatedly confirm this point.

As already mentioned, the most popular physical concepts allow us to describe observations as succinctly and as accurately as possible. They are formed with the aim of having the largest possible amount of understanding with the smallest possible amount of effort. Both Occam's razor – the requirement not to introduce unnecessary concepts – and the drive for unification automatically reduce the number and the type of concepts used in physics. In other words, the progress of physical science was and is based on a programme that reduces the possible choice of concepts as drastically as possible.

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In summary, we found that physical concepts are the same for everybody and are free of beliefs and personal choices: they are, first of, all *boring*. Moreover, as they could stem from machines instead of people, concepts are *born of laziness*. These human analogies – not meant to be taken too seriously – confirm that physical concepts are *not* created; they are discovered. If a teacher tells you the opposite, he is lying.

Having handled the case of physical concepts, let us now turn to physical statements. The situation is somewhat similar: physical statements must be *correct*, *lazy*, *arrogant* and *boring*. Let us see why.

* Anna Wierzbicka concludes that her research clearly indicates that semantic primitives are *discovered*, in particular that they are deduced from the fundamentals of human experience, and not invented.

Ref. 237

TABLE 24 The 'scientific method'.

Normal description **Curiosity**	Lobbyist description **Scientific method**
1. look around a lot	1. interact with the world
2. don't believe anything told	2. forget unproven statements
3. choose something interesting and explore it yourself	3. observe and measure
4. make up your own mind and describe precisely what you saw	4. use reason, build hypothesis
5. check if you can also describe similar situations in the same way	5. analyse hypothesis
6. increase the precision of observation until the checks either fail or are complete	6. perform experiments to check hypothesis
7. depending on the case, continue with step 4 or 1	7. ask authority for more money

> Wo der Glaube anfängt, hört die Wissenschaft auf.*
>
> Ernst Haeckel, *Natürliche Schöpfungsgeschichte*, 1879.

How do we find physical concepts, patterns and rules?

> Grau, theurer Freund, ist alle Theorie,
> Und grün des Lebens goldner Baum.**
>
> J.W. v. Goethe, *Faust*.

> Physics is usually presented as an objective science, but I notice that physics changes and the world stays the same, so there must be something subjective about physics.
>
> Richard Bandler

Progressing through the study of motion reflects a young child's attitude towards life: a child is driven by curiosity. The progress follows the simple programme on the left of Table 24. Adult scientists do the same, except that they use more fashionable terms, given on the right of the table. Adults also have specialized professions to make money from their curiosity. The experts of step 7, who request more money, are variously called *lobbyists* or *fund raisers*; instead of calling this program *curiosity*, they call it the *scientific method*.

Physics being the talk about motion,*** and motion being a vast topic, a lot can be explored and told. The experts of step 6 are called *experimental physicists* or simply *exper-*

* 'Where belief starts, science ends.'

** 'Grey, dear friend, is all theory, and green the golden tree of life.' Johann Wolfgang von Goethe (b. 1749 Frankfurt am Main, d. 1832 Weimar), the influential German poet.

*** Several sciences have the term 'talk' as part of their name, namely all those whose name finishes in '-logy', such as e.g. biology. The ending stems from ancient Greek and is deduced from λήγηιν meaning 'to say, to talk'. Physics as the science of motion could thus be called 'kinesiology' from κίνησις, meaning 'motion'; but for historical reasons this term has a different meaning, namely the study of human muscular activity – and

imentalists, a term derived from the Latin 'experiri', meaning 'to try out'. Most of them are part of the category 'graduate students'. The experts of steps 5 and 4 are called *theoretical physicists* or simply *theoreticians*.* This is a rather modern term; the first professors of theoretical physics were appointed around the start of the twentieth century. The term 'theory' is derived from the Greek θεωρία meaning 'observation, contemplation'. Finally, there are the people who focus on steps 1 to 3, and who induce others to work on steps 4 to 6; they are called *geniuses*. The geniuses are those people who introduce the concepts that best help to describe nature.

Obviously an important point is hidden in step 6: how do all these people know whether their checks fail? How do they know if a concept or statement applies to nature? How do they recognize truth?

> All professions are conspiracies against laymen.
> George Bernard Shaw

What is a lie?

> Get your facts straight, and then you can distort them at your leisure.
> Mark Twain

> The pure truth is always a lie.
> Bert Hellinger

In most countries, every person must know what 'truth' is, since in a law court for example, telling an untruth can lead to a prison sentence. And the courts are full of experts in lie detection. **

In court, a *lie* is a statement that knowingly contrasts with observations.*** The truth of a statement is thus checked by observation. The check itself is sometimes called the *proof* of the statement. For law courts, and for physics, we thus have

▷ ***Truth*** is the correspondence with facts.

▷ ***Facts*** are observations shared with other people or machines.

Therefore,

▷ A ***lie*** is a statement in contrast with facts.

also, unfortunately, a lot of esoteric nonsense. The term 'physics' is either derived from the Greek φύσικη (τέχνη is understood) meaning '(the art of) nature', or from the title of Aristotle' works τά φυσικά meaning 'natural things'. Both expressions are derived from φύσις, meaning 'nature'.

* If you like theoretical physics, have a look at the refreshingly candid web page by Nobel Prize winner Gerard 't Hooft with the title *How to become a good theoretical physicist*. It can be found at www.phys.uu.nl/~thooft/theorist.html.

** Some scholars have spent most of their research career on lies and lying. A well-known example is Paul Ekman, whose fascinating website at www.paulekman.com tells how to spot lies from the behaviour of the person telling it.

*** Statements not yet checked are variously called *speculations*, *conjectures*, *hypotheses*, or – wrongly – simply *theses*. Statements that are in correspondence with observations are called *correct* or *true*; statements that contrast with observations are called *wrong* or *false*.

Except in court, lies are fun statements, because we can draw any imaginable conclusion from them. A well-known discussion between two Cambridge professors early in the twentieth century makes the point. McTaggart asked: 'If $2 + 2 = 5$, how can you prove that I am the pope?' Godfrey Hardy: 'If $2 + 2 = 5$, then $4 = 5$; subtract 3; then $1 = 2$; but McTaggart and the pope are two; therefore McTaggart and the pope are one.' As noted long ago, *ex falso quodlibet*; from what is wrong, anything imaginable can be deduced. Therefore, in our mountain ascent we need to build on previously deduced results and our trip could not be completed if we had a false statement somewhere in our chain of arguments.

Nevertheless, lying is such an important activity that one should learn to perform it well – in order to learn to discover it in others. The art of lying has three stages: the animal stage, the child stage and the adult stage. Many animals have been shown to deceive their kin. Children start lying just before their third birthday, by hiding experiences. Psychological research has even shown that children who lack the ability to lie cannot complete their personal development towards a healthy human being. Ref. 207

Adults are habitual liars. Many adults cheat on taxes. Others lie to cover up their wrongdoings. If you ever lie in court, you better do it well; indeed, experience shows that you might get away with many criminal activities. The worst examples of liars are those violent contemporaries – often politicians or intellectuals – who claim that truth "does not exist".

What is a good lie?

Since a lie is a statement in contrast with facts – or shared observations – a *good* lie is a lie whose contrast with facts is hard to discover. Let us explore the art of good lies.

The first way of lying is to put an emphasis on the sharedness only. Populists and polemics do this regularly. ('Every foreigner is a danger for the values of our country.') Since almost any imaginable opinion, however weird, is held by some group – and thus shared – one can always claim it as true.* Unfortunately, it is no secret that ideas also get shared because they are fashionable, imposed or opposed to somebody who is generally disliked. Often a sibling in a family has this role – remember Cassandra.** For a good lie we thus need more than sharedness, more than *intersubjectivity* alone.

A good lie should be, like a true statement, really independent of the listener and the observer and, in particular, independent of their age, their sex, their education, their civilization or the group to which they belong. For example, it is especially hard – but not impossible – to lie with mathematics. The reason is that the basic concepts of mathematics, be they 'set', 'relation' or 'number', are taken from observation and are intersubjective, so that statements about them are easily checked. Therefore, good lies avoid mathemat-

* The work of the sociologist Gabriel Tarde (b. 1843 Sarlat, d. 1903 Paris), especially his concepts of *imitation* and group mind, already connects to this fact.

** The implications of birth order on creativity in science and on acceptance of new ideas has been studied in the fascinating book by Frank J. Sulloway, *Born to Rebel – Birth Order, Family Dynamics and Creative Lives*, Panthon Books, 1996. This exceptional book tells the result of a life-long study correlating the personal situations in the families of thousands of people and their receptivity to about twenty revolutions in the recent history. The book also includes a test in which the reader can deduce their own propensity to rebel, on a scale from 0 to 100 %. Darwin scores 96 % on this scale.

ics.*

Thirdly, a 'good' lie should avoid statements about observations and use *interpretations* instead. For example, some people like to talk about other universes, which implies talking about fantasies, not about observations. However, a really good lie has to avoid to make statements which are meaningless; the most destructive comment that can be made about a statement is the one used by the great Austrian physicist Wolfgang Pauli:

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'That is not even wrong.'

Fourthly, a good lie avoids talking about observations, but focuses on imagination. Only truth needs to be *empirical*; *speculative* statements differ from truth by not caring about observations. If you want to lie 'well' even with empirical statements, you need to pay attention. There are two types of empirical statements: *specific* statements and *universal* statements. For example, 'On the 2nd of June 1960 I saw a green swan swimming on the northern shore of the lake of Varese' is specific, whereas 'All ravens are black' is universal, since it contains the term 'all'. There is a well-known difference between the two, which is important for lying well: specific statements cannot be falsified, they are only

Ref. 257

verifiable, and universal statements cannot be verified, they are only falsifiable. Why is this so?

Universal statements, such as 'the speed of light is constant', cannot be tested for *all* possible cases. (Note that if they could, they would not be universal statements, but just a list of specific ones.) However, they can be reversed by a counter-example. Another example of the universal type is: 'Apples fall upwards.' Since it is falsified by an observation conducted by Newton several centuries ago, or by everyday experience, it qualifies as an (easily detectable) lie. In general therefore, lying by stating the opposite of a theory is usually unsuccessful. If somebody insists on doing so, the lie becomes a *superstition*, a *belief*, a *prejudice* or a *doctrine*. These are the low points in the art of lying. A famous case of insistence on a lie is that of the colleagues of Galileo, who are said to have refused to look through his telescope to be convinced that Jupiter has moons, an observation that would have shaken their belief that everything turns around the Earth. Obviously these astronomers were amateurs in the art of lying. A good universal lie is one whose counter-example is not so easily spotted.

There should be no insistence on lies in physics. Unfortunately, classical physics is full of lies. We will dispel them during the rest of our walk.

Lying by giving specific instead of universal statements is much easier. ('I can't remember.') Even a specific statement such as 'yesterday the Moon was green, cubic and smelled of cheese' can never be completely falsified: there is no way to show with absolute certainty that this is wrong. The only thing that we can do is to check whether the statement is compatible with other observations, such as whether the different shape affected the tides as expected, whether the smell can be found in air collected that day, etc. A *good* specific lie is thus not in evident contrast with other observations.**

* In mathematics, 'true' is usually specified as 'deducible' or 'provable'; this is in fact a special case of the usual definition of truth, namely 'correspondence with facts', if one remembers that mathematics studies the properties of classifications.

** It is often difficult or tedious to verify statements concerning the past, and the difficulty increases with the distance in time. That is why people can insist on the occurrence of events which are supposed to be exceptions to the patterns of nature ('miracles'). Since the advent of rapid means of communication these checks are becoming increasingly easy, and no miracles are left over. This can be seen in Lourdes in France,

Incidentally, universal and specific statements are connected: the *opposite* of a universal statement is always a specific statement, and vice versa. For example, the opposite of the general statement 'apples fall upwards', namely 'some apples fall downwards', is specific. Similarly, the specific statement 'the Moon is made of green cheese' is in opposition to the universal statement 'the Moon is solid for millions of years and has almost no smell or atmosphere.'

In other words, law courts and philosophers disagree. Law courts have no problem with calling theories true, and specific statements lies. Many philosophers avoid this. For example, the statement 'ill-tempered gaseous vertebrates do not exist' is a statement of the universal type. If a universal statement is in agreement with observations, and if it is falsifiable, law courts call it *true*. The opposite, namely the statement: 'ill-tempered gaseous vertebrates do exist', is of the *specific* type, since it means 'Person X has observed an ill-tempered gaseous vertebrate in some place Y at some time Z'. To verify this, we need a record of the event. If such a record, for example a photographs or testimony does not exist, and if the statement *can* be falsified by other observations, law courts call the specific statement a *lie*. Even though these are the rules for everyday life and for the law, there is no agreement among philosophers and scientists that this is acceptable. Why? Intellectuals are a careful lot, because many of them have lost their lives as a result of exposing lies too openly.

In short, specific lies, like all specific statements, can never be falsified with certainty. This is what makes them so popular. Children learn specific lies first. ('I haven't eaten the jam.') General lies, like all general statements, can always be corroborated by examples. This is the reason for the success of ideologies. But the criteria for recognizing lies, even general lies, have become so commonplace that beliefs and lies try to keep up with them. It became fashionable to use expressions such as 'scientific fact' – there are no non-scientific facts –, or 'scientifically proven' – observations cannot be proven otherwise – and similar empty phrases. These are not 'good' lies; whenever we encounter a sentence beginning with 'science says ...' or 'science and religion do ...' we just need to replace 'science' by 'knowledge' or 'experience' to check whether such a sentence are to be taken seriously or not.*

Lies differ from true statements in their emotional aspect. Specific statements are usually boring and fragile, whereas specific lies are often sensational and violent. In contrast, general statements are often daring and fragile whereas general lies are usually boring and violent. The truth is fragile. True statements require the author to stick his neck out

where even though today the number of visitors is much higher than in the past, no miracles have been seen in decades. (In fact there is one exception that has with several witnesses. In 1998, a man in a wheelchair was pushed into the holy water. When he came out again, miraculously, his wheelchair had new tires.)

In fact, all modern so-called 'miracles' are kept alive only by consciously eschewing checks, such as the supposed yearly liquefaction of blood in Napoli, the milk supposedly drunk by statues in temples, the supposed healers in television evangelism, etc. Most miracles only remain because many organizations make money out of the difficulty of falsifying specific statements. For example, when the British princess Diana died in a car crash in 1997, even though the events were investigated in extreme detail, the scandal press could go on almost without end about the 'mysteries' of the accident. Ref. 258

* To clarify the vocabulary usage of this text: *religion* is spirituality plus a varying degree of beliefs and power abuse. The mixture depends on each person's history, background and environment. *Spirituality* is the open participation in the whole of nature. Most, maybe all, people with a passion for physics are spiritual. Most are not religious.

to criticism. Researchers know that if one doesn't stick the neck out, it can't be an observation or a theory. (A *theory* is another name for one or several connected, not yet falsified universal statements about observations.)* Telling the truth does make vulnerable. For this reason, theories are often *daring*, *arrogant* or *provoking*; at the same time they have to be *fragile* and *vulnerable*. For many men, theories thus resemble what they think about women. Darwin's *The origin of species* illustrates the stark contrast between the numerous boring and solid facts that Darwin collected and the daring theory that he deduced. Boredom of facts is a sign of truth.

In contrast, the witch-hunters propagating 'creationism' or so-called 'intelligent design' are examples of liars. The specific lies they propagate, such as 'the world was created in October 4004 BCE', are sensational, whereas the general lies they propagate, such as 'there have not been big changes in the past', are boring. This is in full contrast with common sense. Moreover, lies, in contrast to true statements, make people violent. The worse the lie, the more violent the people. This connection can be observed regularly in the news. In other words, 'creationism' and 'intelligent design' are not only lies, they are bad lies. A 'good' *general lie*, like a good physical theory, seems crazy and seems vulnerable, such as 'people have free will'. A 'good' *specific lie* is boring, such as 'this looks like bread, but for the next ten minutes it is not'. Good lies do not induce violence. Feelings can thus be a criterion to judge the quality of lies, if we pay careful attention to the type of statement. A number of common lies are discussed later in this chapter.

An important aspect of any 'good' lie is to make as few *public* statements as possible, so that critics can check as little as possible. (For anybody sending corrections of mistakes in this text, I provide a small reward.) To detect lies, public scrutiny is important, though not always reliable. Sometimes, even scientists make statements which are not based on observations. However, a 'good' lie is always well prepared and told on purpose; accidental lies are frowned upon by experts. Examples of good lies in science are 'aether', 'UFOs', 'creation science', or 'cold fusion'. Sometimes it took many decades to detect the lies in these domains.

To sum up, the central point of the art of lying without being caught is simple: do not divulge details. Be *vague*. All the methods used to verify a statement ask for details, for *precision*. For any statement, its degree of precision allows one to gauge the degree to which the author is sticking his neck out. The more precision that is demanded, the weaker a statement becomes, and the more likely a fault will be found, if there is one. This is the main reason that we chose an increase in precision as a guide for our mountain ascent: we want to avoid lies completely. By the way, the same method is used in criminal trials. To discover the truth, investigators typically ask all the witnesses a large number

* In other words, a set of not yet falsified patterns of observations on the same topic is called a *(physical) theory*. The term 'theory' will always be used in this sense in this walk, i.e., with the meaning 'set of correct general statements'. This use results from its Greek origin: 'theoria' means 'observation'; its original meaning, 'passionate and emphatic contemplation', summarizes the whole of physics in a single word. ('Theory', like 'theatre', is formed from the root θέ, meaning 'the act of contemplating'.) Sometimes, however, the term 'theory' is used – being confused with 'hypothesis' – with the meaning of 'conjecture', as in 'your theory is wrong', sometimes with the meaning of 'model', as in 'Chern–Simons' theory and sometimes with the meaning of 'standard procedure', as in 'perturbation theory'. These incorrect uses are avoided here. To bring the issue to a point: the *theory of evolution* is not a conjecture, but a set of correct statements based on observation.

of questions, allowing as many *details* as possible come to light. When sufficient details are collected, and the precision is high enough, the situation becomes clear. Telling 'good' lies is much more difficult than telling the truth; it requires an excellent imagination.

> Truth is an abyss.
>
> Democritus

> To teach superstitions as truth is a most terrible thing.
>
> Hypatia of Alexandria (*c.* 355–415)

> [Absolute truth:] It is what scientists say it is when they come to the end of their labors.
>
> Ref. 259 Charles Peirce

Is this statement true? – A bit about nonsense

> There are three types of people: those who believe in Father Christmas, those who do not believe in Father Christmas, and those who are Father Christmas.
>
> Anonymous

> Truth is a rhetorical concept.
>
> Paul Feyerabend

Not all statements can be categorized as true or false. There is a third option: statements can simply make *no sense*. There are even such statements in mathematics, where they are called *undecidable*. However, 'undecidable' and 'nonsense' is really the same thing! An example is the continuum hypothesis. This hypothesis is undecidable because it makes a statement that depends on the precise meaning of the term 'set'; in standard mathematical usage the term is not defined sufficiently precisely so that a truth value can be assigned to the continuum hypothesis. In short, statements can be undecidable because the concepts contained in them are not sharply defined.

Statements can also be undecidable for other reasons. Phrases such as 'This statement is not true' illustrate the situation. The phrase is undecidable because it references to itself. Kurt Gödel* has even devised a general way of constructing such undecidable statements in the domain of logic and mathematics. The different variations of these *self-referential* statements, especially popular both in the field of logic and computer science, have captured a large public.** Similarly undecidable statements can be constructed with terms Ref. 260 such as 'calculable', 'provable' and 'deducible'.

In fact, self-referential statements are undecidable because they are meaningless. If the usual definition of 'true', namely corresponding to facts, is substituted into the sentence 'This statement is not true', we quickly see that it has no meaningful content. The

* Kurt Gödel (b. 1906 Brünn, d. 1978 Princeton), famous logician.
** A general introduction is given in the beautiful books by Raymond Smullyan: *Satan, Cantor and Infinity and Other Mind-boggling Puzzles*, Knopf, 1992; *What is the Name of This Book? The Riddle of Dracula and Other Logical Puzzles*, Touchstone, 1986, and *The Lady or the Tiger? And Other Puzzles*, Times Books, 1982. Also definitions can have no content, such as David Hilbert's 'smallest number that has not been mentioned this century' or 'the smallest sequence of numbers that is described by more signs than this sentence'.

most famous meaningless sentence of them all was constructed by the linguist Noam Chomsky:

Colorless green ideas sleep furiously.

Ref. 215 It is often used as an example for the language processing properties of the brain, but nobody sensible elevates it to the status of a paradox and writes philosophical discussions about it. To do that with the title of this section is a similar waste of energy.

The main reason for the popular success of self-reference is the difficulty in perceiving the lack of meaning.* A good example is the statement:

This statement is false or you are an angel.

Challenge 284 s We can actually deduce from it that 'you are an angel.' Can you see how? If you want, you can change the second half and get even more interesting statements. Such examples show that statements referring to themselves have to be treated with great care when under investigation. In short, whenever you meet somebody who tries to use the self-referential construction by Kurt Gödel to deduce another statement, take a step back, or better, a few more. Self-reference, especially the type defined by Gödel, is a hard but common path – especially amongst wannabe-intellectuals – to think, tell and write *nonsense*. Nothing useful can be deduced from nonsense. Well, not entirely; it does help to meet psychiatrists on a regular basis.

In physics, in the other natural sciences, and in legal trials these problems do not emerge, because self-referential statements are not used.** In fact, the work of logicians confirms, often rather spectacularly, that there is no way to extend the term 'truth' beyond the definition of 'correspondence with facts.'

> Ein Satz kann unmöglich von sich selbst aussagen, daß er wahr ist.***
>
> Ludwig Wittgenstein, *Tractatus*, 4.442

Curiosities and fun challenges about lies and nonsense

> A man is his own easiest dupe, for what he wishes to be true he generally believes to be true.
>
> Ref. 262 Demosthenes, 349 BCE.

* A well-known victim of this difficulty is Paulus of Tarsus. The paradox of the Cretan poet *Epimenedes* (6th century BCE) who said 'All Cretans lie' is too difficult for the notoriously humour-impaired Paulus, who in his letter to Titus (chapter 1, verses 12 and 13, in the christian bible) calls Epimenedes a 'prophet', adds some Ref. 261 racist comments, and states that this 'testimony' is true. But wait! There is a final twist to this story. The statement 'All Cretans lie' is *not* a paradox at all; a truth value can actually be ascribed to it, because the Challenge 283 s statement is not really self-referential. Can you confirm this? The only *genuine* paradox is 'I am lying', to which it is indeed impossible to ascribe a truth value.

Challenge 285 s ** Why are circular statements, like those of Galilean physics, not self-referential?

*** 'It is quite impossible for a proposition to state that it itself is true.'

“Quator vero sunt maxima comprehendendae veritatis offendicula, quae omnem quemcumque sapientem impediunt, et vix aliquem permittunt ad verum titulum sapientiae pervenire: videlicet fragilis et indignae auctoritatis exemplum, consuetudinis diurnitatis, vulgi sensus imperiti, et propriae ignorantiae occultatio cum ostentatione sapientiae apparentis.*”
Roger Bacon, *Opus majus*, 1267.

“Es ist ja nicht alles, was ich den Bürgern sage, gelogen.**”
Konrad Adenauer, 1962, West German Chancellor.

Some lies are entertaining and funny – and are better called jokes –, some are signs of psychic disturbance, and some are made with criminal intent. Some statements are not lies, but simply nonsense. Have fun distinguishing them.

* *

During a church sermon, a man fell asleep. He dreamt about the French revolution: he was being brought to the guillotine. At that moment, his wife noticed that he was sleeping. In the same moment in which the man dreamt that the knife was hitting him, his wife gave him a tap on his neck with her fan. The shock instantly killed the man. – Is this story true or false?

Challenge 286 e

* *

A well-known bad lie: ‘Yesterday I drowned.’

* *

Starting in the 1990s, so-called *crop circles* were produced by people walking with stilts, a piece of wood and some rope into fields of crops. Nevertheless, many pretended and even more believed that these circles were made by extraterrestrial beings. Can you find some reasons why this is impossible?

Challenge 287 s

* *

Often one hears or reads statements like: ‘mind (or spirit or soul) is stronger than matter.’ Beware of anybody who says this; he wants something from you. Can you show that such statements are all and always wrong?

Challenge 288 e

* *

In certain countries, two lies were particularly frequent in the early twenty-first century. The first: global warming does not exist. The second: global warming is not due to human causes. Are these good or bad lies?

Challenge 289 s

* ‘There are four stumbling blocks to truth and knowledge: weak and unworthy authority, custom, popular prejudice, and the concealment of ignorance with apparent knowledge.’
** ‘Indeed, not everything that I tell the people is a lie.’

* *

Sometimes it is heard that a person whose skin is completely covered with finest metal powder will die, due to the impossibility of the skin to breathe. Can you show from you own observation that this is wrong?

Challenge 290 s

* *

A famous mixture of hoax and belief premises that the Earth was created about six thousand years ago. (Some believers even use this lie as justification for violence against non-believers.) Can you explain why the number is wrong?

Challenge 291 s

* *

A famous provocation: the world has been created last Saturday. Can you decide whether this is wrong?

Challenge 292 s

* *

Hundreds of hoaxes are found on the www.museumofhoaxes.com website. It gives an excellent introduction into the art of lying; of course it exposes only those who have been caught. Enjoy the science stories, especially those about archaeology. Several other sites with similar content can be found on the internet.

* *

In the 1990s, many so-called 'healers' in the Philippines earned large amounts of money by suggesting patients that they were able to extract objects from their bodies without operating. Why is this not possible? (For more information on health lies, see the www.quackwatch.com website.)

Challenge 293 e

* *

Is homoeopathy a lie?

Challenge 294 s

* *

Since the 1980s, certain persons have claimed that it is possible to acquire knowledge simply from somebody 1000 km away, without any communication between the two people. However, the assumed 'morphogenetic fields' realizing this feat cannot exist. Why not?

Challenge 295 e

* *

It is claimed that a Fire Brigade building in a city in the US hosts a light bulb that has been burning without interruption since 1901 (at least this was the case in 2005). Can this be true? Hundreds of such stories, often called 'urban legends,' can be found on the www.snopes.com website. However, some of the stories are not urban legends, but true, as the site shows.

Challenge 296 s

* *

'This statement has been translated from French into English.' Is the statement true, false or neither?

* *

Challenge 297 s Aeroplanes have no seat row 13. Many tall hotels have no floor 13. What is the lie behind this habit? What is the truth behind it? Once the author asked a singer in Napoli to sing 'Fenesta che lucive', a beautiful song performed by Enrico Caruso and many others since. The singer refused, explaining that the local public would run out in rage, and that the owners of the place would be forced to clean the whole place with salt, to get rid of bad luck. Many superpositions are found across the world.

* *

Challenge 298 s For about a thousand years, certain people pretend that they have been stigmatized, i.e., that they have 'miraculously' suffered wounds that are similar to those of Jesus's crucifixion. How can one prove by a one-second observation that all of these people, without exception, produced the wounds by themselves?

* *

'In the middle age and in antiquity, people believed in the flat Earth.' This is a famous lie that is rarely questioned. The historian Reinhard Krüger has shown that the lie is most of all due to the writers Thomas Paine (1794) and Washington Irving (1928). Fact is that since Aristotle, everybody believed in a spherical Earth.

* *

Challenge 299 s Is the term 'multiverse', a claimed opposite to 'universe', a lie or a belief?

* *

The following is not a lie. A good way to suppress curiosity in children is used in many environments: let the child watch television!killing curiosity whenever it wants. Do it for a few weeks and you will not recognize the child any more. Do it for a few years, and its curiosity will not come back at all. The internet and smart phones have the same effect.

* *

Challenge 300 e How would you show that 'Earth rays' are a lie?

* *

Challenge 301 s How would you show that the statement 'the laws of nature could change any time' is a lie?

* *

Challenge 302 e 'I can generate energy from the vacuum.' Show that this is a lie.

* *

Challenge 303 e 'Not everything that exists can be measured.' Show that this frequent statement is a lie – without exception.

* *

'Not everything is known.' This statement is quite interesting: modern physics indeed claims the opposite in many domains. For example, all forms of energy are known; so are all forms of moving entities. In short, even though this statement is correct – indeed,

not everything is known – it is often used by liars. Be careful when you hear it; if the statement is made without evidence, it is made by a crook.

* *

Here is a lie that uses mathematics, from a journalist: 'Your university exams treat women applicants worse than men; your statistics show that only 41 % of all female, but 57 % of all male applicants are admitted.' The university is small and has only two faculties; so it checks its numbers.

Faculty 1 admitted 60 % of all males (60 of 100 applicants) and 65 % of all applicant females (13 of 20 applicants). Faculty 2 admitted 30 % of all males (3 of 10 applicants) and 32 % of all females (16 of 50 applicants).

In total, the university thus admitted 63 of 110 male applicants (or 57 %) and 29 of 70 female applicants (or 41 %). In other words, even though in each faculty the percentage of admitted females was *higher*, the *total* admission percentage for females was *lower*.
Challenge 304 e Why? In fact, this is a true story; in this version, the numbers are simplified, to make the situation as clear as possible. But a large university once got in trouble with journalists in this way, despite preferring women in each of its departments. Some journalists are excellent liars.

* *

A domain in which lies are common is the food industry. It is now possible to buy artificial eggs, artificial tomato, or artificial shrimps. Many bread products contain cysteine; for decades, cysteine was extracted from human hair! In Europe, most food products also do not tell the country of origin or the content of genetic engineering. Most Bavarian pretzels are made in China, for example.

* *

A famous lie: genetically engineered crops are good for the food supply. In fact, they increase the use of pesticides, have reduced fertility, cost more and increased food problems. Biofuel for cars has produced the same disastrous effects.

* *

Physicists have helped to reveal that many common statements are lies. Examples are: "astrology holds" - "creation did occur" - "perpetuum mobiles are possible" - "vacuum is an energy source" - "lightning is thrown by Zeus" - "certain actions bring bad luck" - - "energy speeds faster than light exist" - "telepathy is possible" - "more than three spatial dimensions exist" - "there are things that cannot be measured" - "miracles contradict the laws or rules of nature" - - "exceptions to the rules of nature exist" - "quantum theory implies many worlds" - "there are no measurement limits" - "infinite quantities exist in nature" - "supersymmetry is valid" - "particles are membranes" - "a multiverse exists" - "mind is stronger than matter". Other lies and many funny prejudices and superstitions are mentioned throughout our adventure.

* *

The British Broadcasting Corporation is famous for its April 1st pranks. One of the best ever is its documentary on flying penguins. Simply search on the internet for the beauti-

Challenge 305 e ful film showing how a species of penguins takes off and flies.

OBSERVATIONS AND THEIR COLLECTION

> Knowledge is a sophisticated statement of ignorance.
>
> Attributed to Karl Popper

The collection of a large number of true statements about a type of observations, i.e., of a large number of facts, is called *knowledge*. Where the domain of observations is sufficiently extended, one speaks of a *science*. A *scientist* is thus somebody who collects knowledge.* We found above that an observation is classified input in the memory of several people. Since there is motion all around, to describe all these observations is a mammoth task. As for every large task, to a large extent the use of appropriate tools determines the degree of success that can be achieved. These tools, in physics and in all other sciences, fall in three groups: tools for the collection of observations, tools to communicate observations and tools to communicate relations between observations. The latter group has been already discussed in the section on language and on mathematics. We just touch on the other two.

Ref. 263

Have enough observations been collected?

> Every generation is inclined to define 'the end of physics' as coincident with the end of their scientific contributions.
>
> Julian Schwinger**

Physics is an experimental science; it rests on the *collection* of observations. To realize this task effectively, all sorts of *instruments*, i.e., tools that facilitate observations, have been developed and built. Microscopes, telescopes, oscilloscopes, as well as thermometers, hygrometers, manometers, pyrometers, spectrometers amongst others are familiar examples. The precision of many of these tools is being continuously improved even today; their production is a sizeable part of modern industrial activity, examples being electrical measuring apparatus and diagnostic tools for medicine, chemistry and biology. Instruments can be as small as a tip of a few tungsten atoms to produce an electron beam of a few volts, and as large as 27 km in circumference, producing an electron beam with

* The term 'scientist' is a misnomer peculiar to the English language. Properly speaking, a 'scientist' is a follower of *scientism*, an extremist philosophical school that tried to resolve all problems through science. For this reason, some religious sects have the term in their name. Since the English language did not have a shorter term to designate 'scientific persons', as they used to be called, the term 'scientist' started to appear in the United States, from the eighteenth century onwards. Nowadays the term is used in all English-speaking countries – but not outside them, fortunately.

** Julian Seymour Schwinger (b. 1918 New York City, d. 1994 Los Angeles), child prodigy and physicist, was famous for his clear thinking and his excellent lectures. He worked on waveguides and synchroton radiation, made contributions to nuclear physics and developed quantum electrodynamics. For the latter he received the 1965 Nobel Prize in Physics together with Tomonaga and Feynman. He was a thesis advisor to many famous physicists and wrote several excellent and influential textbooks. Nevertheless, at the end of his life, he became strangely interested in a hoax turned sour: cold fusion.

Ref. 264

more than 100 GV effective accelerating voltage. Instruments have been built that contain and measure the coldest known matter in the universe. Other instruments can measure length variations of far less than a proton diameter over kilometre long distances. Instruments have been put deep inside the Earth, on the Moon, on several planets, and have been sent outside the solar system.

In this walk, instruments are not described in detail; many good textbooks on this topic are available. Many observations collected by instruments are not mentioned in our adventure. The most important measurement results in physics are recorded in standard publications, such as the Landolt–Börnstein series and the physics journals. (Appendix E gives a general overview of information sources.)

Ref. 265, Ref. 266

Ref. 267

Will there be significant new future observations in the domain of the fundamentals of motion? At present, *in this specific domain*, even though the number of physicists and publications is at an all-time high, the number of new experimental discoveries has been steadily diminishing for many years and is now fairly small. The sophistication and investment necessary to obtain new results has become extremely high. In many cases, measuring instruments have reached the limits of technology, of budgets or even those of nature. The number of new experiments that produce results showing no deviation from theoretical predictions is increasing steadily. The number of historical papers that try to enliven dull or stalled fields of enquiry are increasing. Claims of new effects and discoveries which turn out to be false, due to measurement errors, self-deceit or even fraud have become so frequent that scepticism to new results has become a common response.

Most importantly, no difference between observations and the theory of motion are known, as we will discover in the next two volumes. Although in many domains of science, including physics, discoveries are still expected, new observations on the fundamentals of motion are only a remote possibility.

In short, the task of collecting observations on the foundations of motion (though not on other topics of physics) seems to be *complete*. Indeed, the vast majority of observations described in this adventure were obtained before the end of the twentieth century. We are not too early with our walk.

> “Measure what is measurable; make measurable what is not.”
>
> Wrongly attributed to Galileo.

Ref. 268

Are all physical observables known?

> “Scientists have odious manners, except when you prop up their theory; then you can borrow money from them.”
>
> Mark Twain

The most practical way to *communicate* observations was developed a long time ago: by measurements. A measurement allows effective communication of an observation to other times and places. This is not always as trivial as it sounds; for example, in the Middle Ages people were unable to compare precisely the ‘coldness’ of the winters of two different years! The invention of the thermometer provided a reliable solution to this requirement. A *measurement* is thus the classification of an observation into a standard set

of observations. To put it simply:

▷ A measurement is a *comparison with a standard*.

This definition of a measurement is precise and practical, and has therefore been universally adopted. For example, when the length of a house is measured, this aspect of the house is classified into a certain set of standard lengths, namely the set of lengths defined by multiples of a unit. A *unit* is the abstract name of the standard for a certain observable. Numbers and units allow the most precise and most effective communication of measurement results.

For all measurable quantities, practical standard units and measurement methods have been defined; the main ones are listed and defined in Appendix A. All units are derived from a few fundamental ones; this is ultimately due to our limited number of senses: length, time and mass are related to sight, hearing and touch. Our limited number of senses is, in turn, due to the small number of observables of nature. Animals and machines share the same fundamental senses.

We call *observables* the different measurable aspects of a system. Most observables, such as size, speed, position, etc. can be described by numbers, and in this case they are *quantities*, i.e., multiples of some standard unit. Observables are usually abbreviated by *(mathematical) symbols*, usually letters from some alphabet. For example, the symbol c commonly specifies the velocity of light. For most observables, standard symbols have been defined by international bodies.* The symbols for the observables that describe the state of an object are also called *variables*. Variables on which other observables depend are often called *parameters*. (Remember: a parameter is a variable constant.) For example, the speed of light is a constant, the position a variable, and the temperature is often a parameter, on which the length of an object, for example, can depend. Note that not all observables are quantities; in particular, parities are not multiples of any unit.

Physical observables are tools to communicate observations. Is it possible to talk about observations at all? Yes, as we do it every day. But it is many a philosopher's hobby to discuss whether there actually is an example for an 'Elementarsatz' – an atomic fact – mentioned by Wittgenstein in his *Tractatus*. There seems to be at least one that fits: *Differences exist*. It is a simple sentence; in the final part of our walk, it will play a central role.

Today, all physical observables are known. The task of defining tools for the communication of observations can thus be considered *complete*. This is a simple and strong statement. It shows that the understanding of the fundamentals of motion is near completion.

Indeed, the BIPM, the Bureau International des Poids et Mesures, has stopped adding new units. The last unit, the katal, was introduced in 1999 as an abrreviation of or mol/s. The full list of physical units is presented in Appendix A.

* All mathematical symbols used in this walk, together with the alphabets from which they are taken, are listed in Appendix A on notation. They follow international standards whenever they are defined. The standard symbols of the physical quantities, as defined by the International Standards Organization (ISO), the International Union of Pure and Applied Physics (IUPAP) and the International Union of Pure and Applied Chemistry (IUPAC), can be found for example in the *bible*, i.e., the *CRC Handbook of Chemistry and Physics*, CRC Press, 1992.

No new observables are expected to be found. In the past, the importance of a physicist could be ranked by the number of observables he or she had discovered. Discovering obervables had always been less common than discovering new patterns, or 'laws' of nature. Even a great scientist such as Einstein, who discovered several pattern of nature, only introduced one new observable, namely the metric tensor for the description of gravity. Following this criterion – as well as several others – Maxwell might be the most important physicist, having introduced several material dependent observables. For Schrödinger, the wave function describing electron motion could be counted as an observable (even though it is a quantity necessary to calculate measurement results, and not itself an observable). Incidentally, the introduction of *any* term that is taken up by others is a rare event; 'gas', 'entropy' or 'kinetic energy' are such examples. Usually, observables were developed by many people cooperating together. Indeed, almost no observables bear people's names, whereas many 'laws' do.

Given that the list of observables necessary to describe nature is complete, does this mean that all the patterns or rules of nature are known? Not necessarily; in the history of physics, observables were usually defined and measured long *before* the precise rules connecting them were found. For example, all observables used in the description of motion itself – such as time, position and its derivatives, momentum, energy and all the thermodynamic quantities – were defined before or during the nineteenth century, whereas the most precise versions of the patterns or 'laws' of nature connecting them, *special relativity* and *non-equilibrium thermodynamics*, have been found only in the twentieth century. The same is true for all observables connected to electromagnetic interaction. The correct patterns of nature, *quantum electrodynamics*, was discovered long after the corresponding observables. The observables that were discovered last were the fields of the strong and the weak nuclear interactions. Also, in this case, the patterns of nature were formulated much later.

In summary, all observables about the fundamentals of motion have been discovered. We are, at this moment of history, in a fortunate situation: we can talk with precision about all motion observed in nature. The last part of our adventure will explore the tiny possibility for errors or loopholes in this statement.

Do observations take time?

An observation is an interaction with some part of nature leading to the production of a record, such as a memory in the brain, data on a tape, ink on paper, or any other fixed pattern applied to a support. The necessary irreversible interaction process is often called *writing* the record. Obviously, writing takes a certain amount of time; zero interaction time would give no record at all. Therefore any recording device, including our brain, always records some *time average* of the observation, however short it may be.

In summary, what we call a fixed image, be it a mental image or a photograph, is always the time average of a moving situation. Without time averaging, we would have no fixed memories. On the other hand, any time averaging introduces a blur that hides certain details; and in our quest for precision, at a certain moment, these details are bound to become important. The discovery of these details will begin in the upcoming part of the walk, the volume that explores quantum theory.

In the final part of our mountain ascent we will discover that there is a shortest possi-

ble averaging time. Observations of that short duration show so many details that even the distinction between particles and empty space is lost. In contrast, our concepts of everyday life appear only after relatively long time averages. The search for an average-free description of nature is one of the big challenges of our adventure.

Is induction a problem in physics?

> Nur *gesetzmäßige* Zusammenhänge sind *denkbar*.*
>
> Ludwig Wittgenstein, *Tractatus*, 6.361

> There is a tradition of opposition between adherents of induction and of deduction. In my view it would be just as sensible for the two ends of a worm to quarrel.
>
> Alfred North Whitehead

Induction is the usual term used for the act of making, from a small and finite number of experiments, general conclusions about the outcome of *all* possible experiments performed in other places, or at other times. In a sense, it is the technical term for sticking out one's neck, which is necessary in every scientific statement. Universal statements, including the so-called 'laws' and patterns of nature, rely on induction. Induction has been a major topic of discussion for science commentators. Frequently one finds the remark that knowledge in general, and physics in particular, relies on induction for its statements. According to some, induction is a type of hidden belief that underlies all sciences but at the same time contrasts with them.

To avoid wasting energy, we make only a few remarks. The first can be deduced from a simple experiment. Try to convince a critic of induction to put their hand into a fire. Nobody who honestly calls induction a belief should conclude from a few unfortunate experiences in the past that such an act would also be dangerous in the future... In short, somehow induction works.

A second point is that physical universal statements are always openly stated; they are never hidden. The refusal to put one's hand into a fire is a consequence of the invariance of observations under time and space translations. Indeed, general statements of this type form the very basis of physics. However, no physical statement is a belief only because it is universal; it always remains open to experimental checks. Physical induction is not a hidden method of argumentation, it is an explicit part of experimental statements. In fact, the complete list of 'inductive' statements used in physics is well known: we gave it in the first part of our adventure. These statements are so important that they have been given a special name: they are called *symmetries*. The list of all known symmetries of nature is the list of all inductive statements used in physics.

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Perhaps the best argument for the use of induction is that there is no way to avoid it when one is thinking. There is no way to think, to talk or to remember without using concepts, i.e., without assuming that most objects or entities or processes have the same properties over time. There is also no way to communicate with others without assuming that the observations made from the other's viewpoint are similar to one's own. There is

* 'Only connexions that are *subject to law* are *thinkable*.'

no way to think without symmetry and induction. Indeed, the concepts related to symmetry and induction, such as space and time, belong to the fundamental concepts of language. In fact, the only sentences which do not use induction, the sentences of logic, do not have any content (*Tractatus*, 6.11). Without induction, we cannot classify observations at all! Evolution has given us memory and a brain because induction works. To criticize induction is not to criticize natural sciences, it is to criticize the use of thought in general. We should never take too seriously people who themselves do what they criticize in others; sporadically pointing out the ridicule of this endeavour is just the right amount of attention they deserve.

Page 258
Ref. 263
Challenge 306 s

The topic could be concluded here, were it not for some interesting developments in modern physics that put two additional nails in the coffin of arguments against induction. First, in physics whenever we make statements about all experiments, all times or all velocities, such statements are actually about a *finite number* of cases. We know today that infinities, both in size and in number, do not occur in nature. The infinite number of cases appearing in statements in classical physics and in quantum mechanics are apparent, not real, and due to human simplifications and approximations. Statements that a certain experiment gives the same result 'everywhere' or that a given equation is correct for 'all times', always encompass only a *finite* number of examples. A great deal of otherwise often instinctive repulsion to such statements is avoided in this way. In the sciences, as well as in this book, 'all' *never* means an infinite number of cases.

Secondly, it is well known that extrapolating from a few cases to many is false when the few cases are independent of each other. However, this conclusion is correct if the cases are interdependent. From the observation that somebody found a penny on the street on two subsequent months, cannot follow that he will find one the coming month. Induction is only correct if we know that all cases have similar behaviour, e.g. because they follow from the same origin. For example, if a neighbour with a hole in his pocket carries his salary across that street once a month, and the hole always opens at that point because of the beginning of stairs, then the conclusion would be correct.

The results of modern physics encountered in the final part of our walk show that all situations in nature are indeed interdependent, and thus we will prove that what is called 'induction' is in fact a logically correct conclusion.

> In the progress of physics, the exception often turned out to be the general case.

THE QUEST FOR PRECISION AND ITS IMPLICATIONS

> Der Zweck der Philosophie ist die logische Klärung der Gedanken.*
>
> Ludwig Wittgenstein, *Tractatus*, 4.112

To talk well about motion means to talk precisely. Precision requires avoiding three common mistakes in the description of nature.

* 'The object of philosophy is the logical clarification of thoughts.'

First, concepts must be *consistent*. Concepts should never have a contradiction built into their definition. For example, any phenomenon occurring in nature evidently is a 'natural' phenomenon; therefore, to talk about either 'supernatural' phenomena or 'unnatural' phenomena is a mistake that nobody interested in motion should let go unchallenged; such terms contain a logical contradiction. Naturally, *all* observations are natural. Incidentally, there is a reward of more than a million dollars for anybody proving the opposite. In over twenty years, nobody has yet been able to collect it.

Ref. 269

Second, concepts must be *fixed*. Concepts should not have unclear or constantly changing definitions. Their content and their limits must be kept constant and explicit. The opposite of this is often encountered in crackpots or populist politicians; it distinguishes them from more reliable thinkers. Physicists also fall into the trap; for example, there is, of course, only one *single* (physical) universe, as even the name says. To talk about more than one universe is an increasingly frequent error.

Ref. 270

Third, concepts must be *used as defined*. Concepts should not be used outside their domain of application. It is easy to succumb to the temptation to transfer results from physics to philosophy without checking the content. An example is the question: 'Why do particles follow the laws of nature?' The flaw in the question is due to a misunderstanding of the term 'laws of nature' and to a confusion with the laws of the state. If nature were governed by 'laws', they could be changed by parliament. Remembering that 'laws of nature' simply means 'pattern', 'property' or 'description of behaviour', and rephrasing the question correctly as 'Why do particles behave in the way we describe their behaviour?' one can recognize its senselessness.

In the course of our walk, we will often be tempted by these three mistakes. A few such situations follow, with the ways of avoiding them.

> “Consistency is the last refuge of the unimaginative.”
>
> Oscar Wilde

What are interactions? – No emergence

> “The whole is always more than the sum of its parts.”
>
> Aristotle, *Metaphysica*, 10f–1045a.

In the physical description of nature, the whole is always *more* than the sum of its parts. Actually, the difference has a special name:

▷ The the difference between a whole and the sum of its parts is called the *interaction between the parts.*

For example, the energy of the whole minus the sum of the energies of its parts is called the energy of interaction. The study of interactions is the main topic of physics. In other words, physics is concerned *primarily* with the difference between the parts and the whole, contrary to what is often suggested by bad journalists or other sloppy thinkers.

Note that the term 'interaction' is based on the general observation that anything that affects anything else is, in turn, affected by it:

▷ Interactions are reciprocal.

For example, if one body changes the momentum of another, then the second changes the momentum of the first by the same (negative) amount. The reciprocity of interactions is a result of conservation 'laws'. The reciprocity is also the reason that somebody who uses the term 'interaction' is considered a heretic by monotheistic religions, as theologians Ref. 271 regularly point out. These belief experts regularly stress that such a reciprocity implicitly Challenge 307 s denies the immutability of the deity. (Are they correct?)

The simple definition of interaction given above sounds elementary, but it leads to surprising conclusions. Take the atomic idea of Democritus in its modern form: nature is made of vacuum and of particles. The first consequence is the *paradox of incomplete description*: experiments show that there are interactions between vacuum and particles. However, interactions are differences between parts and the whole, in this case between vacuum and particles on the one hand, and the whole on the other. We thus have deduced that nature is not made of vacuum and particles alone.

The second consequence is the *paradox of overcomplete description*. It starts from the Vol. IV, page 195 result that is deduced later on:

▷ Experiments also show that interactions happen through exchange of particles.

However, we have counted particles already as basic building blocks of nature. Does this mean that the description of nature by vacuum and particles is an overdescription, counting things twice? Challenge 308 e Vol. VI, page 80 We will resolve both paradoxes in the last part of our mountain ascent.

The application of the definition of interaction also settles the frequently heard question of whether in nature there are 'emergent' properties, i.e., properties of systems that cannot be deduced from the properties of their parts and interactions. By this definition, there are no emergent properties. 'Emergent' properties can only appear if interactions Vol. I, page 395 are approximated or neglected. The idea of 'emergent' properties is a product of minds with restricted horizons, unable to see or admit the richness of consequences that general Ref. 272 principles can produce. In defending the idea of emergence, one belittles the importance of interactions, working, in a seemingly innocuous, maybe unconscious, but in fact sneaky way, against the use of reason in the study of nature. 'Emergence' is a superstition.

What is existence?

> “You know what I like most? Rhetorical questions.”

Ref. 273 Assume a friend tells you 'I have seen a *grampus* today!' You would naturally ask what it looks like. What answer do we expect? We expect something like 'It's an animal with a certain number of heads similar to a X, attached to a body like a Y, with wings like a Z, it make noises like a U and it felt like a V' – the letters denoting some other animal or object. Generally speaking, in the case of an object, this scene from Darwin's voyage to South America shows that in order to talk to each other, we first need certain basic, common

concepts ('animal', 'head', 'wing', etc.).* In addition, for the definition of a new entity we need a characterization of its parts ('size', 'colour'), of the way these parts relate to each other, and of the way that the whole interacts with the outside world ('feel', 'sound'). In other words, for an object to exist, we must be able to give a list of relations with the outside world. An object exists if we can interact with it. (Is observation sufficient to determine existence?)

Challenge 309 s

For an abstract concept, such as 'time' or 'superstring', the definition of existence has to be refined only marginally:

▷ *(Physical) existence* is the effectiveness to describe interactions accurately.

This definition applies to trees, time, virtual particles, imaginary numbers, entropy and so on. It is thus pointless to discuss whether a physical concept 'exists' or whether it is 'only' an abstraction used as a tool for descriptions of observations. The two possibilities coincide. The point of dispute can only be whether the description provided by a concept is or is not *precise*.

For mathematical concepts, existence has a somewhat different meaning: a mathematical concept is said to exist if it has no built-in contradictions. This is a much weaker requirement than physical existence. It is thus incorrect to deduce physical existence from mathematical existence. This is a frequent error; from Pythagoras' times onwards it was often stated that since mathematical concepts exist, they must therefore also exist in nature. Historically, this error occurred in the statements that planet orbits 'must' be circles, that planet shapes 'must' be spheres or that physical space 'must' be Euclidean. Today this is still happening with the statements that space and time 'must' be continuous and that nature 'must' be described by sets. In all these cases, the reasoning is wrong. In fact, the continuous attempts to deduce physical existence from mathematical existence hide that the opposite is correct: a short reflection shows that mathematical existence is a special case of physical existence.

Challenge 310 s

We note that there is also a different type of existence, namely *psychological existence*. A concept can be said to exist psychologically if it describes human internal experience. Thus a concept can exist psychologically even if it does not exist physically. It is easy to find examples from the religions or from systems that describe inner experiences. Also myths, legends and comic strips define concepts that only exist psychologically, not physically. In our walk, whenever we talk about existence, we mean physical existence only.

Challenge 311 s

Do things exist?

> Wer Wissenschaft und Kunst besitzt,
> Hat auch Religion;
> Wer jene beiden nicht besitzt,
> Der habe Religion.**
> Johann Wolfgang von Goethe, *Zahme Xenien, IX*

* By the way, a grampus was the old name for what is called an 'orca' today.
** He who possesses science and art, also has religion; he who does not possess the two, better have religion.

Using the above definition of existence, the question becomes either trivial or imprecise. It is trivial in the sense that things necessarily exist if they describe observations, since they were defined that way. But perhaps the questioner meant to ask: Does reality exist independently of the observer?

Using the above, this question can be rephrased: 'Do the things we observe exist independently of observation?' After thousands of years of extensive discussion by professional philosophers, logicians, sophists and amateurs the answer is the same: it is 'Yes', because the world did not change after great-grandmother died. The disappearance of observers does not seem to change the universe. These experimental findings can be corroborated by inserting the definition of 'existence' into the question, which then becomes: 'Do the things we observe interact with other aspects of nature when they do not interact with people?' The answer is evident. Several popular books on quantum mechanics fantasize about the importance of the 'mind' of observers – whatever this term may mean; they provide pretty examples of authors who see themselves as irreplaceable, seemingly having lost the ability to see themselves as part of a larger entity.

Of course there are other opinions about the existence of things. The most famous is that of the unmarried George Berkeley (b. 1685 Kilkennys, d. 1753 Oxford) who rightly understood that thoughts based on observation alone, if spread, would undermine the basis of the religious organization of which he was one of the top managers. To counteract this tendency, in 1710 he published *A Treatise Concerning the Principles of Human Knowledge*, a book denying the existence of the material world. This reactionary book became widely known in like-minded circles (it was a time when few books were written) even though it is based on a fundamentally flawed idea: it assumes that the concept of 'existence' and that of 'world' can be defined independently. (You may be curious to try the feat.)

Challenge 312 e

Berkeley had two aims when he wrote his book. First, he tried to deny the capacity of people to arrive at judgements on nature or on any other matter *from their own experience*. Second, he also tried to deny the *ontological reach* of science, i.e., the conclusions one can draw from experience on the questions about human existence. (Later, a university was not ashamed to use his name.) Even though Berkeley is generally despised nowadays, he actually achieved his main aim: he was the originator of the statement that science and religion do not contradict, but *complement* each other. By religion, Berkeley did not mean either morality or spirituality; every scientist is a friend of both of these. By religion, Berkeley meant that the standard set of beliefs for which he stood is above the deductions of reason. This widely cited statement, itself a belief, is still held dearly by many even to this day. However, when searching for the origin of motion, all beliefs stand in the way, including this one. Carrying beliefs is like carrying oversized baggage: it prevents one from reaching the top of Motion Mountain.

Does the void exist?

> “Teacher: 'What is found between the nucleus and the electrons?'
> Student: 'Nothing, only air.'”

In philosophical discussions '*void*' is usually defined as 'non-existence'. It then becomes a game of words to ask for a yes or no answer to the question 'Does the void exist?' The expression 'the existence of non-existence' is either a contradiction of terms or is at least unclearly defined; the topic would not seem to be of great interest. However, similar questions do appear in physics, and a physicist should be prepared to notice the difference of this from the previous one. Does a vacuum exist? Does empty space exist? Or is the world 'full' everywhere, as the more conservative biologist Aristotle maintained? In the past, people have been killed for giving an answer that was unacceptable to authorities.

It is not obvious, but it is nevertheless important, that the modern physical concepts of 'vacuum' and 'empty space' are not the same as the philosophical concept of 'void'. 'Vacuum' is not defined as 'non-existence'; on the contrary, it is defined as the absence of matter and radiation. Vacuum is an entity with specific observable properties, such as its number of dimensions, its electromagnetic constants, its curvature, its vanishing mass, its interaction with matter through curvature and through its influence on decay, etc. (A table of the properties of a physical vacuum is given on page 133.) Historically, it took a long time to clarify the distinction between a physical vacuum and a philosophical void. People confused the two concepts and debated the existence of the vacuum for more than two thousand years. The first to state that it existed, with the courage to try to look through the logical contradiction at the underlying physical reality, were Leucippus and Democritus, the most daring thinkers of antiquity. Their speculations in turn elicited the reactionary response of Aristotle, who rejected the concept of vacuum. Aristotle and his disciples propagated the belief about nature's *horror of the vacuum*.

The discussion changed completely in the seventeenth century, when the first experimental method to realize a vacuum was devised by Torricelli.* Using mercury in a glass
Challenge 313 s
tube, he produced the first laboratory vacuum. Can you guess how? Arguments against the existence of the vacuum again appeared around 1900, when it was argued that light needed 'aether' for its propagation, using almost the same arguments that had been used two hundred years earlier, but in different words. However, experiments failed to detect any of the supposed properties of this unclearly defined concept. Experiments in the field of general relativity showed that a vacuum can move – though in a completely different way from the way in which the aether was expected to move – that the vacuum can be bent, but it then tends to return to its shape. Then, in the late twentieth century, quantum field theory again argued against the existence of a true vacuum and in favour of a space full of virtual particle–antiparticle pairs, culminating in the discussions around
Vol. VI, page 54
the cosmological constant.

In short, the vacuum exists. The question 'Does the void exist?' is settled conclusively
Vol. VI, page 82
only in the last part of this walk, in a rather surprising way.

> Natura abhorret vacuum.
>
> Antiquity

* Evangelista Torricelli (b. 1608 Faenza, d. 1647 Florence), physicist, pupil and successor to Galileo. The (non-SI) pressure unit 'torr' is named after him.

Is nature infinite?

> “It is certain and evident to our senses, that in the world some things are in motion. Now whatever is moved is moved by another... If that by which it is moved be itself moved, then this also needs to be to be moved by another, and that by another again. But this cannot go on to infinity, because then there would be no first mover and consequently, no other mover, seeing that subsequent movers move only inasmuch as they are moved by the first mover, as the staff moves only because it is moved by the hand. Therefore it is necessary to arrive at a first mover, moved by no other; and this everyone understands to be god.”
>
> Thomas Aquinas (b. *c.* 1225 Aquino, d. 1274 Fossanova) *Summa Theologiae*, I, q. 2.

Most of the modern discussions about set theory centre on ways to defining the term 'set' for various types of infinite collections. For the description of motion this leads to two questions: Is the universe infinite? Is it a set? We begin with the first one. Illuminating the question from various viewpoints, we will quickly discover that it is both simple and imprecise.

Do we need infinite quantities to describe nature? Certainly, in classical and quantum physics we do, e.g. in the case of space-time. Is this necessary? We can say already a few things.

Any set can be finite in one aspect and infinite in another. For example, it is possible to proceed along a finite mathematical distance in an infinite amount of time. It is also possible to travel along any distance whatsoever in a given amount of mathematical time, making infinite speed an option, even if relativity is taken into account, as was explained
Vol. II, page 47
earlier.

Despite the use of infinities, scientists are still limited. We saw above that many types
Page 264
of infinities exist. However, no infinity larger than the cardinality of the real numbers plays a role in physics. No space of functions or phase space in classical physics and no
Ref. 274
Hilbert space in quantum theory has higher cardinality. Despite the ability of mathematicians to define much larger kinds of infinities, the description of nature does not need them. Even the most elaborate descriptions of motion use only the infinity of the real numbers.

But is it possible at all to say *of nature* or of one of its aspects that it is indeed *infi-*
Challenge 314 s
nite? Can such a statement be compatible with observations? No. It is evident that every statement that claims that something in nature is infinite is a belief, and is not backed by observations. We shall patiently eliminate this belief in the following.

The possibility of introducing false infinities make any discussion on whether human-
Ref. 259
ity is near the 'end of science' rather difficult. The amount of knowledge and the time required to discover it are unrelated. Depending on the speed with which one advances through it, the end of science can be near or unreachable. In practice, scientists have thus the power to *make* science infinite or not, e.g. by reducing the speed of progress. As scientists need funding for their work, one can guess the stand that they usually take.

In short, the universe cannot be proven to be infinite. But can it be *finite*? At first sight, this would be the only possibility left. (It is not, as we shall see.) But even though many have tried to describe the universe as finite in all its aspects, no one has yet been successful. In order to understand the problems that they encountered, we continue with the other question mentioned above:

Is the universe a set?

A simple observation leads us to question whether the universe is a set. For 2500 years it has been said that the universe is made of vacuum and particles. This implies that the universe is made of a certain *number* of particles. Perhaps the only person to have taken this conclusion to the limit was the astrophysicist Arthur Eddington (b. 1882 Kendal, d. 1944 Cambridge), who wrote:

Ref. 275

> I believe there are 15,747,724,136,275,002,577,605,653,961,181,555,468,044, 717,914,527,116,709,366,231,425,076,185,631,031,296 protons in the universe and the same number of electrons.

Ref. 276

Eddington was ridiculed over and over again for this statement and for his beliefs that lead up to it. His arguments were indeed based on his personal preferences for certain pet numbers. However, we should not laugh too loudly. In fact, for 2500 years almost all scientists have thought along the same line, the only difference being that they have left the precise number unspecified! In fact, *any other number* put into the above sentence would be equally ridiculous. Avoiding specifying it is just a coward's way of avoiding looking at this foggy aspect of the particle description of nature.

Is there a particle number at all in nature? If you smiled at Eddington's statement, or if you shook your head over it, it may mean that you instinctively believe that nature is not a set. Is this so? Whenever we define the universe as the totality of events, or as the totality of all space-time points and objects, we imply that space-time points can be distinguished, that objects can be distinguished and that both can be distinguished from each other. We thus assume that nature is separable and a set. But is this correct? The question is important. The ability to distinguish space-time points and particles from each other is often called *locality*. Thus the universe is separable or is a set if and only if our description of it is local.* And in everyday life, locality is observed without exception.

In daily life we also observe that nature is separable and a whole at the same time. It is a 'many that can be thought as one': in daily life nature is a set. Indeed, the basic characteristic of nature is its diversity. In the world around us we observe changes and differences; we observe that nature is separable. Furthermore, all aspects of nature belong together: there are relations between these aspects, often called 'laws,' stating that the different aspects of nature form a whole, usually called the universe.

In other words, the possibility of describing observations with the help of 'laws' follows from our experience of the separability of nature. The more precisely the separability is specified, the more precisely the 'laws' can be formulated. Indeed, if nature were not

* In quantum mechanics also other, more detailed definitions of locality are used. We will mention them in the quantum part of this text. The issue mentioned here is a different, more fundamental one, and not connected with that of quantum theory.

Vol. IV, page 149

separable or were not a unity, we could not explain why stones fall downwards. Thus we are led to speculate that we should be able to deduce all 'laws' from the observation that nature is separable.

In addition, only the separability allows us to describe nature at all. A description is a classification, that is, a mapping between certain aspects of nature and certain concepts. All concepts are sets and relations. Since the universe is separable, it can be described with the help of sets and relations. Both are separable entities with distinguishable parts. A precise description is commonly called an understanding. In short, the universe is comprehensible only because it is separable.

Moreover, only the separability of the universe makes our brain such a good instrument. The brain is built from a large number of connected components, and only the brain's separability allows it to function. In other words, thinking is only possible because nature is separable.

Finally, only the separability of the universe allows us to distinguish reference frames, and thus to define all symmetries at the basis of physical descriptions. And in the same way that separability is thus necessary for *covariant* descriptions, the unity of nature is necessary for *invariant* descriptions. In other words, the so-called 'laws' of nature are based on the experience that nature is both separable and unifiable – that it is a set.

These arguments seem overwhelmingly to prove that the universe is a set. However, these arguments apply only to everyday experience, everyday dimensions and everyday energies. Is nature a set also *outside* the domains of daily life? Are objects different at all energies, even when they are looked at with the highest precision possible? We have three open issues left: the issue of the number of particles in the universe; the circular definition of space, time and matter; and the issue as to whether describing nature as made of particles and void is an overdescription, an underdescription, or neither. These three issues make us doubt whether objects are countable at all energies. We will discover
Vol. VI, page 101
in the final part of our mountain ascent that indeed, objects in nature cannot be counted at high energy. The consequences will be extensive and fascinating. As an example, try to answer the following: if the universe is not a set, what does that mean for space and
Challenge 315 s
time?

Does the universe exist?

> Each progressive spirit is opposed by a thousand men appointed to guard the past.
>
> Maurice Maeterlink

Following the definition above, existence of a concept means its usefulness to describe interactions. There are two common definitions of the concept of 'universe'. The first is the totality of all matter, energy and space-time. But this usage results in a strange consequence: since nothing can interact with this totality, we cannot claim that the universe exists.

So let us take the more restricted view, namely that the universe is only the totality of all matter and energy. But also in this case it is impossible to interact with the universe.
Challenge 316 s
Can you give a few arguments to support this?

In short, we arrive at the conclusion that the universe does not exist. We will indeed
Vol. VI, page 106
confirm this result in more detail later on in our walk. In particular, since the universe

does not exist, it does not make sense to even try to answer *why* it exists. The best answer
Ref. 215 might be: because of furiously sleeping, colourless green ideas.

What is creation?

> (Gigni) De nihilo nihilum, in nihilum nil posse reverti.*
>
> Persius, *Satira*, III, v. 83-84.

Ref. 277

> Anaxagoras, discovering the ancient theory that nothing comes from nothing, decided to abolish the concept of creation and introduced in its place that of discrimination; he did not hesitate to state, in effect, that all things are mixed to the others and that discrimination produces their growth.
>
> *Anonymous fragment*, Middle Ages.

Ref. 278

The term 'creation' is often heard when talking about nature. It is used in various contexts with different meanings.

One speaks of 'creation as the characterization of human actions, such as observed in an artist painting or a secretary typing. Obviously, this is one type of change. In the classification of change introduced at the beginning of our walk, the changes cited are movements of objects, such as the electrons in the brain, the molecules in the muscles, the material of the paint, or the electrons inside the computer. This type of creation is thus a special case of motion.

One also speaks of creation in the biological or social sense, such as in 'the creation of life', or 'creation of a business', or 'the creation of civilization'. These events are forms of growth or of self-organization; again, they are special cases of motion.

Physicists often say that a lamp 'creates' light or that a stone falling into a pond 'creates' water ripples. Similarly, they talk of 'pair creation' of matter and antimatter. It was one of
Vol. IV, page 188 the important discoveries of physics that all these processes are special types of motion,
Vol. V, page 108 namely excitation of fields.

In popular writing on cosmology, 'creation' is also a term commonly applied, or better misapplied, to the *big bang*. However, the expansion of the universe is a pure example of motion, and contrary to a frequent misunderstanding, the description of the big bang contains only processes that fall into one of the previous three categories, as shown
Vol. II, page 240 in the relevant chapter in general relativity. The big bang is not an example of creation. Quantum cosmology provides more reasons to show why the naive term 'creation' is not
Vol. II, page 239 applicable to the big bang. First, it turns out that the big bang was not an event. Second, it was not a beginning. Third, it did not provide a *choice* from a large set of possibilities. The big bang does not have any properties attributed to the term 'creation'.

In summary, we conclude that in all cases, *creation is a type of motion*. (The same applies to the notions of 'disappearance' and 'annihilation'.) No other type of creation is observed in nature. In particular, the naive sense of 'creation', namely 'appearance from nothing' – *ex nihilo* in Latin – is never observed in nature. All observed types of 'creation' require space, time, forces, energy and matter for their realization. Creation re-

* Nothing (can appear) from nothing, nothing can disappear into nothing.

quires something to exist already, in order to take place. In addition, precise exploration shows that no physical process and no example of motion has a beginning. Our walk will show us that nature does not allow us to pinpoint beginnings. This property alone is sufficient to show that 'creation' is not a concept applicable to what happens in nature. Worse still, creation is applied only to physical systems; we will discover that nature is not a system and worse, that systems do not exist at all.

The opposite of creation is *conservation*. The central statements of physics are conservation theorems: for energy, mass, linear momentum, angular momentum, charge, etc. In fact, every conservation 'law' is a detailed and accurate rejection of the concept of creation. The ancient Greek idea of atoms already contains this rejection. Atomists stated that there is no creation and no disappearance, but only motion of atoms. Every transformation of matter is a motion of atoms. In other words, the idea of the atom was a direct consequence of the negation of creation. It took humanity over 2000 years before
Vol. I, page 311 it stopped locking people in jail for talking about atoms, as had happened to Galileo.

However, there is one exception in which the naive concept of creation does apply: it describes what magicians do on stage. When a magician makes a rabbit appear from nowhere, we indeed experience 'creation' from nothing. At its best such magic is a form of entertainment, at its worst, a misuse of gullibility. The idea that the universe results from either of these two does not seem appealing; on second thought though, maybe looking at the universe as the ultimate entertainment could open up a fresh and more productive approach to life.

Voltaire (b. 1694 Paris, d. 1778 Paris) popularized an argument against creation often used in the past: we do not know whether creation has taken place or not. Today the situation is different: we *do* know that it has *not* taken place, because creation is a type of motion and, as we will see in the concluding part of our mountain ascent, motion did not exist near the big bang.

Have you ever heard the expression 'creation of the laws of nature'? It is one of the most common examples of disinformation. First of all, this expression confuses the 'laws' with nature itself. A description is not the same as the thing itself; everybody knows that giving their beloved a description of a rose is different from giving an actual rose. Second, the expression implies that nature is the way it is because it is somehow 'forced' to follow the 'laws' – a rather childish and, what is more, incorrect view. And third, the expression assumes that it is possible to 'create' descriptions of nature. But a 'law' is a description, and a description by definition cannot be created: so the expression makes no sense at all. The expression 'creation of the laws of nature' is the epitome of confused thinking.

It may well be that calling a great artist 'creative' or 'divine', as was common during the Renaissance, is not blasphemy, but simply an encouragement to the gods to try to do as well. In fact, whenever one uses the term 'creation' to mean anything other than some form of motion, one is discarding both observations and human reason. It is one of the last pseudo-concepts of our modern time; no expert on motion should forget this. It is impossible to escalate Motion Mountain without getting rid of 'creation'. This is not
Vol. V, page 312 easy. We will encounter the next attempt to bring back creation in the study of quantum theory.

> “Every act of creation is first of all an act of destruction.”
>
> Pablo Picasso

Is nature designed?

> In the beginning the universe was created. This has made a lot of people very angry and has been widely regarded as a bad move.
>
> Douglas Adams, *The Restaurant at the End of the Universe.*

The tendency to infer the intentional creation of an object from its simple existence is widespread. Some people jump to this conclusion every time they see a beautiful landscape. This habit stems from the triple prejudice that a beautiful scene implies a complex description, in turn implying complex building instructions, and therefore pointing to an underlying *design*.

This chain of thought contains several mistakes. First, in general, beauty is not a consequence of complexity. Usually it is the opposite: indeed, the study of chaos and of self-organization demonstrates how beautifully complex shapes and patterns can be generated with extremely simple descriptions. Ref. 279 Vol. I, page 385

True, for most human artefacts, complex descriptions indeed imply complex building processes; a personal computer is a good example of a complex object with a complex production process. But in nature, this connection does not apply. We have seen above that even the amount of information needed to construct a human body is about a million times smaller than the information stored in the brain alone. Similar results have been found for plant architecture and for many other examples of patterns in nature. The simple descriptions behind the apparent complexities of nature have been and are still being uncovered by the study of self-organization, chaos, turbulence and fractal shapes. In nature, complex structures derive from *simple* processes. Beware of anyone who says that nature has 'infinite' or 'high complexity': first of all, complexity is not a measurable entity, despite many attempts to quantify it. In addition, all known complex system can be described by (relatively) few parameters and simple equations. Finally, nothing in nature is infinite.

The second mistake in the argument for design is to link a description with an 'instruction', and maybe even to imagine that some unknown 'intelligence' is somehow pulling the strings of the world's stage. The study of nature has consistently shown that there is no hidden intelligence and no instruction behind the processes of nature. An instruction is a list of orders to an executioner. But there are no orders in nature, and no executioners. There are no 'laws' of nature, only descriptions of processes. Nobody is building a tree; the tree is an outcome of the motion of molecules making it up. The genes in the tree do contain information; but no molecule is given any instructions. What seem to be instructions to us are just natural movements of molecules and energy, described by the same patterns taking place in non-living systems. The whole idea of instruction – like that of 'law' of nature – is an ideology, born from an analogy with monarchy or even tyranny, and a typical anthropomorphism.

The third mistake in the argument for design is the suggestion that a complex description for a system implies an underlying design. This is not correct. A complex description only implies that the system has a long evolution behind it. The correct deduction is: something of large complexity exists; therefore it has *grown*, i.e., it has been transformed through input of (moderate) energy over time. This deduction applies to flowers, moun-

tains, stars, life, people, watches, books, personal computers and works of art; in fact it applies to all objects in the universe. The complexity of our environment thus points out the considerable age of our environment and reminds us of the shortness of our own life.

The lack of basic complexity and the lack of instructions in nature confirm a simple result: there is not a single observation in nature that implies or requires design or creation. On the other hand, the variety and intensity of nature's phenomena fills us with deep awe. The wild beauty of nature shows us how *small* a part of nature we actually are, both in space and in time.* We shall explore this experience in detail. We shall find that remaining open to nature's phenomena in all their overwhelming intensity is central to the rest of our adventure.

Vol. V, page 311

> There is a separation between state and church, but not yet between state and science.
>
> Paul Feyerabend

What is a description?

> In theory, there is no difference between theory and practice. In practice, there is.

Following standard vocabulary usage, a description of an observation is a list of the details. The above example of the grampus showed this clearly. In other words, a description of an observation is the act of categorizing it, i.e., of comparing, by identifying or distinguishing, the observation with all the other observations already made.

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▷ A description is a classification.

In short, *to describe means to see as an element of a larger set*.

A description can be compared to the 'you are here' sign on a city tourist map. Out of a set of possible positions, the 'you are here' sign gives the actual one. Similarly, a description highlights the given situation in comparison with all other possibilities. For example, the formula $a = GM/r^2$ is a description of the observations relating motion to gravity, because it classifies the observed accelerations a according to distance to the central body r and to its mass M; indeed such a description sees each specific case as an example of a general pattern. The habit of generalizing is one reason for the often disturbing dismissiveness of scientists: when they observe something, their professional training usually makes them classify it as a special case of a known phenomenon and thus keeps them from being surprised or from being exited about it.

A description is thus the opposite of a *metaphor*; the latter is an analogy relating an observation with another *special* case; a description relates an observation with a *general* case, such as a physical theory.

* The search for a 'sense' in life or in nature is a complicated (and necessary) way to try to face the smallness of human existence.

> Felix qui potuit rerum cognoscere causas,
> atque metus omnis et inexorabile fatum
> subjecit pedibus strepitumque acherontis avari.
> Vergilius*

Reason, purpose and explanation

> Der ganzen modernen Weltanschauung liegt
> die Täuschung zugrunde, daß die sogenannten
> Naturgesetze die Erklärungen der
> Naturerscheinungen seien.**
> Ludwig Wittgenstein, *Tractatus*, 6.371

Compare the following two types of questions and answers:

1. Why are the leaves of most trees green? Because they absorb red and blue light. Why do they absorb those colours? Because they contain chlorophyll. Why is chlorophyll green? Because all chlorophyll types contain magnesium between four pyrrole groups, and this chemical combination gives the green colour, as a result of its quantum mechanical energy levels. Why do plants contain chlorophyll? Because this is what land plants can synthesize. Why only this? Because all land plants originally evolved from the green algae, who are only able to synthesize this compound, and not the compounds found in the blue or in the red algae, which are also found in the sea.
2. Why do children climb trees, and why do some people climb mountains? Because of the sensations they experience during their activity: the feelings of achievement, the symbolic act to go upwards, the wish to get a wider view of the world are part of this type of adventure.

The two types of 'why'-questions show the general difference between reasons and purposes (although the details of these two terms are not defined in the same way by everybody). A *purpose* or *intention* is a classification applied to the actions of humans or animals; strictly speaking, it specifies the quest for a feeling, namely for achieving some type of satisfaction after completion of the action. On the other hand, a *reason* is a specific relation of a fact with the rest of the universe, usually its past. What we call a reason always rests outside the observation itself, whereas a purpose is always internal to it.

Reasons and purposes are the two possibilities of explanations, i.e., the two possible answers to questions starting with 'why'. Usually, physics is not concerned with purpose or with people's feeling, mainly because its original aim, to talk about motion with precision, does not seem to be achievable in this domain. Therefore, *physical* explanations of facts are never purposes, but are always reasons. A *physical explanation* of an observation
Ref. 280
is always the description of its relation with the rest of nature.***

* 'Happy he who can know the causes of things and who, free of all fears, can lay the inexorable fate and the noise of Acheron to his feet.' *Georgica*, book II, verses 490 ss.) Publius Vergilius Maro (b. 70 Mantua, d. 19 bce Brindisi), the great Roman poet, is author of the Aeneid. Acheron was the river crossed by those who had just died and were on their way to the Hades.

** 'The whole modern conception of the world is founded on the illusion that the so-called laws of nature are the explanations of natural phenomena.'

*** It is important to note that purposes are *not* put aside because they pertain to the future, but because they are inadmissible anthropomorphisms. In fact, for deterministic systems, we can equally say that the future is actually a *reason* for the present and the past, a fact often forgotten.

This means that – contrary to common opinion – a question starting with 'why' is accessible to physical investigation as long as it asks for a reason and not for a purpose. In particular, questions such as 'why do stones fall downwards and not upwards?' or 'why do electrons have that value of mass, and why do they have mass at all?' or 'why does space have three dimensions and not thirty-six?' can be answered, as these ask for the connection between specific observations and more general ones. Of course, not all demands for explanation have been answered yet, and there are still problems to be solved. Our present trail only leads from a few answers to some of the more fundamental questions about motion.

The most general quest for an explanation derives from the question: why is the universe the way it is? The topic is covered in our mountain ascent using the two usual approaches, namely:

Unification and demarcation

> "Tout sujet est un ; et, quelque vaste qu'il soit, il peut être renfermé dans un seul discours.*"
> Buffon, *Discours sur le style*.

Studying the properties of motion, constantly paying attention to increase the accuracy of description, we find that explanations are generally of two types:**

1. 'It is like all such cases; also this one is described by ...' The situation is recognized as a *special case* of a general behaviour.
2. 'If the situation were different, we would have a conclusion in contrast with observations.' The situation is recognized as the *only possible case*.***

In other words, the first procedure to find explanations is to formulate patterns, rules or 'laws' that describe larger and larger numbers of observations, and compare the observation with them. This endeavour is called the *unification* of physics – by those who like it; those who don't like it, call it 'reductionism'. For example, the same rule describes the flight of a tennis ball, the motion of the tides at the sea shore, the timing of ice ages, and the time at which the planet Venus ceases to be the evening star and starts to be the morning star. These processes are all consequences of universal gravitation. Similarly, it is not evident that the same rule describes the origin of the colour of the eyes, the formation of lightning, the digestion of food and the working of the brain. These processes are described by quantum electrodynamics.

Unification has its most impressive successes when it predicts an observation that has not been made before. A famous example is the existence of antimatter, predicted

* Every subject is one and, however vast it is, it can be comprised in a single discourse.

Challenge 317 s ** Are these the only possible ones?

*** These two cases have not to be confused with similar sentences that *seem* to be explanations, but that aren't:

— 'It is like the case of ...' A similarity with another *single* case is *not* an explanation.
— 'If it were different, it would contradict the idea that ...' A contradiction with an *idea* or with a theory is *not* an explanation.

by Dirac when he investigated the solutions of an equation that describes the precise behaviour of common matter.

The second procedure in the search for explanations is to eliminate all other imaginable alternatives in favour of the actually correct one. This endeavour has no commonly accepted name: it could be called the *demarcation* of the 'laws' of physics – by those who like it; others call it 'anthropocentrism', or simply 'arrogance'.

When we discover that light travels in such a way that it takes the shortest possible time to its destination, when we describe motion by a principle of least action, or when we discover that trees are branched in such a way that they achieve the largest effect with the smallest effort, we are using a demarcation viewpoint.

In summary, unification, answering 'why' questions, and demarcation, answering 'why not' questions, are typical for the progress throughout the history of physics. We can say that the dual aspects of unification and demarcation form the composing and the opposing traits of physics. They stand for the desire to *know everything*.

However, neither demarcation nor unification can explain the universe as a whole.
Challenge 318 s Can you see why? In fact, apart from unification and demarcation, there is a third possibility that merges the two and allows one to say more about the universe. Can you find
Challenge 319 s it? Our walk will automatically lead to it later.

Pigs, apes and the anthropic principle

> Das wichtigste Hilfsmittel des Wissenschaftlers ist der Papierkorb.*
>
> Several authors

The wish to achieve demarcation of the patterns of nature is most interesting when we follow the consequences of different rules of nature until we find them in contradiction with the most striking observation: our own human existence. In this special case the program of demarcation is often called the *anthropic principle* – from the Greek ἄνθρωπος, meaning 'man'.

For example, if the Sun–Earth distance were different from what it is, the resulting temperature change on the Earth would have made impossible the appearance of life, which needs liquid water. Similarly, our brain would not work if the Moon did not circle the Earth. It is also well-known that if there were fewer large planets in the solar system, the evolution of humans would have been impossible. The large planets divert large numbers of comets, preventing them from hitting the Earth. The spectacular collision of comet Shoemaker–Levy-9 with Jupiter, the astronomical event of July 1994, was an example of this diversion of a comet.**

Also the anthropic principle has its most impressive successes when it predicts unknown observations. The most famous example stems from the study of stars. Carbon atoms, like all other atoms except most hydrogen, helium or lithium atoms, are formed in stars through fusion. While studying the mechanisms of fusion in 1953, the well-known astrophysicist Fred Hoyle*** found that carbon nuclei could not be formed from the alpha particles present inside stars at reasonable temperatures, unless they had an excited

* 'The most important instrument of a scientist is the waste paper basket.'
** For a collection of pictures of this event, see e.g. the garbo.uwasa.fi/pc/gifslevy.html website.
*** Fred Hoyle (b. 1915 Bingley, d. 2001 Bournemouth), important astronomer and astrophysicist, was the

state with an increased cross-section. From the fact of our existence, which is based on carbon, Hoyle thus predicted the existence of a previously unknown excited state of the carbon nucleus. And, indeed, the excited state was found a few months later by Willy Fowler.*

Ref. 281

In its *serious* form, the anthropic principle is therefore the quest to deduce the description of nature from the experimental fact of our own existence. In the popular literature, however, the anthropic principle is often changed from a simple experimental method to deduce the patterns of nature, to its *perverted* form, a melting pot of absurd metaphysical ideas in which everybody mixes up their favourite beliefs. Most frequently, the experimental observation of our own existence has been perverted to reintroduce the idea of 'design', i.e., that the universe has been constructed with the aim of producing humans; often it is even suggested that the anthropic principle is an *explanation* – a gross example of disinformation.

Ref. 282

How can we distinguish between the serious and the perverted form? We start with an observation. We would get exactly the same rules and patterns of nature if we used the existence of pigs or monkeys as a starting point. In other words, if we would reach *different* conclusions by using the *porcine principle* or the *simian principle*, we are using the perverted form of the anthropic principle, otherwise we are using the serious form. (The carbon-12 story is thus an example of the serious form.) This test is effective because there is no known pattern or 'law' of nature that is particular to humans but unnecessary for apes or pigs.**

> Er wunderte sich, daß den Katzen genau an den Stellen Löcher in den Pelz geschnitten wären, wo sie Augen hätten.***
>
> Georg Christoph Lichtenberg

Does one need cause and effect in explanations?

> There are in nature neither rewards nor punishments – there are only consequences.
>
> Robert Ingersoll

> The world owes you nothing. It was there first.
>
> Mark Twain

---

first and maybe only physicist who ever made a specific prediction – namely the existence of an excited state of the carbon nucleus – from the simple fact that humans exist. A permanent maverick, he coined the term 'big bang' even though he did not accept the evidence for it, and proposed another model, the 'steady state'. His most important and well-known research was on the formation of atoms inside stars. He also propagated the belief that life was brought to Earth from extraterrestrial microbes.

* William A. Fowler (b. 1911 Pittsbrugh, d. 1995 Pasadena) shared the 1983 Nobel Prize in Physics with Subramanyan Chandrasekhar for this and related discoveries.

** Though apes do not seem to be good physicists, as described in the text by D. J. Povinelli, *Folk Physics for Apes: the Chimpanzee's Theory of How the World Works*, Oxford University Press, 2000.

*** 'He was amazed that cats had holes cut into their fur precisely in those places where they had eyes.' Georg Christoph Lichtenberg (b. 1742 Ober-Ramstadt, d. 1799 Göttingen), physicist and intellectual, professor in Göttingen, still famous today for his extremely numerous and witty aphorisms and satires. Among others of his time, Lichtenberg made fun of all those who maintained that the universe was made exactly to the measure of man, a frequently encountered idea in the foggy world of the anthropic principle.

> No matter how cruel and nasty and evil you may be, every time you take a breath you make a flower happy.
>
> Mort Sahl

Historically, the two terms 'cause' and 'effect' have played an important role in philosophical discussions. In particular, during the birth of modern mechanics, it was important to point out that every effect has a cause, in order to distinguish precise thought from thought based on beliefs, such as 'miracles', 'divine surprises' or 'evolution from nothing'. It was equally essential to stress that effects are different from causes; this distinction avoids pseudo-explanations such as the famous example by Molière where the doctor explains to his patient in elaborate terms that sleeping pills work because they contain a dormitive virtue.

Ref. 283

But in physics, the concepts of cause and effect are not used at all. That miracles do not appear is expressed every time we use symmetries and conservation theorems. The observation that cause and effect differ from each other is inherent in any evolution equation. Moreover, the concepts of cause and effect are not clearly defined; for example, it is especially difficult to define what is meant by one cause as opposed to several of them, and the same for one or several effects. Both terms are impossible to quantify and to measure. In other words, useful as 'cause' and 'effect' may be in personal life for distinction between events that regularly succeed each other, they are not necessary in physics. In physical explanations, they play no special roles.

> Ἀγαθον καὶ ξαξόν · ἕν καὶ ταὐτό.*
>
> Heraclitus

> Wenn ein Arzt hinter dem Sarg seines Patienten geht, so folgt manchmal tatsächlich die Ursache der Wirkung.**
>
> Robert Koch

Is consciousness required?

> Variatio delectat.***
>
> Cicero

A lot of mediocre discussions are going on about consciousness, and we will skip them here. What is consciousness? Most simply and concretely, consciousness means the possession of a small part of oneself that is watching what the rest of oneself is perceiving, feeling, thinking and doing. In short, consciousness is the ability to observe oneself, and in particular one's inner mechanisms and motivations.

Ref. 284

▷ *Consciousness* is the ability of introspection.

* 'Good and bad – one and the same.'
** 'When a doctor walks behind the coffin of his patient, indeed the cause sometimes follows the effect.'
*** 'Change pleases.' Marcus Tullius Cicero (b. 106 Arpinum, d. 43 bce Formiae), important lawyer, orator and politician at the end of the Roman republic.

For this reason, consciousness is *not* a prerequisite for studying motion. Indeed, animals, plants or machines are also able to observe motion. For the same reason, consciousness is not necessary to observe quantum mechanical motion. On the other hand, both the study of motion and that of oneself have a lot in common: the need to observe carefully, to overcome preconceptions, to overcome fear and the fun of doing so.

For the time being, we have put enough emphasis on the precision of concepts. Talking about motion is also something to be deeply enjoyed. Let us see why.

> Precision and clarity obey the indeterminacy relation: their product is constant.
>
> Niels Bohr

Curiosity

> Precision is the child of curiosity.

Like the history of every person, also the history of mankind charts a long struggle to avoid the pitfalls of accepting the statements of authorities as truth, without checking the facts. Indeed, whenever curiosity leads us to formulate a question, there are always two general ways to proceed. One is to check the facts personally, the other is to ask somebody. However, the last way is dangerous: it means to give up a part of oneself. Healthy people, children whose curiosity is still alive, as well as scientists, choose the first way. After all, science is due to adult curiosity.

Curiosity, also called the *exploratory drive*, plays strange games with people. Starting with the original experience of the world as a big 'soup' of interacting parts, curiosity can drive one to find *all* the parts and *all* the interactions. It drives not only people. It has been observed that when rats show curious behaviour, certain brain cells in the hypothalamus get active and secrete hormones that produce positive feelings and emotions. If a rat has the possibility, via some implanted electrodes, to excite these same cells by pressing a Ref. 285 switch, it does so voluntarily: rats get *addicted* to the feelings connected with curiosity. Like rats, humans are curious because they enjoy it. They do so in at least four ways: because they are artists, because they are fond of pleasure, because they are adventurers and because they are dreamers. Let us see how.

Originally, curiosity stems from the desire to interact in a positive way with the environment. Young children provide good examples: curiosity is a natural ingredient of their life, in the same way that it is for other mammals and a few bird species; incidentally, the same taxonomic distribution is found for play behaviour. In short, all animals that play are curious, and vice versa. Curiosity provides the basis for learning, for creativity and thus for every human activity that leaves a legacy, such as art or science. The sculptor and art theoretician Joseph Beuys had as his own guiding principle that *every* Ref. 286 creative act is a form of art. Humans, and especially children, enjoy curiosity because they feel its importance for creativity, and for growth in general.

Curiosity regularly leads one to exclaim: 'Oh!', an experience that leads to the second reason to be curious: relishing feelings of wonder and surprise. Epicurus (Epikuros) (b. 341 Samos, d. 271 BCE Athens) maintained that this experience, θαυμάζειν, is the origin of philosophy. These feelings, which nowadays are variously called religious, spiri-

tual, numinous, etc., are the same as those to which rats can become addicted. Among these feelings, Rudolf Otto has introduced the now classical distinction into the fascinating and the frightening. He named the corresponding experiences 'mysterium fascinans' and 'mysterium tremendum'.* Within these distinctions, physicists, scientists, children and connoisseurs take a clear stand: they choose the fascinans as the starting point for their actions and for their approach to the world. Such feelings of fascination induce some children who look at the night sky to dream about becoming astronomers, some who look through a microscope to become biologists or physicists, and so on. (It could also be that genetics plays a role in this pleasure of novelty seeking.) Ref. 287

Perhaps the most beautiful moments in the study of physics are those appearing after new observations have shaken our previously held thinking habits, have forced us to give up a previously held conviction, and have engendered the feeling of being lost. When, in this moment of crisis, we finally discover a more adequate and precise description of the observations, which provide a better insight into the world, we are struck by a feeling usually called illumination. Anyone who has kept alive the memory and the taste for these magic moments knows that in these situations one is pervaded by a feeling of union between oneself and the world.** The pleasure of these moments, the adventures of the change of thought structures connected with them, and the joy of insight following them provides the drive for many scientists. Little talk and lots of pleasure is their common denominator. In this spirit, the important physicist Victor Weisskopf (b. 1908 Vienna, d. 2002 Newton) liked to say jokingly: 'There are two things that make life worth living: Mozart and quantum mechanics.'

The choice of moving away from the tremendum towards the fascinans stems from an innate desire, most obvious in children, to reduce uncertainty and fear. This drive is the father of all adventures. It has a well-known parallel in ancient Greece, where the first men studying observations, such as Epicurus, stated explicitly that their aim was to free people from unnecessary fear by deepening knowledge and transforming people from frightened passive victims into fascinated, active and responsible beings. Those ancient thinkers started to popularize the idea that, like the common events in our life, the rarer events also follow rules. For example, Epicurus underlined that lightning is a natural phenomenon caused by interactions between clouds, and stressed that it was a natural process, i.e., a process that followed rules, in the same way as the falling of a stone or any other familiar process of everyday life.

Investigating the phenomena around them, philosophers and later scientists succeeded in freeing people from most of their fears caused by uncertainty and a lack of knowledge about nature. This liberation played an important role in the history of human culture and still pervades in the personal history of many scientists. The aim to arrive at stable, rock-bottom truths has inspired (but also hindered) many of them; Al-

* This distinction is the basis of Rudolf Otto, *Das Heilige – Über das Irrationale in der Idee des Göttlichen und sein Verhältnis zum Rationalen*, Beck 1991. This is a new edition of the epoch-making work originally published at the beginning of the twentieth century. Rudolf Otto (b. 1869 Peine, d. 1937 Marburg) was one of the most important theologians of his time.

** Several researchers have studied the situations leading to these magic moments in more detail, notably the physician and physicist Hermann von Helmholtz (b. 1821 Potsdam, d. 1894 Charlottenburg) and the mathematician Henri Poincaré (b. 1854 Nancy, d. 1912 Paris). They distinguish four stages in the conception of an idea at the basis of such a magic moment: saturation, incubation, illumination and verification. Ref. 288

bert Einstein is a well-known example for this, discovering relativity, helping to start up but then denying quantum mechanics.

Interestingly, in the experience and in the development of every human being, curiosity, and therefore the sciences, appears *before* magic and superstition. Magic needs deceit to be effective, and superstition needs indoctrination; curiosity doesn't need either. Conflicts of curiosity with superstitions, ideologies, authorities or the rest of society are thus preprogrammed.

Curiosity is the exploration of limits. For every limit, there are two possibilities: the limit can turn out to be real or apparent. If the limit is real, the most productive attitude is that of acceptance. Approaching the limit then gives strength. If the limit is only apparent and in fact non-existent, the most productive attitude is to re-evaluate the mistaken view, extract the positive role it performed, and then cross the limit. Distinguishing between real and apparent limits is only possible when the limit is investigated with great care, openness and unintentionality. Most of all, exploring limits need courage.

> Das gelüftete Geheimnis rächt sich.*
>
> Bert Hellinger

Courage

> Il est dangereux d'avoir raison dans des choses où des hommes accrédités ont tort.**
>
> Voltaire

> Manche suchen Sicherheit, wo Mut gefragt ist, und suchen Freiheit, wo das Richtige keine Wahl läßt.***
>
> Bert Hellinger

Most of the material in this chapter is necessary in the adventure to get to the top of Motion Mountain. But we need more. Like any enterprise, curiosity also requires courage, and complete curiosity, as aimed for in our quest, requires complete courage. In fact, it is easy to get discouraged on this journey. The quest is often dismissed by others as useless, uninteresting, childish, confusing, damaging, crazy or, above all, evil. For example, between the death of Socrates in 399 BCE and Paul Thierry, Baron d'Holbach, in the eighteenth century, no book was published with the statement 'gods do not exist', because of the threats to the life of anyone who dared to make the point. Even today, this type of attitude still abounds, as the newspapers show. Ref. 291

Through the constant elimination of uncertainty, both curiosity and scientific activity are implicitly opposed to any idea, person or organization that tries to avoid the comparison of statements with observations. These 'avoiders' demand to live with superstitions and beliefs. But superstitions and beliefs produce unnecessary fear. And fear is the basis of all unjust authorities. One gets into a vicious circle: avoiding comparison with observation produces fear – fear keeps unjust authority in place – unjust authority avoids comparison with observation – etc.

* 'The unveiled secret takes revenge.'

** 'It is dangerous to be right in matters where established men are wrong.' Ref. 289

*** 'Some look for security where courage is required and look for freedom where the right way doesn't leave any choice.' Ref. 290

Through the constant drive towards certainty, curiosity and science are fundamentally opposed to unjust authority, a connection that made life difficult for people such as Anaxagoras in ancient Greece, Hypatia in the Christian Roman empire, Galileo Galilei in the former church state, Antoine Lavoisier in revolutionary France and Albert Einstein (and many others) in nazi Germany. In the second half of the twentieth century, victims were Robert Oppenheimer, Melba Phillips and Chandler Davis in the United States, and Andrei Sakharov in the Soviet Union. Each of them tell a horrible but instructive story, as have, more recently, Fang Lizhi, Xu Liangying, Liu Gang and Wang Juntao in China, Kim Song-Man in South Corea, Otanazar Aripov in Uzbekistan, Ramadan al-Hadi al-Hush in Libya, Bo Bo Htun in Burma, Sami Kilani and Salman Salman in Palestine, Abdus Salam in Pakistan, as well as many hundreds of others. In many authoritarian societies the antagonism between curiosity and injustice has hindered or even completely suppressed the development of physics and other sciences, with extremely negative economic, social and cultural consequences.

When embarking on the adventure to understand motion, we need to be conscious of what we are doing. In fact, external obstacles can be avoided or at least largely reduced by keeping the project to oneself. Other difficulties still remain, this time of personal nature. Many have tried to embark on this adventure with some hidden or explicit intention, usually of an ideological nature, and then have got entangled by it before reaching the end. Some have not been prepared to accept the humility required for such an endeavour. Others were not prepared for the openness required, which can shatter deeply held beliefs. Still others were not ready to turn towards the unclear, the dark and the unknown, confronting them at every occasion.

On the other hand, the dangers are worth it. By taking curiosity as a maxim, facing disinformation and fear with all one's courage, one achieves freedom from all beliefs. In exchange, you come to savour the fullest pleasures and the deepest satisfaction that life has to offer.

We thus continue our hike. At this point, the trail towards the top of Motion Mountain is leading us towards the next adventure: discovering the origin of sizes, shapes and colours in nature.

> “And the gods said to man: 'Take what you want, and pay the price.'”
>
> Popular saying

> “It is difficult to make a man miserable while he feels he is worthy of himself.”
>
> Abraham Lincoln

Chapter 10

CLASSICAL PHYSICS IN A NUTSHELL

Classical electrodynamics, with mechanics, thermodynamics and relativity, completes our walk through classical physics. In the structure of physics, classical physics encompasses four of the eight points that make up all of physics, the science of motion. As a whole, classical physics describes the motion of everyday Page 8 bodies, the motion of heat, the motion of extremely fast objects, the motion of empty space, and the motion of light and electric charge. By completing classical physics, we have covered one half of our adventure. Let us summarize what we have found out about motion so far – and what we did not.

What can move?

In nature, four entities can move: *objects*, *radiation*, *space-time* and *horizons*. In all cases, their motion happens in such a way as to minimize change. Change is also called (physical) action. In short, all motion minimizes action.

In all cases of motion, we distinguish the permanent or *intrinsic properties* from the varying *state*. We learned to distinguish and to characterize the possible intrinsic properties and the possible states of each moving entity.

About *objects*, we found that in everyday life, all sufficiently small objects or particles are described completely by their *mass* and their electric *charge*. There is no magnetic charge. Mass and electric charge are thus the only localized intrinsic properties of classical, everyday objects. Both mass and electric charge are defined by the accelerations they produce around them. Both quantities are conserved; thus they can be added (with certain precautions). Mass, in contrast to charge, is always positive. Mass describes the interaction of objects in collisions and in gravitation, charge the interaction with electromagnetic fields.

All *varying* aspects of objects, i.e., their *state*, can be described using *momentum* and *position*, as well as *angular momentum* and *orientation*. These four quantities can vary continuously in amount and direction. Therefore the set of all possible states forms a space, the so-called *phase space*. The state of extended, shape-changing objects is given by the states of all its constituent particles. These particles make up all objects by interacting electromagnetically.

The Lagrangian determines the action, or total change, of any kind of motion. Action, or change, is independent of the observer; the state is not. The states found by different observers are related: the relations are called the 'laws' or properties of motion. For different times they are called *evolution equations*, for different places and orientations they

are called *transformation relations*, and for different gauges they are called *gauge transformations*. Motion of each everyday objects is fully described by the principle of least action: motion minimizes action.

Radiation also moves. Everyday types of radiation, such as light, radio waves and their related forms, are travelling electromagnetic waves. They are described by same equations that describe the interaction of charged or magnetic objects. The speed of massless fields is the maximum possible energy speed in nature and is the same for all observers. The motion of radiation describes the motion of images. The *intrinsic properties* of radiation are its dispersion relation and its energy–angular momentum relation. The *state* of radiation is described by its electromagnetic field strength, its phase, its polarization and its coupling to matter. The motion of the electromagnetic field and of radiation minimizes action and change.

Space-time is also able to move, by changing its curvature. The state of space-time is given by the metric, which describes distances and curvature, and thus the local warpedness. The warpedness can oscillate and propagate, so that empty space can move like a wave. Also the motion of space-time minimizes change. The principle of least action is valid. The intrinsic properties of space-time are the number of dimensions, its metric signature and its topology. Experiments show that the large scale topology of space-time is simple.

Horizons can be seen as limit cases of either space-time or matter-radiation. The share the same intrinsic and state properties. The dark night sky, the boundary of the universe, is the most important example of a horizon. Other examples are the boundaries of black holes. The universe, both its space-time and its matter content, shows maximum age and distance values. The history of the universe is long, about three times as long as the history of the Earth. On large scales, all matter in the universe moves away from all other matter: the universe, and its horizon, is expanding.

Properties of classical motion

Around us, we observe motion for objects, radiation, space-time and horizons. In our exploration of classical physics, we distilled six specific properties of all classical – or everyday – motion.

1. Everyday motion is *continuous*. Continuous motion allows defining space and time. All energy moves in the way space-time dictates it, and space moves the way energy dictates it. This relation describes the motion of the stars, of thrown stones, of light beams and of the tides. Rest and free fall are the same, and gravity is curved space-time. Mass breaks conformal symmetry and thus distinguishes space from time.

   The continuity of motion is somewhat limited: The (local) speed of energy, mass and charge is bound from above by a universal constant c, and (local) energy change per time is bound from above by a universal constant $c^5/4G$. The speed value c is realized for the motion of massless particles. It also relates space to time. The power value $c^5/4G$ is realized by horizons. Horizons are found around black holes and at the border of the universe. The maximum power value also relates space-time curvature to energy flow and thus describes the elasticity of space-time.

   The continuity of motion is limited in a second way: No two objects can be at the

same spot at the same time. This is the first statement that humans encounter about electromagnetism. It is due to the repulsion of charges of the same sign found in matter. More detailed investigation shows that electric charge accelerates other charges, that charge is necessary to define length and time intervals, and that charges are the source of electromagnetic fields. Also light is such a field. Light travels at the maximum possible velocity c. In contrast to objects, light and electromagnetic fields can interpenetrate.
2. Everyday motion *conserves* mass, electric charge, energy, linear momentum and angular momentum. For these quantities, nothing appears out of nothing. Conservation applies to all kinds of motion: to linear motion, to rotational motion, and to motion of matter, radiation, space-time and horizons. Energy and momentum are similar to continuous substances: they are never destroyed, never created, but always redistributed. Not even heat, growth, transformations, biological evolution or friction are exceptions conservation.
3. Everyday motion is *relative*: motion depends on the observer. Not even the firm floor below our feet contradicts relativity.
4. Everyday motion is *reversible*: everyday motion can occur backwards. Not even friction, the breaking of objects or death are exceptions to reversibility.
5. Everyday motion is *mirror-invariant*: everyday motion can occur in a mirror-reversed way. In short, we found that the classical motion of objects, radiation and space-time is right–left symmetric. Human-made objects, such as writing, are no exceptions to mirror-invariance.
6. Everyday motion is *lazy*: motion happens in a way that minimizes change, i.e., physical action. In Galilean physics and electrodynamics, action is the time average of the difference between kinetic and potential energy. In general relativity, action takes into account the curvature and elasticity of space-time. The principle of least action – or cosmic laziness – hold for all cases.

In short, our exploration of classical physics showed us that motion is *predictable* and *limited*: nature follows patterns and rules and there are *no* surprises in nature. Nature cannot do what it likes to do.

We will discover later that some rare examples of non-everyday motion violate reversibility and mirror-invariance in a subtle way. The subtle violations disappear if the terms are properly extended in their meaning. Also mass conservation is violated separately, but becomes, in relativity, part of energy conservation. In short, the general statements about motion, suitably corrected, remain valid across the nature.

Above all, we saw that *motion minimizes action*. This deep result remains valid throughout our adventure. In other terms, the universe has no freedom to determine what occurs inside it.

After completing the classical parts of the adventure, you might think that you know classical physics well. If you do so, read the excellent collection by Friedrich Herrmann, *Historical Burdens on Physics*, available for free download, at www.physikdidaktik.uni-karlsruhe.de/index_en.html. If the topics presented there – all simple to understand – are clear to you, you have become a real expert on classical physics.

Challenge 320 e

The future of planet Earth

Maybe nature shows no surprises, but it still provides many adventures. On the 2nd of March 2009, a small asteroid 'almost' hit the Earth. It passed at a distance of only 63 500 km from our planet. On impact, it would have destroyed a region the size of London. Such events occur regularly.* Several other adventures can be predicted by classical physics; they are listed in Table 25. Several items are problems facing humanity in the distant future, but some, such as volcanic eruptions or asteroid impacts, could happen at any time. All are research topics.

Ref. 292

TABLE 25 Examples of disastrous motion of possible future importance.

Critical situation	Years from now
Giant tsunami from volcanic eruption at Canary islands	*c.* 10-200
End of fundamental physics, with a definite proof that nature is simple	*c.* 20 (around year 2030)
Major nuclear material accident or weapon use	unknown
Explosion of volcano in Greenland, Italy or elsewhere, leading to long darkening of sky	unknown
Explosion of Yellowstone or other giant volcano leading to year-long volcanic winter	0 to 100 000
Earth's mantle instability leading to massive volcanic activity	unknown
Mini ice age due to collapse of gulf stream	unknown
Ozone shield reduction	*c.* 100
Rising ocean levels due to greenhouse warming	> 100
End of applied physics	> 200
Several magnetic north and south poles appear, allowing solar storms to disturb radio and telecommunications, to interrupt electricity supplies, to increase animal mutations and to disorient migrating animals such as wales, birds and tortoises	*c.* 800
Our interstellar gas cloud detaches from the solar systems, changing the size of the heliosphere, and thus expose us more to aurorae and solar magnetic fields	*c.* 3 000
Reversal of Earth's magnetic field, implying a time with almost no magnetic field, with increased cosmic radiation levels and thus more skin cancers and miscarriages	unknown
Atmospheric oxygen depletion due to forest reduction and exaggerated fuel consumption	> 1000
Upcoming ice age	*c.* 15 000
Possible collision with interstellar gas cloud assumed to be crossed by the Earth every 60 million years, maybe causing mass extinctions	*c.* 50 000
Possible genetic degeneration of homo sapiens due to Y chromosome reduction	*c.* 200 000

* The web pages around cfa-www.harvard.edu/iau/lists/Closest.html provide more information on such events.

TABLE 25 (Continued) Examples of disastrous motion of possible future importance.

Critical situation	Years from now
Africa collides with Europe, transforming the Mediterranean into a lake that starts evaporating	around $3 \cdot 10^6$
Gamma-ray burst from within our own galaxy, causing radiation damage to many living beings	between 0 and $5 \cdot 10^6$
Asteroid hitting the Earth, generating tsunamis, storms, darkening sunlight, etc.	between 0 and $50 \cdot 10^6$
Neighbouring star approaching, starting comet shower through destabilization of Oort cloud and thus risk for life on Earth	$> 10^6$
American continent collides with Asia	$> 100 \cdot 10^6$
Molecular cloud engulfs the solar system	unknown
Instability of solar system	$> 100 \cdot 10^6$
Low atmospheric CO_2 content stops photosynthesis	$> 100 \cdot 10^6$
Collision of Milky Way with star cluster or other galaxy	$> 150 \cdot 10^6$
Sun ages and gets hotter, evaporating seas	$> 250 \cdot 10^6$
Ocean level increase due to Earth rotation slowing/stopping (if not evaporated before)	$> 10^9$
Temperature rise/fall (depending on location) due to Earth rotation stop	$> 10^9$
Sun runs out of fuel, becomes red giant, engulfs Earth	$5.0 \cdot 10^9$
Sun stops burning, becomes white dwarf	$5.2 \cdot 10^9$
Earth core solidifies, removing magnetic field and thus Earth's cosmic radiation shield	$10.0 \cdot 10^9$
Nearby nova (e.g. Betelgeuse) bathes Earth in annihilation radiation	unknown
Nearby supernova (e.g. Eta Carinae) blasts over solar system	unknown
Galaxy centre destabilizes rest of galaxy	unknown
Universe recollapses – if ever (see page 132, volume II)	$> 20 \cdot 10^9$
Matter decays into radiation – if ever (see Appendix B in vol. V)	$> 10^{33}$
Problems with naked singularities	only in science fiction
Vacuum becomes unstable	only in science fiction

Despite the fascination of the predictions (all made in the year 2000), we leave aside these literally tremendous issues and continue on our adventure.

> “I'm an old man and I've known many troubles. Most of them never happened.”
>
> Anonymous wisdom

The essence of classical physics – the infinitely small and the lack of surprises

We can summarize classical physics with two simple statements: First, *classical physics is the description of motion using the concept of the infinitely small*. Secondly, *nature lacks*

surprises.

All concepts used so far, be they for motion, space, time or observables, assume that the infinitely small exists. Special relativity, despite the speed limit, still allows infinitely small velocities; general relativity, despite its black hole limit, still allows infinitely small force and power values. Similarly, in the description of electrodynamics and gravitation, both integrals and derivatives are abbreviations of mathematical processes that use and assume infinitely small distances and time intervals. In other words, the classical description of nature introduces and is based on the infinitely small in the description of motion.

Using the infinitely small as a research tool, the classical description of motion discovers that energy, momentum, angular momentum and electric charge are conserved. They are conserved also for infinitely small dimensions or time intervals. In other words, there are no surprises in motion.

The detailed study of conservation has lead us to a strong conclusion: the infinitely small shows us that *motion is deterministic*. The existence of real surprises would contradict determinism.

The lack of surprises implies the lack miracles. Indeed, some people argue that infinity is the necessary ingredient to perform miracles. Classical physics shows that this is not the case. Conservation and the lack of surprises also imply that motion and nature are not described by concepts such as 'punishment' or 'reward' or 'fairness'. This is also the case for disasters and catastrophes. Conservation and the lack of surprises also imply that motion and nature are not designed and have no aim. Some people claim the opposite.; they are mistaken.

Classical physics is the absence of surprises. As reassuring as this result may be, it leaves us with a doubt. Both special and general relativity have eliminated the existence of the infinitely large. There is no infinitely large force, power, size, age or speed. Why should the infinitely small exist, but the infinitely large not? In fact, there are still more open questions about motion.

Summary: Why have we not yet reached the top of the mountain?

> “The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote... Our future discoveries must be looked for in the sixth place of decimals.”
>
> Albert Michelson, 1894.*

We might think that we know nature now, as did Albert Michelson at the end of the nineteenth century. He claimed that electrodynamics and Galilean physics implied that the major laws of physics were well known. The statement is often quoted as an example of flawed predictions, since it reflects an incredible mental closure to the world around him. The origin of every size, shape and colour – from the atoms to humans and up to the universe – was unknown when Michelson, who later even earned the Nobel Prize in

* From his address at the dedication ceremony for the Ryerson Physical Laboratory at the University of Chicago.

physics, gave his speech. Indeed, not only was general relativity still unknown; above all, *quantum theory* still needed to be discovered.

Many physicists in Michelson's time knew that important changes in the description of nature were necessary. Michelson had overlooked three contradiction between electrodynamics and nature for which he had no excuse. First of all, we found above that clocks and metre bars are necessarily made of matter and necessarily based on electromagnetism. But as we saw, classical electrodynamics does not explain the stability and properties of matter and atoms. Matter is made of small particles, but the relation between these particles, electricity and the smallest charges is not clear. If we do not understand matter, we do not yet understand space and time, since we defined space and time using measurement devices made of matter.

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Secondly, Michelson knew that the origin of no colour observed in nature is described by classical electrodynamics. Classical electrodynamics can only explain colour differences and colour changes, but it cannot describe absolute colour values.

Worse, Michelson overlooked a third aspect: the classical description of nature does not allow us to understand *life*. The abilities of living beings – growing, seeing, hearing, feeling, thinking, being healthy or sick, reproducing and dying – are all unexplained by classical physics. In fact, all these abilities *contradict* classical physics.

At the end of the nineteenth century, the progress in technology due to the use of electricity, chemistry and vacuum technology allowed better and better machines and apparatuses to be built. All were built with classical physics in mind. In the years between 1890 and 1920, these classical machines completely destroyed the foundations of classical physics. Experiments with these apparatuses showed that matter is made of atoms of finite and constant size, that electrical charge comes in smallest amounts, that there is a smallest entropy value, a smallest angular momentum value and a smallest action value in nature, and that particles and light behave randomly. In short, precise experiments show that in nature, the existence of the infinitely small is wrong in many cases: many observables come in *quanta*. Like an old empire, the reign of classical physics collapsed. Classical physics does *not* describe nature correctly at small scales.

In summary, understanding light, matter and its interactions, including life itself, is the aim of the upcoming parts of our ascent of Motion Mountain. And to understand life we need to understand the size, shape, colour and material properties of all things. And this understanding takes place at small scales. More specifically, in order to understand light, matter and life, we need to study *particles*. A lot is still left to explore. And this exploration will lead us from wonder to wonder.

Appendix A

UNITS, MEASUREMENTS AND CONSTANTS

Measurements are comparisons with standards. Standards are based on *units*. Many different systems of units have been used throughout the world. Most of these standards confer power to the organization in charge of them. Such power can be misused; this is the case today, for example in the computer industry, and was so in the distant past. The solution is the same in both cases: organize an independent and global standard. For measurement units, this happened in the eighteenth century: in order to avoid misuse by authoritarian institutions, to eliminate problems with differing, changing and irreproducible standards, and – this is not a joke – to simplify tax collection and to make it more just, a group of scientists, politicians and economists agreed on a set of units. It is called the *Système International d'Unités*, abbreviated *SI*, and is defined by an international treaty, the 'Convention du Mètre'. The units are maintained by an international organization, the 'Conférence Générale des Poids et Mesures', and its daughter organizations, the 'Commission Internationale des Poids et Mesures' and the 'Bureau International des Poids et Mesures' (BIPM). All originated in the times just before the French revolution. Ref. 293

SI units

All SI units are built from seven *base units*, whose official definitions, translated from French into English, are given below, together with the dates of their formulation:

- 'The *second* is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.' (1967)*
- 'The *metre* is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.' (1983)*
- 'The *kilogram* is the unit of mass; it is equal to the mass of the international prototype of the kilogram.' (1901)*
- 'The *ampere* is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to $2 \cdot 10^{-7}$ newton per metre of length.' (1948)*
- 'The *kelvin*, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.' (1967)*
- 'The *mole* is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12.' (1971)*

▪ 'The *candela* is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \cdot 10^{12}$ hertz and has a radiant intensity in that direction of (1/683) watt per steradian.' (1979)*

In the near future, it is planned to change the definition of the SI units by basing them on the cube diagram of page 8, as explained on http://www.bipm.org/en/si/new_si/.

We note that both time and length units are defined as certain properties of a standard example of motion, namely light. In other words, also the Conférence Générale des Poids et Mesures makes the point that the observation of motion is a *prerequisite* for the definition and construction of time and space. *Motion is the fundament of every observation and of all measurement.* By the way, the use of light in the definitions had been proposed already in 1827 by Jacques Babinet.**

From these basic units, all other units are defined by multiplication and division. Thus, all SI units have the following properties:

▪ SI units form a system with *state-of-the-art precision*: all units are defined with a precision that is higher than the precision of commonly used measurements. Moreover, the precision of the definitions is regularly being improved. The present relative uncertainty of the definition of the second is around 10^{-14}, for the metre about 10^{-10}, for the kilogram about 10^{-9}, for the ampere 10^{-7}, for the mole less than 10^{-6}, for the kelvin 10^{-6} and for the candela 10^{-3}.

▪ SI units form an *absolute* system: all units are defined in such a way that they can be reproduced in every suitably equipped laboratory, independently, and with high precision. This avoids as much as possible any misuse by the standard-setting organization. (The kilogram, still defined with the help of an artefact, is the last exception to this requirement; extensive research is under way to eliminate this artefact from the definition – an international race that will take a few more years. There are two approaches: counting particles, or fixing $\hbar$. The former can be achieved in crystals, e.g., crystals made of pure silicon, the latter using any formula where $\hbar$ appears, such as the formula for the de Broglie wavelength or that of the Josephson effect.)

▪ SI units form a *practical* system: the base units are quantities of everyday magnitude. Frequently used units have standard names and abbreviations. The complete list includes the seven base units just given, the supplementary units, the derived units and the admitted units.

The *supplementary* SI units are two: the unit for (plane) angle, defined as the ratio of arc length to radius, is the *radian* (rad). For solid angle, defined as the ratio of the subtended area to the square of the radius, the unit is the *steradian* (sr).

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* The respective symbols are s, m, kg, A, K, mol and cd. The international prototype of the kilogram is a platinum–iridium cylinder kept at the BIPM in Sèvres, in France. For more details on the levels of the caesium atom, consult a book on atomic physics. The Celsius scale of temperature θ is defined as: $\theta/°\mathrm{C} = T/\mathrm{K} - 273.15$; note the small difference with the number appearing in the definition of the kelvin. SI also states: 'When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.' In the definition of the mole, it is understood that the carbon 12 atoms are unbound, at rest and in their ground state. In the definition of the candela, the frequency of the light corresponds to 555.5 nm, i.e., green colour, around the wavelength to which the eye is most sensitive.

** Jacques Babinet (b. 1794 Lusignan, d. 1874 Paris) was a physicist who published important work in optics.

The *derived* units with special names, in their official English spelling, i.e., without capital letters and accents, are:

Name	Abbreviation	Name	Abbreviation
hertz	$\mathrm{Hz} = 1/\mathrm{s}$	newton	$\mathrm{N} = \mathrm{kg\,m/s^2}$
pascal	$\mathrm{Pa} = \mathrm{N/m^2} = \mathrm{kg/m\,s^2}$	joule	$\mathrm{J} = \mathrm{Nm} = \mathrm{kg\,m^2/s^2}$
watt	$\mathrm{W} = \mathrm{kg\,m^2/s^3}$	coulomb	$\mathrm{C} = \mathrm{As}$
volt	$\mathrm{V} = \mathrm{kg\,m^2/As^3}$	farad	$\mathrm{F} = \mathrm{As/V} = \mathrm{A^2 s^4/kg\,m^2}$
ohm	$\Omega = \mathrm{V/A} = \mathrm{kg\,m^2/A^2 s^3}$	siemens	$\mathrm{S} = 1/\Omega$
weber	$\mathrm{Wb} = \mathrm{Vs} = \mathrm{kg\,m^2/As^2}$	tesla	$\mathrm{T} = \mathrm{Wb/m^2} = \mathrm{kg/As^2} = \mathrm{kg/Cs}$
henry	$\mathrm{H} = \mathrm{Vs/A} = \mathrm{kg\,m^2/A^2 s^2}$	degree Celsius	°C (see definition of kelvin)
lumen	$\mathrm{lm} = \mathrm{cd\,sr}$	lux	$\mathrm{lx} = \mathrm{lm/m^2} = \mathrm{cd\,sr/m^2}$
becquerel	$\mathrm{Bq} = 1/\mathrm{s}$	gray	$\mathrm{Gy} = \mathrm{J/kg} = \mathrm{m^2/s^2}$
sievert	$\mathrm{Sv} = \mathrm{J/kg} = \mathrm{m^2/s^2}$	katal	$\mathrm{kat} = \mathrm{mol/s}$

We note that in all definitions of units, the kilogram only appears to the powers of 1, 0 and −1. Can you try to formulate the reason? Challenge 321 s

The *admitted* non-SI units are *minute*, *hour*, *day* (for time), *degree* $1° = \pi/180\,\mathrm{rad}$, *minute* $1' = \pi/10\,800\,\mathrm{rad}$, *second* $1'' = \pi/648\,000\,\mathrm{rad}$ (for angles), *litre*, and *tonne*. All other units are to be avoided.

All SI units are made more practical by the introduction of standard names and abbreviations for the powers of ten, the so-called *prefixes*:*

Power	Name		Power	Name		Power	Name		Power	Name	
10^{1}	deca	da	10^{-1}	deci	d	10^{18}	Exa	E	10^{-18}	atto	a
10^{2}	hecto	h	10^{-2}	centi	c	10^{21}	Zetta	Z	10^{-21}	zepto	z
10^{3}	kilo	k	10^{-3}	milli	m	10^{24}	Yotta	Y	10^{-24}	yocto	y
10^{6}	Mega	M	10^{-6}	micro	µ	unofficial:			Ref. 295		
10^{9}	Giga	G	10^{-9}	nano	n	10^{27}	Xenta	X	10^{-27}	xenno	x
10^{12}	Tera	T	10^{-12}	pico	p	10^{30}	Wekta	W	10^{-30}	weko	w
10^{15}	Peta	P	10^{-15}	femto	f	10^{33}	Vendekta	V	10^{-33}	vendeko	v
						10^{36}	Udekta	U	10^{-36}	udeko	u

* Some of these names are invented (yocto to sound similar to Latin *octo* 'eight', zepto to sound similar to Latin *septem*, yotta and zetta to resemble them, exa and peta to sound like the Greek words ἑξάκις and πεντάκις for 'six times' and 'five times', the unofficial ones to sound similar to the Greek words for nine, ten, eleven and twelve); some are from Danish/Norwegian (atto from *atten* 'eighteen', femto from *femten* 'fifteen'); some are from Latin (from *mille* 'thousand', from *centum* 'hundred', from *decem* 'ten', from *nanus* 'dwarf'); some are from Italian (from *piccolo* 'small'); some are Greek (micro is from μικρός 'small', deca/deka from δέκα 'ten', hecto from ἑκατόν 'hundred', kilo from χίλιοι 'thousand', mega from μέγας 'large', giga from γίγας 'giant', tera from τέρας 'monster').

Translate: I was caught in such a traffic jam that I needed a microcentury for a picoparsec and that my car's fuel consumption was two tenths of a square millimetre. Challenge 322 e

▪ SI units form a *complete* system: they cover in a systematic way the full set of observables of physics. Moreover, they fix the units of measurement for all other sciences as well.

▪ SI units form a *universal* system: they can be used in trade, in industry, in commerce, at home, in education and in research. They could even be used by extraterrestrial civilizations, if they existed.

▪ SI units form a *self-consistent* system: the product or quotient of two SI units is also an SI unit. This means that in principle, the same abbreviation, e.g. 'SI', could be used for every unit.

The SI units are not the only possible set that could fulfil all these requirements, but they are the only existing system that does so.* In the near future, the BIPM plans to use the cube of physical constants, shown in Figure 1, to define SI units. This implies fixing the values of e and k in addition to the already fixed value for c. The only exception will remain the fixing of a basic time unit with the help of an atomic transition, not with the constant G, because this constant cannot be measured with high precision.

The meaning of measurement

Challenge 323 e

Every measurement is a comparison with a standard. Therefore, any measurement requires *matter* to realize the standard (even for a speed standard), and *radiation* to achieve the comparison. The concept of measurement thus assumes that matter and radiation exist and can be clearly separated from each other.

Every measurement is a comparison. Measuring thus implies that space and time exist, and that they differ from each other.

Every measurement produces a measurement result. Therefore, every measurement implies the *storage* of the result. The process of measurement thus implies that the situation before and after the measurement can be distinguished. In other terms, every measurement is an *irreversible* process.

Every measurement is a process. Thus every measurement takes a certain amount of time and a certain amount of space.

All these properties of measurements are simple but important. Beware of anybody who denies them.

Precision and accuracy of measurements

Measurements are the basis of physics. Every measurement has an *error*. Errors are due to lack of precision or to lack of accuracy. *Precision* means how well a result is reproduced when the measurement is repeated; *accuracy* is the degree to which a measurement corresponds to the actual value.

* Apart from international units, there are also *provincial* units. Most provincial units still in use are of Roman origin. The mile comes from *milia passum*, which used to be one thousand (double) strides of about 1480 mm each; today a nautical mile, once defined as minute of arc on the Earth's surface, is defined exactly as 1852 m. The inch comes from *uncia/onzia* (a twelfth – now of a foot). The pound (from *pondere* 'to weigh') is used as a translation of *libra* – balance – which is the origin of its abbreviation lb. Even the habit of counting in dozens instead of tens is Roman in origin. These and all other similarly funny units – like the system in which all units start with 'f', and which uses furlong/fortnight as its unit of velocity – are now officially defined as multiples of SI units.

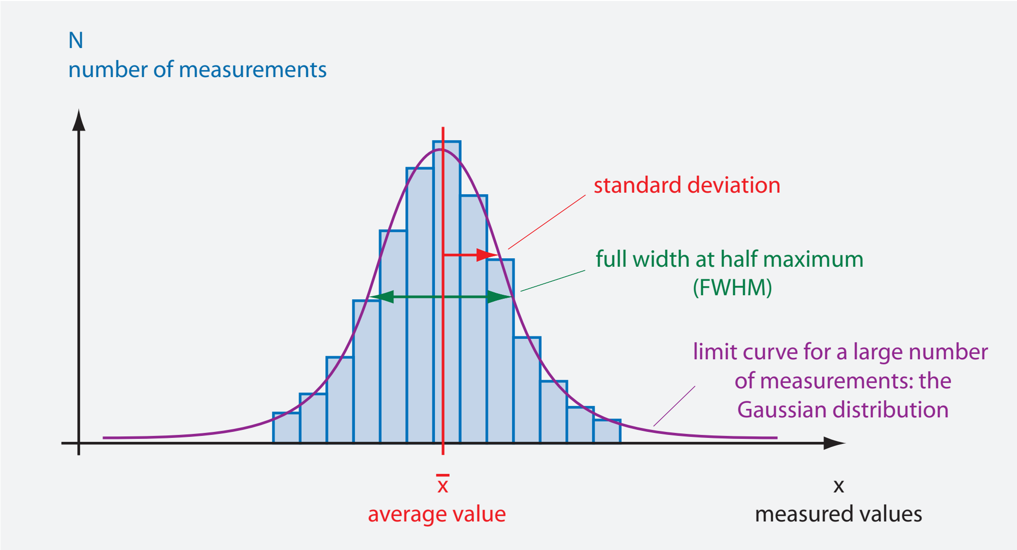

FIGURE 169 A precision experiment and its measurement distribution. The precision is high if the width of the distribution is narrow; the accuracy is high if the centre of the distribution agrees with the actual value.

Lack of precision is due to accidental or *random errors*; they are best measured by the *standard deviation*, usually abbreviated σ; it is defined through

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 , \tag{107}$$

where $\bar{x}$ is the average of the measurements x_i. (Can you imagine why $n - 1$ is used in the formula instead of n?)

Challenge 324 s

For most experiments, the distribution of measurement values tends towards a normal distribution, also called *Gaussian distribution*, whenever the number of measurements is increased. The distribution, shown in Figure 169, is described by the expression

$$N(x) \approx \mathrm{e}^{-\frac{(x-\bar{x})^2}{2\sigma^2}} . \tag{108}$$

The square σ^2 of the standard deviation is also called the *variance*. For a Gaussian distribution of measurement values, 2.35σ is the full width at half maximum.

Challenge 325 e

Lack of accuracy is due to *systematic errors*; usually these can only be estimated. This estimate is often added to the random errors to produce a *total experimental error*, sometimes also called *total uncertainty*. The *relative* error or uncertainty is the ratio between the error and the measured value.

Ref. 296

For example, a professional measurement will give a result such as 0.312(6) m. The number between the parentheses is the standard deviation σ, in units of the last digits. As above, a Gaussian distribution for the measurement results is assumed. Therefore, a value of 0.312(6) m implies that the actual value is expected to lie

Challenge 326 e

— within 1σ with 68.3 % probability, thus in this example within 0.312 ± 0.006 m;
— within 2σ with 95.4 % probability, thus in this example within 0.312 ± 0.012 m;
— within 3σ with 99.73 % probability, thus in this example within 0.312 ± 0.018 m;
— within 4σ with 99.9937 % probability, thus in this example within 0.312 ± 0.024 m;
— within 5σ with 99.999 943 % probability, thus in this example within 0.312 ± 0.030 m;
— within 6σ with 99.999 999 80 % probability, thus within 0.312 ± 0.036 m;
— within 7σ with 99.999 999 999 74 % probability, thus within 0.312 ± 0.041 m.

Challenge 327 s (Do the latter numbers make sense?)

Note that standard deviations have one digit; you must be a world expert to use two, and a fool to use more. If no standard deviation is given, a (1) is assumed. As a result, among professionals, 1 km and 1000 m are *not* the same length!

What happens to the errors when two measured values A and B are added or subtracted? If the all measurements are independent – or uncorrelated – the standard deviation of the sum *and* that of difference is given by $\sigma = \sqrt{\sigma_A^2 + \sigma_B^2}$. For both the product or ratio of two measured and uncorrelated values C and D, the result is $\rho = \sqrt{\rho_C^2 + \rho_D^2}$, where the ρ terms are the *relative* standard deviations.

Challenge 328 s Assume you measure that an object moves 1.0 m in 3.0 s: what is the measured speed value?

Limits to precision

What are the limits to accuracy and precision? There is no way, even in principle, to measure a length x to a *precision* higher than about 61 digits, because in nature, the ratio between the largest and the smallest measurable length is $\Delta x/x > l_{Pl}/d_{horizon} = 10^{-61}$.
Challenge 329 e (Is this ratio valid also for force or for volume?) In the final volume of our text, studies
Vol. VI, page 89 of clocks and metre bars strengthen this theoretical limit.

But it is not difficult to deduce more stringent practical limits. No imaginable machine can measure quantities with a higher precision than measuring the diameter of the Earth within the smallest length ever measured, about 10^{-19} m; that is about 26 digits of precision. Using a more realistic limit of a 1000 m sized machine implies a limit of 22 digits. If, as predicted above, time measurements really achieve 17 digits of precision, then they are nearing the practical limit, because apart from size, there is an additional practical restriction: cost. Indeed, an additional digit in measurement precision often means an additional digit in equipment cost.

Physical constants

In physics, general observations are deduced from more fundamental ones. As a consequence, many measurements can be deduced from more fundamental ones. The most fundamental measurements are those of the physical constants.

The following tables give the world's best values of the most important physical constants and particle properties – in SI units and in a few other common units – as published in the standard references. Ref. 297 The values are the world averages of the best measurements made up to the present. As usual, experimental errors, including both random and estimated systematic errors, are expressed by giving the standard deviation in the last digits. In fact, behind each of the numbers in the following tables there is a long

Ref. 298 story which is worth telling, but for which there is not enough room here.

Ref. 297 In principle, all quantitative properties of matter can be calculated with quantum theory and the values of certain physical constants. For example, colour, density and elastic Vol. V, page 254 properties can be predicted using the equations of the standard model of particle physics and the values of the following basic constants.

TABLE 27 Basic physical constants.

Quantity	Symbol	Value in SI units	Uncert.[a]
Constants that define the SI measurement units			
Vacuum speed of light[c]	c	299 792 458 m/s	0
Vacuum permeability[c]	μ_0	$4\pi \cdot 10^{-7}$ H/m	0
		= 1.256 637 061 435 ... μH/m	0
Vacuum permittivity[c]	$\varepsilon_0 = 1/\mu_0 c^2$	8.854 187 817 620 ... pF/m	0
Original Planck constant	h	$6.626\,069\,57(52) \cdot 10^{-34}$ Js	$4.4 \cdot 10^{-8}$
Reduced Planck constant, quantum of action	$\hbar$	$1.054\,571\,726(47) \cdot 10^{-34}$ Js	$4.4 \cdot 10^{-8}$
Positron charge	e	0.160 217 656 5(35) aC	$2.2 \cdot 10^{-8}$
Boltzmann constant	k	$1.380\,6488(13) \cdot 10^{-23}$ J/K	$9.1 \cdot 10^{-7}$
Gravitational constant	G	$6.673\,84(80) \cdot 10^{-11}\ \mathrm{Nm^2/kg^2}$	$1.2 \cdot 10^{-4}$
Gravitational coupling constant	$\kappa = 8\pi G/c^4$	$2.076\,50(25) \cdot 10^{-43}\ \mathrm{s^2/kg\,m}$	$1.2 \cdot 10^{-4}$
Fundamental constants (of unknown origin)			
Number of space-time dimensions		3 + 1	0[b]
Fine-structure constant[d] or	$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$	1/137.035 999 074(44)	$3.2 \cdot 10^{-10}$
e.m. coupling constant	$= g_{\mathrm{em}}(m_e^2 c^2)$	= 0.007 297 352 5698(24)	$3.2 \cdot 10^{-10}$
Fermi coupling constant[d] or	$G_F/(\hbar c)^3$	$1.166\,364(5) \cdot 10^{-5}\ \mathrm{GeV^{-2}}$	$4.3 \cdot 10^{-6}$
weak coupling constant	$\alpha_w(M_Z) = g_w^2/4\pi$	1/30.1(3)	$1 \cdot 10^{-2}$
Weak mixing angle	$\sin^2 \theta_W(\overline{MS})$	0.231 24(24)	$1.0 \cdot 10^{-3}$
	$\sin^2 \theta_W$ (on shell)	0.2224(19)	$8.7 \cdot 10^{-3}$
	$= 1 - (m_W/m_Z)^2$		
Strong coupling constant[d]	$\alpha_s(M_Z) = g_s^2/4\pi$	0.118(3)	$25 \cdot 10^{-3}$
CKM quark mixing matrix	$\lvert V \rvert$	$\begin{pmatrix} 0.97428(15) & 0.2253(7) & 0.00347(16) \\ 0.2252(7) & 0.97345(16) & 0.0410(11) \\ 0.00862(26) & 0.0403(11) & 0.999152(45) \end{pmatrix}$	
Jarlskog invariant	J	$2.96(20) \cdot 10^{-5}$	
PMNS neutrino mixing m.	P	$\begin{pmatrix} 0.82 & 0.55 & -0.15 + 0.038i \\ -0.36 + 0.020i & 0.70 + 0.013i & 0.61 \\ 0.44 + 0.026i & -0.45 + 0.017i & 0.77 \end{pmatrix}$	
Elementary particle masses (of unknown origin)			
Electron mass	m_e	$9.109\,382\,91(40) \cdot 10^{-31}$ kg	$4.4 \cdot 10^{-8}$
		$5.485\,799\,0946(22) \cdot 10^{-4}$ u	$4.0 \cdot 10^{-10}$
		0.510 998 928(11) MeV	$2.2 \cdot 10^{-8}$

TABLE 27 (Continued) Basic physical constants.

Quantity	Symbol	Value in SI units	Uncert.[a]
Muon mass	m_μ	$1.883\,531\,475(96) \cdot 10^{-28}$ kg	$5.1 \cdot 10^{-8}$
		$0.113\,428\,9267(29)$ u	$2.5 \cdot 10^{-8}$
		$105.658\,3715(35)$ MeV	$3.4 \cdot 10^{-8}$
Tau mass	m_τ	$1.776\,82(16)\,\mathrm{GeV}/c^2$	
El. neutrino mass	m_{ν_e}	$< 2\,\mathrm{eV}/c^2$	
Muon neutrino mass	m_{ν_e}	$< 2\,\mathrm{eV}/c^2$	
Tau neutrino mass	m_{ν_e}	$< 2\,\mathrm{eV}/c^2$	
Up quark mass	u	1.8 to $3.0\,\mathrm{MeV}/c^2$	
Down quark mass	d	4.5 to $5.5\,\mathrm{MeV}/c^2$	
Strange quark mass	s	$95(5)\,\mathrm{MeV}/c^2$	
Charm quark mass	c	$1.275(25)\,\mathrm{GeV}/c^2$	
Bottom quark mass	b	$4.18(17)\,\mathrm{GeV}/c^2$	
Top quark mass	t	$173.5(1.4)\,\mathrm{GeV}/c^2$	
Photon mass	γ	$< 2 \cdot 10^{-54}$ kg	
W boson mass	$W^\pm$	$80.385(15)\,\mathrm{GeV}/c^2$	
Z boson mass	Z^0	$91.1876(21)\,\mathrm{GeV}/c^2$	
Higgs mass	H	$126(1)\,\mathrm{GeV}/c^2$	
Gluon mass	$g_{1...8}$	$c.\ 0\,\mathrm{MeV}/c^2$	
Composite particle masses			
Proton mass	m_p	$1.672\,621\,777(74) \cdot 10^{-27}$ kg	$4.4 \cdot 10^{-8}$
		$1.007\,276\,466\,812(90)$ u	$8.9 \cdot 10^{-11}$
		$938.272\,046(21)$ MeV	$2.2 \cdot 10^{-8}$
Neutron mass	m_n	$1.674\,927\,351(74) \cdot 10^{-27}$ kg	$4.4 \cdot 10^{-8}$
		$1.008\,664\,916\,00(43)$ u	$4.2 \cdot 10^{-10}$
		$939.565\,379(21)$ MeV	$2.2 \cdot 10^{-8}$
Atomic mass unit	$m_\mathrm{u} = m_{^{12}\mathrm{C}}/12 = 1$ u	$1.660\,538\,921(73)$ yg	$4.4 \cdot 10^{-8}$

a. Uncertainty: standard deviation of measurement errors.
b. Only measured from to 10^{-19} m to 10^{26} m.
c. Defining constant.
d. All coupling constants depend on the 4-momentum transfer, as explained in the section on renormalization. *Fine-structure constant* is the traditional name for the electromagnetic coupling constant g_em in the case of a 4-momentum transfer of $Q^2 = m_\mathrm{e}^2 c^2$, which is the smallest one possible. At higher momentum transfers it has larger values, e.g., $g_\mathrm{em}(Q^2 = M_\mathrm{W}^2 c^2) \approx 1/128$. In contrast, the strong coupling constant has lover values at higher momentum transfers; e.g., $\alpha_\mathrm{s}(34\,\mathrm{GeV}) = 0.14(2)$.

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Why do all these constants have the values they have? For any constant *with a dimension*, such as the quantum of action $\hbar$, the numerical value has only historical meaning. It is $1.054 \cdot 10^{-34}$ Js because of the SI definition of the joule and the second. The question why the value of a dimensional constant is not larger or smaller therefore always requires

one to understand the origin of some dimensionless number giving the ratio between the constant and the corresponding *natural unit* that is defined with c, G, $\hbar$ and α. More details and the values of the natural units are given later. Vol. IV, page 205 Understanding the sizes of atoms, people, trees and stars, the duration of molecular and atomic processes, or the mass of nuclei and mountains, implies understanding the ratios between these values and the corresponding natural units. The key to understanding nature is thus the understanding of all ratios, and thus of all dimensionless constants. The quest of understanding all ratios, including the fine structure constant α itself, is completed only in the final volume of our adventure.

The basic constants yield the following useful high-precision observations.

TABLE 28 Derived physical constants.

Quantity	Symbol	Value in SI units	Uncert.
Vacuum wave resistance	$Z_0 = \sqrt{\mu_0/\varepsilon_0}$	376.730 313 461 77... Ω	0
Avogadro's number	N_A	$6.022\,141\,29(27) \cdot 10^{23}$	$4.4 \cdot 10^{-8}$
Loschmidt's number at 273.15 K and 101 325 Pa	N_L	$2.686\,7805(24) \cdot 10^{23}$	$9.1 \cdot 10^{-7}$
Faraday's constant	$F = N_A e$	96 485.3365(21) C/mol	$2.2 \cdot 10^{-8}$
Universal gas constant	$R = N_A k$	8.314 4621(75) J/mol K	$9.1 \cdot 10^{-7}$
Molar volume of an ideal gas at 273.15 K and 101 325 Pa	$V = RT/p$	22.413 968(20) l/mol	$9.1 \cdot 10^{-7}$
Rydberg constant [a]	$R_\infty = m_e c\alpha^2/2h$	10 973 731.568 539(55) m^{-1}	$5 \cdot 10^{-12}$
Conductance quantum	$G_0 = 2e^2/h$	77.480 917 346(25) μS	$3.2 \cdot 10^{-10}$
Magnetic flux quantum	$\varphi_0 = h/2e$	2.067 833 758(46) pWb	$2.2 \cdot 10^{-8}$
Josephson frequency ratio	$2e/h$	483.597 870(11) THz/V	$2.2 \cdot 10^{-8}$
Von Klitzing constant	$h/e^2 = \mu_0 c/2\alpha$	25 812.807 4434(84) Ω	$3.2 \cdot 10^{-10}$
Bohr magneton	$\mu_B = e\hbar/2m_e$	9.274 009 68(20) yJ/T	$2.2 \cdot 10^{-8}$
Classical electron radius	$r_e = e^2/4\pi\varepsilon_0 m_e c^2$	2.817 940 3267(27) fm	$9.7 \cdot 10^{-10}$
Compton wavelength	$\lambda_C = h/m_e c$	2.426 310 2389(16) pm	$6.5 \cdot 10^{-10}$
of the electron	$\lambda\!\!\!^-_c = \hbar/m_e c = r_e/\alpha$	0.386 159 268 00(25) pm	$6.5 \cdot 10^{-10}$
Bohr radius [a]	$a_\infty = r_e/\alpha^2$	52.917 721 092(17) pm	$3.2 \cdot 10^{-10}$
Quantum of circulation	$h/2m_e$	$3.636\,947\,5520(24) \cdot 10^{-4}\ m^2/s$	$6.5 \cdot 10^{-10}$
Specific positron charge	e/m_e	$1.758\,820\,088(39) \cdot 10^{11}$ C/kg	$2.2 \cdot 10^{-8}$
Cyclotron frequency of the electron	$f_c/B = e/2\pi m_e$	27.992 491 10(62) GHz/T	$2.2 \cdot 10^{-8}$
Electron magnetic moment	μ_e	$-9.284\,764\,30(21) \cdot 10^{-24}$ J/T	$2.2 \cdot 10^{-8}$
	μ_e/μ_B	−1.001 159 652 180 76(27)	$2.6 \cdot 10^{-13}$
	μ_e/μ_N	$-1.838\,281\,970\,90(75) \cdot 10^3$	$4.1 \cdot 10^{-10}$
Electron g-factor	g_e	−2.002 319 304 361 53(53)	$2.6 \cdot 10^{-13}$
Muon–electron mass ratio	m_μ/m_e	206.768 2843(52)	$2.5 \cdot 10^{-8}$
Muon magnetic moment	μ_μ	$-4.490\,448\,07(15) \cdot 10^{-26}$ J/T	$3.4 \cdot 10^{-8}$
muon g-factor	g_μ	−2.002 331 8418(13)	$6.3 \cdot 10^{-10}$

TABLE 28 (Continued) Derived physical constants.

QUANTITY	SYMBOL	VALUE IN SI UNITS	UNCERT.
Proton–electron mass ratio	m_p/m_e	1 836.152 672 45(75)	$4.1 \cdot 10^{-10}$
Specific proton charge	e/m_p	$9.578\,833\,58(21) \cdot 10^7$ C/kg	$2.2 \cdot 10^{-8}$
Proton Compton wavelength	$\lambda_{C,p} = h/m_p c$	1.321 409 856 23(94) fm	$7.1 \cdot 10^{-10}$
Nuclear magneton	$\mu_N = e\hbar/2m_p$	$5.050\,783\,53(11) \cdot 10^{-27}$ J/T	$2.2 \cdot 10^{-8}$
Proton magnetic moment	μ_p	$1.410\,606\,743(33) \cdot 10^{-26}$ J/T	$2.4 \cdot 10^{-8}$
	μ_p/μ_B	$1.521\,032\,210(12) \cdot 10^{-3}$	$8.1 \cdot 10^{-9}$
	μ_p/μ_N	2.792 847 356(23)	$8.2 \cdot 10^{-9}$
Proton gyromagnetic ratio	$\gamma_p = 2\mu_p/\hbar$	$2.675\,222\,005(63) \cdot 10^8$ Hz/T	$2.4 \cdot 10^{-8}$
Proton g factor	g_p	5.585 694 713(46)	$8.2 \cdot 10^{-9}$
Neutron–electron mass ratio	m_n/m_e	1 838.683 6605(11)	$5.8 \cdot 10^{-10}$
Neutron–proton mass ratio	m_n/m_p	1.001 378 419 17(45)	$4.5 \cdot 10^{-10}$
Neutron Compton wavelength	$\lambda_{C,n} = h/m_n c$	1.319 590 9068(11) fm	$8.2 \cdot 10^{-10}$
Neutron magnetic moment	μ_n	$-0.966\,236\,47(23) \cdot 10^{-26}$ J/T	$2.4 \cdot 10^{-7}$
	μ_n/μ_B	$-1.041\,875\,63(25) \cdot 10^{-3}$	$2.4 \cdot 10^{-7}$
	μ_n/μ_N	−1.913 042 72(45)	$2.4 \cdot 10^{-7}$
Stefan–Boltzmann constant	$\sigma = \pi^2 k^4/60\hbar^3 c^2$	56.703 73(21) nW/m^{2}K^4	$3.6 \cdot 10^{-6}$
Wien's displacement constant	$b = \lambda_{max} T$	2.897 7721(26) mmK	$9.1 \cdot 10^{-7}$
		58.789 254(53) GHz/K	$9.1 \cdot 10^{-7}$
Electron volt	eV	$1.602\,176\,565(35) \cdot 10^{-19}$ J	$2.2 \cdot 10^{-8}$
Bits to entropy conversion const.	$k \ln 2$	10^{23} bit = 0.956 994 5(9) J/K	$9.1 \cdot 10^{-7}$
TNT energy content		3.7 to 4.0 MJ/kg	$4 \cdot 10^{-2}$

a. For infinite mass of the nucleus.

Some useful properties of our local environment are given in the following table.

TABLE 29 Astronomical constants.

QUANTITY	SYMBOL	VALUE
Tropical year 1900 [a]	a	31 556 925.974 7 s
Tropical year 1994	a	31 556 925.2 s
Mean sidereal day	d	$23^h 56' 4.090\,53''$
Average distance Earth–Sun [b]		149 597 870.691(30) km
Astronomical unit [b]	AU	149 597 870 691 m
Light year, based on Julian year [b]	al	9.460 730 472 5808 Pm
Parsec	pc	30.856 775 806 Pm = 3.261 634 al
Earth's mass	$M_♁$	$5.973(1) \cdot 10^{24}$ kg
Geocentric gravitational constant	GM	$3.986\,004\,418(8) \cdot 10^{14}$ m^3/s^2
Earth's gravitational length	$l_♁ = 2GM/c^2$	8.870 056 078(16) mm
Earth's equatorial radius [c]	$R_{♁eq}$	6378.1366(1) km

TABLE 29 (Continued) Astronomical constants.

Quantity	Symbol	Value
Earth's polar radius [c]	$R_{♁\mathrm{p}}$	6356.752(1) km
Equator–pole distance [c]		10 001.966 km (average)
Earth's flattening [c]	$e_{♁}$	1/298.25642(1)
Earth's av. density	$\rho_{♁}$	$5.5\ \mathrm{Mg/m^3}$
Earth's age	$T_{♁}$	4.50(4) Ga = 142(2) Ps
Earth's normal gravity	g	$9.806\,65\ \mathrm{m/s^2}$
Earth's standard atmospher. pressure	p_0	101 325 Pa
Moon's radius	$R_{☾\mathrm{v}}$	1738 km in direction of Earth
Moon's radius	$R_{☾\mathrm{h}}$	1737.4 km in other two directions
Moon's mass	$M_{☾}$	$7.35 \cdot 10^{22}$ kg
Moon's mean distance [d]	$d_{☾}$	384 401 km
Moon's distance at perigee [d]		typically 363 Mm, historical minimum 359 861 km
Moon's distance at apogee [d]		typically 404 Mm, historical maximum 406 720 km
Moon's angular size [e]		average 0.5181° = 31.08′, minimum 0.49°, maximum 0.55°
Moon's average density	$\rho_{☾}$	$3.3\ \mathrm{Mg/m^3}$
Moon's surface gravity	$g_{☾}$	$1.62\ \mathrm{m/s^2}$
Moons's atmospheric pressure	$p_{☾}$	from 10^{-10} Pa (night) to 10^{-7} Pa (day)
Jupiter's mass	$M_{♃}$	$1.90 \cdot 10^{27}$ kg
Jupiter's radius, equatorial	$R_{♃}$	71.398 Mm
Jupiter's radius, polar	$R_{♃}$	67.1(1) Mm
Jupiter's average distance from Sun	$D_{♃}$	778 412 020 km
Jupiter's surface gravity	$g_{♃}$	$24.9\ \mathrm{m/s^2}$
Jupiter's atmospheric pressure	$p_{♃}$	from 20 kPa to 200 kPa
Sun's mass	$M_{\odot}$	$1.988\,43(3) \cdot 10^{30}$ kg
Sun's gravitational length	$2GM_{\odot}/c^2$	2.953 250 08(5) km
Heliocentric gravitational constant	$GM_{\odot}$	$132.712\,440\,018(8) \cdot 10^{18}\ \mathrm{m^3/s^2}$
Sun's luminosity	$L_{\odot}$	384.6 YW
Solar equatorial radius	$R_{\odot}$	695.98(7) Mm
Sun's angular size		0.53° average; minimum on fourth of July (aphelion) 1888″, maximum on fourth of January (perihelion) 1952″
Sun's average density	$\rho_{\odot}$	$1.4\ \mathrm{Mg/m^3}$
Sun's average distance	AU	149 597 870.691(30) km
Sun's age	$T_{\odot}$	4.6 Ga
Solar velocity around centre of galaxy	$v_{\odot\mathrm{g}}$	220(20) km/s
Solar velocity	$v_{\odot\mathrm{b}}$	370.6(5) km/s

TABLE 29 (Continued) Astronomical constants.

Quantity	Symbol	Value
against cosmic background		
Sun's surface gravity	$g_\odot$	$274\,\mathrm{m/s^2}$
Sun's lower photospheric pressure	$p_\odot$	15 kPa
Distance to Milky Way's centre		8.0(5) kpc = 26.1(1.6) kal
Milky Way's age		13.6 Ga
Milky Way's size		*c.* 10^{21} m or 100 kal
Milky Way's mass		10^{12} solar masses, *c.* $2 \cdot 10^{42}$ kg
Most distant galaxy cluster known	SXDF-XCLJ 0218-0510	$9.6 \cdot 10^9$ al

a. Defining constant, from vernal equinox to vernal equinox; it was once used to define the second. (Remember: π seconds is about a nanocentury.) The value for 1990 is about 0.7 s less, corresponding to a slowdown of roughly 0.2 ms/a. (Watch out: why?) There is even an empirical formula for the change of the length of the year over time.

Challenge 330 s

Ref. 299

b. The truly amazing precision in the average distance Earth–Sun of only 30 m results from time averages of signals sent from Viking orbiters and Mars landers taken over a period of over twenty years. Note that the International Astronomical Union distinguishes the average distance Earth–Sun from the *astronomical unit* itself; the latter is defined as a fixed and exact length. Also the *light year* is a unit defined as an exact number by the IAU. For more details, see www.iau.org/public/measuring.

c. The shape of the Earth is described most precisely with the World Geodetic System. The last edition dates from 1984. For an extensive presentation of its background and its details, see the www.wgs84.com website. The International Geodesic Union refined the data in 2000. The radii and the flattening given here are those for the 'mean tide system'. They differ from those of the 'zero tide system' and other systems by about 0.7 m. The details constitute a science in itself.

d. Measured centre to centre. To find the precise position of the Moon at a given date, see the www.fourmilab.ch/earthview/moon_ap_per.html page. For the planets, see the page www.fourmilab.ch/solar/solar.html and the other pages on the same site.

e. Angles are defined as follows: 1 degree = $1^\circ = \pi/180$ rad, 1 (first) minute = $1' = 1°/60$, 1 second (minute) = $1'' = 1'/60$. The ancient units 'third minute' and 'fourth minute', each 1/60th of the preceding, are not in use any more. ('Minute' originally means 'very small', as it still does in modern English.)

Some properties of nature at large are listed in the following table. (If you want a challenge, can you determine whether any property of the universe itself is listed?)

Challenge 331 s

TABLE 30 Cosmological constants.

Quantity	Symbol	Value
Cosmological constant	Λ	*c.* $1 \cdot 10^{-52}\,\mathrm{m^{-2}}$
Age of the universe [a]	t_0	$4.333(53) \cdot 10^{17}$ s = $13.8(0.1) \cdot 10^9$ a
(determined from space-time, via expansion, using general relativity)		
Age of the universe [a]	t_0	over $3.5(4) \cdot 10^{17}$ s = $11.5(1.5) \cdot 10^9$ a

TABLE 30 (Continued) Cosmological constants.

Quantity	Symbol	Value
(determined from matter, via galaxies and stars, using quantum theory)		
Hubble parameter [a]	H_0	$2.3(2) \cdot 10^{-18}\,\mathrm{s}^{-1} = 0.73(4) \cdot 10^{-10}\,\mathrm{a}^{-1}$
	$= h_0 \cdot 100\,\mathrm{km/s\,Mpc} = h_0 \cdot 1.0227 \cdot 10^{-10}\,\mathrm{a}^{-1}$	
Reduced Hubble parameter [a]	h_0	$0.71(4)$
Deceleration parameter [a]	$q_0 = -(\ddot{a}/a)_0/H_0^2$	$-0.66(10)$
Universe's horizon distance [a]	$d_0 = 3ct_0$	$40.0(6) \cdot 10^{26}\,\mathrm{m} = 13.0(2)\,\mathrm{Gpc}$
Universe's topology		trivial up to $10^{26}\,\mathrm{m}$
Number of space dimensions		3, for distances up to $10^{26}\,\mathrm{m}$
Critical density of the universe	$\rho_c = 3H_0^2/8\pi G$	$h_0^2 \cdot 1.878\,82(24) \cdot 10^{-26}\,\mathrm{kg/m^3}$ $= 0.95(12) \cdot 10^{-26}\,\mathrm{kg/m^3}$
(Total) density parameter [a]	$\Omega_0 = \rho_0/\rho_c$	$1.02(2)$
Baryon density parameter [a]	$\Omega_{B0} = \rho_{B0}/\rho_c$	$0.044(4)$
Cold dark matter density parameter [a]	$\Omega_{CDM0} = \rho_{CDM0}/\rho_c$	$0.23(4)$
Neutrino density parameter [a]	$\Omega_{\nu 0} = \rho_{\nu 0}/\rho_c$	0.001 to 0.05
Dark energy density parameter [a]	$\Omega_{X0} = \rho_{X0}/\rho_c$	$0.73(4)$
Dark energy state parameter	$w = p_X/\rho_X$	$-1.0(2)$
Baryon mass	m_b	$1.67 \cdot 10^{-27}\,\mathrm{kg}$
Baryon number density		$0.25(1)\,/\mathrm{m^3}$
Luminous matter density		$3.8(2) \cdot 10^{-28}\,\mathrm{kg/m^3}$
Stars in the universe	n_s	$10^{22\pm1}$
Baryons in the universe	n_b	$10^{81\pm1}$
Microwave background temperature [b]	T_0	$2.725(1)\,\mathrm{K}$
Photons in the universe	n_γ	10^{89}
Photon energy density	$\rho_\gamma = \pi^2 k^4/15T_0^4$	$4.6 \cdot 10^{-31}\,\mathrm{kg/m^3}$
Photon number density		$410.89\,/\mathrm{cm^3}$ or $400\,/\mathrm{cm^3}(T_0/2.7\,\mathrm{K})^3$
Density perturbation amplitude	$\sqrt{S}$	$5.6(1.5) \cdot 10^{-6}$
Gravity wave amplitude	$\sqrt{T}$	$< 0.71\sqrt{S}$
Mass fluctuations on 8 Mpc	σ_8	$0.84(4)$
Scalar index	n	$0.93(3)$
Running of scalar index	$\mathrm{d}n/\mathrm{d}\ln k$	$-0.03(2)$
Planck length	$l_{Pl} = \sqrt{\hbar G/c^3}$	$1.62 \cdot 10^{-35}\,\mathrm{m}$
Planck time	$t_{Pl} = \sqrt{\hbar G/c^5}$	$5.39 \cdot 10^{-44}\,\mathrm{s}$
Planck mass	$m_{Pl} = \sqrt{\hbar c/G}$	$21.8\,\mu\mathrm{g}$
Instants in history [a]	t_0/t_{Pl}	$8.7(2.8) \cdot 10^{60}$
Space-time points inside the horizon [a]	$N_0 = (R_0/l_{Pl})^3 \cdot (t_0/t_{Pl})$	$10^{244\pm1}$
Mass inside horizon	M	$10^{54\pm1}\,\mathrm{kg}$

a. The index 0 indicates present-day values.
b. The radiation originated when the universe was 380 000 years old and had a temperature of about 3000 K; the fluctuations ΔT_0 which led to galaxy formation are today about $16 \pm 4\,\mu\text{K} = 6(2) \cdot 10^{-6}\, T_0$.

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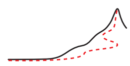
Useful numbers

Ref. 300

π	$3.14159\,26535\,89793\,23846\,26433\,83279\,50288\,41971\,69399\,37510_5$
e	$2.71828\,18284\,59045\,23536\,02874\,71352\,66249\,77572\,47093\,69995_9$
γ	$0.57721\,56649\,01532\,86060\,65120\,90082\,40243\,10421\,59335\,93992_3$
ln 2	$0.69314\,71805\,59945\,30941\,72321\,21458\,17656\,80755\,00134\,36025_5$
ln 10	$2.30258\,50929\,94045\,68401\,79914\,54684\,36420\,76011\,01488\,62877_2$
$\sqrt{10}$	$3.16227\,76601\,68379\,33199\,88935\,44432\,71853\,37195\,55139\,32521_6$



CHALLENGE HINTS AND SOLUTIONS

Challenge 1, page 9: Do not hesitate to be demanding and strict. The next edition of the text will benefit from it.

Challenge 3, page 16: The electric field distorts the flame towards and against the comb. An photograph of the effect is shown in Figure 171. A video of a similar effect in stronger fields is found at www.youtube.com/watch?v=a7_8Gc_Llr8.

Challenge 4, page 20: The liquid drops have to detach from the flow exactly inside the metal counter-electrodes. There is always a tiny charge somewhere on the metal structures (due to cosmic rays, rubbing, previous charging, etc.). In Figure 170, this initial charge is the positive charge drawn on the lower left and upper right metal structure. When the water droplets form, they get a charge that is opposite to that of the metal surrounding the region where they form. The negative droplets fall into the other metal structure. Through the negative charge accumulating there, the positive charge in the first structure increases. When the charge on the metal structure increases, the charge separation in the droplets is more efficient. In other words, water droplet formation inside the metal structures *amplifies* any initial charge. After a while, the charge value and the associated voltage are so high that it leads to a loud bang (if everything is dry, including the air.) Then the process starts again. In fact, a vaguely similar charge separation mechanism leads to cloud charging and to lightning. If you want to build a Kelvin generator at home, have a look at the de.wikipedia.org/wiki/Kelvin-Generator web page, or search for other internet site on the topic.

To avoid the sparks in the fuel tanks of its cars, Opel simply earthed the metal tube at the tank inlet; they had forgotten to ensure electric contact between the tube and the rest of the car.

The explosion of fuel can also occur if you pour fuel into your car from a metal container. Several times, fuel thieves were 'punished' by an explosion triggered by static electricity when they tried to pour stolen fuel into their own car.

On every airport you can see how the problem is avoided: before even attaching the fuel tube to an aeroplane, the worker attaches a conducting cable to connect the truck (or the tank) to the aeroplane.

Challenge 5, page 21: We look at the two sparks through a rapidly rotating mirror. In this way, small timing differences lead to position differences of the two sparks. In the 19th century, the speed values measured in this way varied between 6000 km/s and way over 100 000 km/s, because the speed depends on the effective capacitance and inductance of wire and set-up. Only if these effects can be neglected is the measured speed the same as that of light in vacuum, namely around 300 000 km/s. In modern cables, the speed is often around a third of this value. Page 31

Challenge 6, page 21: A lot of noise appeared while the metal pendulum banged wildly between the two fixed bells.

Challenge 8, page 25: No.

Challenge 9, page 26: The field at a distance of 1 m from an electron is 1.4 nV/m.

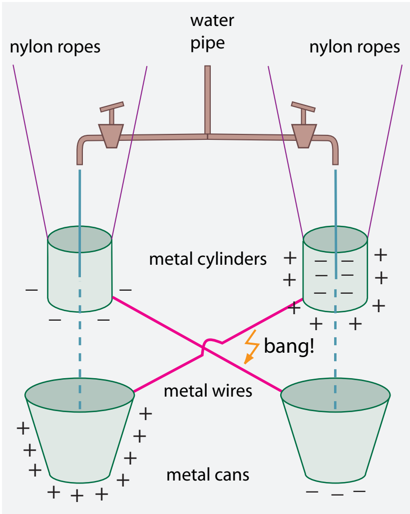


FIGURE 170 The key process in the Kelvin generator: charge separation during droplet formation.

FIGURE 171 The way a flame reacts to a rubbed comb (© Shubham Das and Rakesh Kumar).

Challenge 10, page 25: A simple geometrical effect: anything flowing out homogeneously from a sphere diminishes with the square of the distance.

Challenge 11, page 27: One has $F = \alpha\hbar c N_A^2/4R^2 = 3 \cdot 10^{12}$ N, an enormous force, corresponding to the weight of 300 million tons. It shows the enormous forces that keep matter together.

Obviously, there is no way to keep 1 g of positive charge together, as the repulsive forces among the charges would be even larger.

Challenge 12, page 28: To show the full equivalence of Coulomb's and Gauss's 'laws', first show that it holds for a single point charge. Then expand the result for more than one point charge. That gives Gauss's 'law' in integral form, as given just before this challenge.

To deduce the integral form of Gauss's 'law' for a single point charge, one has to integrate over the closed surface. The essential point here is to note that the integration can be carried out for an inverse square dependence only. This dependence allows transforming the scalar product between the local field and the area element into a normal product between the charge and the solid angle Ω:

$$\boldsymbol{E}\,\mathrm{d}\boldsymbol{A} = \frac{q\mathrm{d}A\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{q\mathrm{d}\Omega}{4\pi\varepsilon_0}\,. \tag{109}$$

In case that the surface is closed the integration is then straightforward.

To deduce the differential form of (the static) Gauss's 'law', namely

$$\nabla\boldsymbol{E} = \frac{\rho}{\varepsilon_0}\,, \tag{110}$$

make use of the definition of the charge density ρ and of the purely mathematical relation

$$\oint_{\text{closed surface}} \boldsymbol{E}\,\mathrm{d}\boldsymbol{A} = \int_{\text{enclosed volume}} \nabla\boldsymbol{E}\,\mathrm{d}\boldsymbol{V}\,, \tag{111}$$

This mathematical relation, valid for any vector field $\boldsymbol{E}$, is called *Gauss's theorem*. It simply states that the flux is the volume integral of the divergence.

To deduce the full form of Gauss's law, including the time-derivative of the magnetic field, include relativistic effects by changing viewpoint to a moving observer.

Challenge 14, page 28: Uncharged bodies can attract each other if they are made of charged constituents neutralizing each other, and if the charges are constrained in their mobility. The charge fluctuations then lead to attraction. Most molecules interact among each other in this way; such forces are also at the basis of surface tension in liquids and thus of droplet formation.

Challenge 15, page 28: No; batteries only separate charges and pump them around.

Challenge 17, page 31: The ratio q/m of electrons and that of the free charges inside metals is not exactly the same.

Challenge 19, page 33: Find out a way to test the issue, perform the experiment, and publish it!

Challenge 20, page 40: If you can, publish the result. Researchers have tried to put people on the ocean during clouded days, have tried experiments in dark rooms, but nothing has been found so far. The experiences of people in magnetic resonance imaging equipment is inconclusive so far.

Challenge 22, page 45: No.

Challenge 24, page 46: The correct version of Ampère's 'law' is

$$\nabla\times\boldsymbol{B} - \frac{1}{c^2}\frac{\partial\boldsymbol{E}}{\partial t} = \mu_0\boldsymbol{j} \tag{112}$$

whereas the expression mentioned in the text misses the term $\frac{\partial\boldsymbol{E}}{\partial t}$.

For another way to state the difference, see Richard P. Feynman, Robert B. Leighton & Matthew Sands, *The Feynman Lectures on Physics*, volume II, Addison Wesley, p. 21-1, 1977. They can be read online for free at http://www.feynmanlectures.info.

Challenge 25, page 47: Only boosts with relativistic speeds mix magnetic and electric fields to an appreciable amount.

Challenge 27, page 48: The dual field $*F$ is defined on page 75.

Challenge 28, page 49: Scalar products of four vectors are always, by construction, Lorentz invariant quantities.

Challenge 29, page 50: X-rays production needs high concentration of energy; such levels are impossible in biological systems.

Challenge 30, page 50: Electric waves of low frequency are produced in nervous systems, and in brains in particular. As mentioned above, various fish communicate via time-varying electric
Ref. 16 dipole fields. But no communication via *radio* waves has ever been found. In fact, there is little hope that such systems exist. Why? (Hint: ponder the involved frequencies, their generation, and the physical properties of water and air.)

Challenge 33, page 53: Almost all neutral particles are made of charged ones. So the argument holds for them as well. There is only one exception: neutrinos. For them, the argument is not valid. However, even neutrinos have charged virtual particles around them, so that the maximum speed also applies to them.

Page 53 **Challenge 34**, page 53: As explained earlier on, for an observer who flies along the wire, the entrance and exit events for charges at the two ends events do not occur simultaneously any more; the wire is charged for a moving observer. Thus there is a magnetic field around a wire for *any* moving observer.

Challenge 35, page 55: The illumination of the sun changes the ionization in the upper atmosphere and provokes convection in the ionosphere. The tides move the ions in the ocean and in
Page 62 the atmosphere. These currents lead to magnetic fields which can be seen in sensitive compass needles.

Challenge 36, page 55: If you find such an effect and are able to demonstrate it, publish it in a didactic journal.

Challenge 37, page 56: Usually, the cables of high voltage lines are too warm to be comfortable.

Challenge 38, page 56: Move them to form a T shape.

Challenge 39, page 56: Hint: a shining bulb is hot.

Challenge 40, page 56: For three and more switches, on uses inverters; an inverter is a switch with two inputs and two outputs which in one position, connects first and second input to first and second output respectively, and in the other position connects the first input to the second output and vice versa. (There are other possibilities, though; wires can be saved using electromagnetic relay switches.)

Challenge 42, page 57: Blond children tend to have the thinnest hair, thus giving the greatest effect. Dry weather is needed to avoid that the moisture in the air discharges the head thus preventing the hair to raise at all.

Challenge 43, page 57: It is possible; however, the systems so far are not small and are dangerous for human health. The idea to collect solar power in deep space and then beam it to the Earth as microwaves has often been aired. Finances and dangers have blocked it so far.

Challenge 45, page 58: Glue two mirrors together at a right angle. Or watch yourself on TV using a video camera.

Challenge 46, page 59: This is again an example of combined triboluminescence and triboelectricity. See also the websites scienceworld.wolfram.com/physics/Triboluminescence.html and www.geocities.com/RainForest/9911/tribo.htm.

Challenge 49, page 61: Pepper is lighter than salt, and thus reacts to the spoon before the salt does.

Challenge 50, page 64: For a wavelength of 546.1 nm (standard green), that is a bit over 18 wavelengths.

Challenge 51, page 64: The angular size of the Sun is too large; diffraction plays no role here.

Challenge 52, page 64: Just use a high speed camera.

Challenge 53, page 64: The current flows perpendicularly to the magnetic field and is thus deflected. It pulls the whole magnet with it.

Challenge 54, page 66: The most simple equivalent to a coil is a rotating mass being put into rotation by the flowing water. A transformer would then be made of two such masses connected through their axis.

Challenge 55, page 66: Light makes seven turns of the Earth in one second.

Challenge 59, page 67: There are no permanent magnets in nature that fit in a floor and that are strong enough to achieve a floating height of 50 to 80 cm. (Note that in one image the floating height is so large that the legs of the woman do not touch the floor.) And anybody who has tried this with an electromagnet knows that such a device would be larger than a complete room.

Looking carefully at the images, you will also note that they are not photographs: there are errors with the shadow and with the reflected image of the woman. And most of all, nobody would cut half the bed out of an image with a woman on the bed. Finally, nobody has ever seen the floating bed shown in the images.

Challenge 61, page 69: The mathematics required to find the solution is fascinating. Explore it!

Challenge 62, page 69: The charged layer has the effect that almost only ions of one charge pass the channels. As a result, charges are separated on the two sides of the liquid, and a current is generated.

Challenge 63, page 69: The attraction at low distances is due to the 'image force', the attraction of a charge to any conducting surface. Measuring the distance d from the centre of the sphere, the repulsion of the point charge starts for values $d > 1.618R$.

Challenge 64, page 69: Leakage currents change the picture. The long term voltage ratio is given by the leakage resistance ratio $V_1/V_2 = R_1/R_2$, as can be easily verified in experiments.

Challenge 65, page 70: The wire parallel to the high voltage line forms a capacitor. The voltage difference that appears is sufficient to trigger the neon lamp.

Challenge 66, page 70: The water disrupts the small discharge sparks, called *aigrettes*. When a new one appears, it makes a small noise. Then, with the arrival of new water, they are disrupted again, and the process repeats. Aigrettes are a form of corona discharge; the also lead to power losses and to radio interference.

Challenge 67, page 70: See above, in the section on invariants.

Challenge 70, page 72: The model does not work in three dimensions. An attempt to correct this is F. De Flaviis, M. Noro & N. G. Alexopoulos, *Diaz-Fitzgerald time domain (D-FTD) method applied to dielectric and lossy materials*, preprint available online.

Challenge 71, page 72: Search on the web, for example on the pages blog.biodiversitylibrary.org/2012/06/narwhal-oceans-one-toothed-wonder.html or narwhalslefttooth.blogspot.de/2011/05/narwhal-tusk-debate.html.

Challenge 76, page 78: Some momentum, usually a very small part, is carried away by the electromagnetic field. Given that the electromagnetic momentum is given by the vector potential, are you able to check whether everything comes out right?

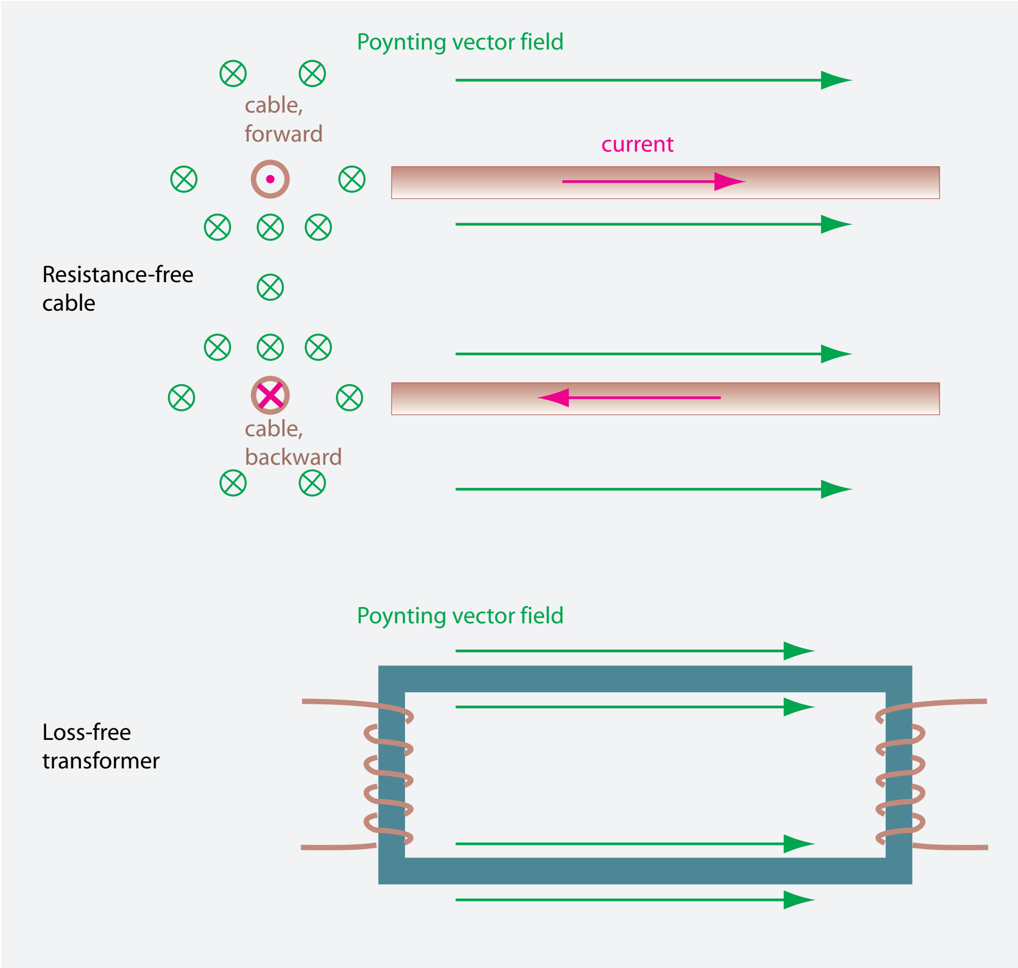


FIGURE 172 The Poynting vector field for a cable without electrical resistance and the situation a long transformer without losses.

Challenge 77, page 79: Field lines and equipotential surfaces are always orthogonal to each other. Thus a field line cannot cross an equipotential surface twice.

Challenge 85, page 83: See Figure 172. If the cable is resistance-free, most of the energy flows just outside the two conductors and parallel to them. If the cable does have resistance, the Poynting vectors point slightly towards the conductors. For the case of a transformer, which can be deduced from the case of the cable via the analogy sketched in the picture, see the beautiful paper by F. Herrmann & G. B. Schmid, *The Ponyting vector field and the energy flow within a transformer*, American Journal of Physics 54, pp. 528–531, 1986.

Challenge 90, page 86: The argument is the same as for the increase in entropy: reverse processes are possible, but the probability is so low that they do not appear in practice. The extremely low probability is due to the fluctuations induced by the environment.

Challenge 91, page 86: Just draw a current through a coil with its magnetic field, then draw the mirror image of the current and redraw the magnetic field.

Challenge 92, page 87: Other asymmetries in nature include the helicity of the DNA molecules

making up the chromosomes and many other molecules in living systems, the right hand preference of most humans, the asymmetry of fish species which usually stay flat on the bottom of the seas.

Challenge 93, page 87: This is not possible at all using gravitational or electromagnetic systems or effects. The only way is to use the weak nuclear interaction, as shown in the chapter on the nucleus.

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Challenge 94, page 87: The Lagrangian does not change if one of the three coordinates is changed by its negative value.

Challenge 95, page 88: The image flips up: a 90 degree rotation turns the image by 180 degrees.

Challenge 96, page 88: Imagine E and B as the unite vectors of two axes in complex space. Then any rotation of these axes is also a generalized duality symmetry.

Challenge 97, page 91: The angular momentum was put into the system when it was formed. If we bring a point charge from infinity along a straight line to its final position close to a magnetic dipole, the magnetic force acting on the charge is not directed along the line of motion. It therefore creates a non-vanishing torque about the origin. See J. M. Aguirregabiria & A. Hernandez, *The Feynman paradox revisited*, European Journal of Physics 2, pp. 168–170, 1981.

Challenge 98, page 92: Show that even though the radial magnetic field of a spherical wave is vanishing by definition, Maxwell's equations would require it to be different from zero. Since electromagnetic waves are transversal, it is also sufficient to show that it is impossible to comb a hairy sphere without having a (double) vortex or two simple vortices. Despite these statements, quantum theory changes the picture somewhat: the emission probability of a photon from an excited atom in a degenerate state is spherically symmetric exactly.

Challenge 99, page 92: If the conservation of linear and angular momentum are taken into account, there is no ambiguity of the Poynting vector. See, for example, W. H. Furry, *Examples of momentum distributions in the electromagnetic field and in matter*, American Journal of Physics 37, pp. 621–636, 1969.

Challenge 100, page 92: The emitted radiation is strongly suppressed because the size of the dipole (the plug) is much smaller than the wavelength of the field.

Challenge 102, page 92: No. Neither electromagnetic motors nor coils have been found in any living system. Muscles, the most powerful actuators in biology, are mainly made of large numbers of electrostatic motors. The fundamental reason for this difference is the low efficiency of microscopic *electromagnetic* motors, which contrasts with the high efficiency of microscopic *electrostatic* motors. At macroscopic sizes, the efficiency advantages switches.

Challenge 104, page 99: In every case of interference, the energy is redistributed into other directions. This is the general rule; sometimes it is quite tricky to discover this other direction.

Challenge 105, page 99: The author regularly sees about 7 lines; assuming that the distance is around 20 μm, this makes about 3 μm per line. The wavelength must be smaller than this value and the frequency thus larger than 100 THz. The actual values for various colours are given in the table of the electromagnetic spectrum.

Challenge 107, page 100: The distance l between the lines of an interference pattern is given by $l = \lambda d/s$, where d is the distance to the screen, and s is the source separation.

To learn more about interference and the conditions for its appearance, explore the concept of Fresnel number. For example, the Fresnel number allows to distinguish the 'far field' from the 'near field', two situations that occur in many wave phenomena.

Challenge 108, page 101: He noted that when a prism produces a rainbow, a thermometer placed in the region after the colour red shows a temperature rise.

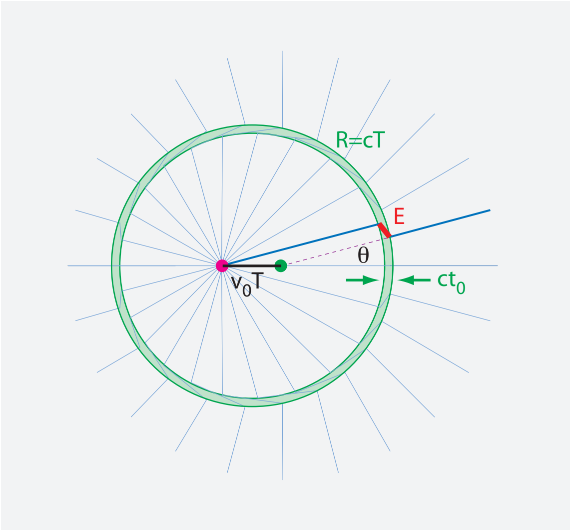

FIGURE 173 Calculating the transverse field of an accelerated charge.

Challenge 111, page 108: Birefringence appears when the refraction is polarization dependent. Only two linear independent polarizations are possible, thus there is no trirefringence in nature. This holds true also for crystals which have three different indices of refraction in three directions!

Challenge 112, page 110: Light reflected form a water surface is partly polarized. Mirages are not.

Challenge 113, page 112: Figure 173 shows electrical field lines. We assume that the charge moves at a initial velocity v_0 that is small compared to c and that it decelerates to zero velocity during a time t_0. After a time T has elapsed, the radiation pulse has travelled a distance $R = cT$, where $T \gg t_0$. The figure shows that at a given kink, drawn in red, the ratio of the transverse field E_t and of the radial field E_r is given by the steepness of the of the kink. (Why?) Geometry then leads to

$$\frac{E_t}{E_r} = \frac{v_0 T \sin\theta}{c t_0} = \frac{a R \sin\theta}{c^2} \ . \tag{113}$$

Inserting Coulomb's expression for the radial field we get

$$E_t = \frac{1}{4\pi\varepsilon_0 c^2} \frac{a \sin\theta}{R} \ . \tag{114}$$

The magnitude of the transversal field thus decreases with $1/R$. In addition, the field depends on the angle θ; this is clearly visible both in Figure 173 and in Figure 67 on page 113. In other words, transmitter antennas have a preferred direction of power emission, namely perpendicularly to the direction of acceleration.

Challenge 114, page 115: Such an observer would experience a wavy but static field, which cannot exist, as the equations for the electromagnetic field show.

Challenge 115, page 115: You would never die. Could you reach the end of the universe?

Challenge 118, page 117: A surface of 1 m^2 perpendicular to the light receives about 1 kW of radiation. It generates the same pressure as the weight of about 0.3 mg of matter. That generates 3 µPa for black surfaces, and the double for mirrors.

Challenge 120, page 118: The shine side gets twice the momentum transfer as the black side, and thus should be pushed backwards.

Challenge 123, page 120: A polarizer can do this.

Challenge 126, page 121: The interference patterns change when colours are changed. Rainbows also appear because different colours are due to different frequencies.

Challenge 129, page 122: Ternary and quaternary rainbows form a bow around the Sun. To see them, typically one has to be behind a building or tree that covers the direct view to the Sun. In 2011, there were only a handful of photographs of a ternary rainbow and only a single photograph of a quaternary rainbow, world-wide.

Challenge 130, page 122: The full rainbow is round like a circle. You can produce one with a garden hose, if you keep the hose in your hand while you stand on a chair, with your back to the evening Sun. (Well, one small part is missing; can you imagine which part?) The circle is due to the spherical shape of droplets. If the droplets were of different shape, *and* if they were all aligned, the rainbow would have a different shape than a simple circle.

Challenge 133, page 129: Take a film of a distant supernova explosion, or better, an optical or gamma-ray burst, and check whether it happens at the same time for each colour separately. This has been done extensively, and no differences have been detected within experimental errors.

Challenge 135, page 131: The first part of the forerunner is a feature with the shortest possible effective wavelength; thus it is given by taking the limit for infinite frequency.

Challenge 136, page 132: The light is pulsed; thus it is the energy velocity.

Challenge 137, page 132: Inside matter, the energy is transferred to atoms, then back to light, then to the next atoms, etc. That takes time and slows down the propagation.

Challenge 139, page 134: For single photons, permeability, permittivity and the wave impedance are not well-defined. Conformal invariance, dimensionality and topology are not are not valid at the tiny Planck scales. Near black holes, if quantum effects are taken into account, there is friction on moving bodies. Quantum field theory shows that vacuum contains and consists of virtual particle–antiparticle pairs. Cosmology shows that the vacuum has non-zero energy content, and the same is suggested by quantum field theory. General relativity shows that curved vacuum can move, and so does quantum gravity. In summary, one can say that vacuum has all the properties that were once ascribed to the aether, but in a way that differs fundamentally from what was discussed by its proponents.

Challenge 140, page 134: Almost no light passes; the intensity of the little light that is transmitted depends exponentially on the ratio between wavelength and hole diameter. One also says that after the hole there is an evanescent wave.

Challenge 141, page 134: The energy density is $1\,\mathrm{kW/m^2}/c = 3.3\,\mathrm{\mu J/m^3}$. Assuming sinusoidal waves, the (root mean square) electric field is $\sqrt{3.3\,\mathrm{\mu J/m^3}/\varepsilon_0} = 610\,\mathrm{V/m}$ – quite a high value. The (root mean square) magnetic field is $610\,\mathrm{V/m}/c = 2.1\,\mathrm{\mu T}$ – a rather low value.

Challenge 142, page 134: Any example of light has only one colour.

Challenge 144, page 135: Too much light is wasted, the wind shields are too expensive, and there is no reason to do something if nobody else does.

Challenge 146, page 137: Three mirrors is the minimum. Two such mirror arrangements are shown in Figure 174. There is also a three-mirror arrangement with parallel input and output beams; can you find it? The ideas behind these arrangements are well explained in the papers by Enrique Galvez and his collaborators. Ref. 97

Challenge 147, page 138: In the left interferometer, light exits in direction B, in the right one, in direction A. The problem can also be generalized to arbitrary interferometer shapes. The way

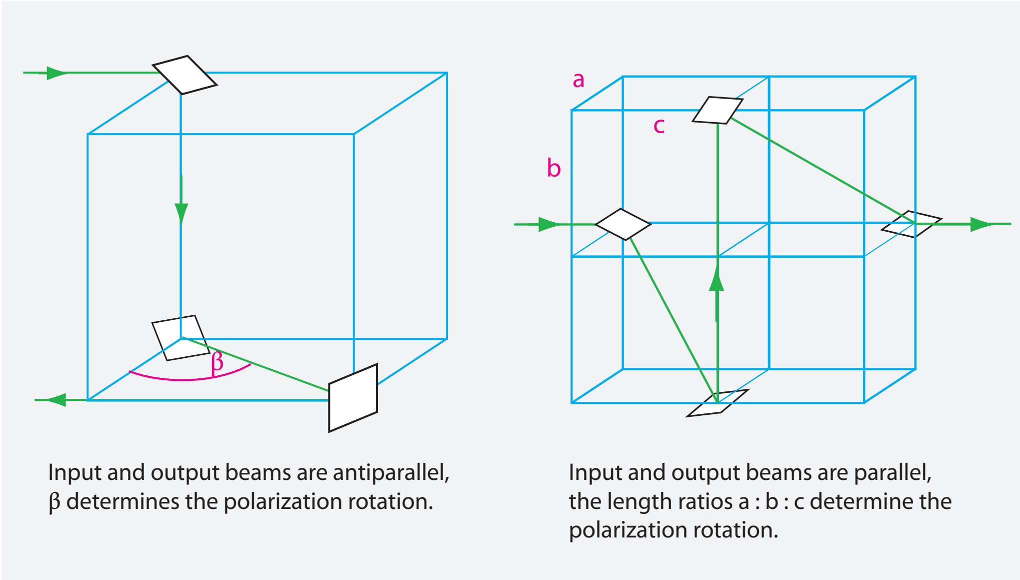

FIGURE 174 Two mirror arrangements that rotate the polarization of a light beam by a predetermined angle.

to solve it in this case is the use of *Berry's phase*. If you are interested, explore this interesting concept with the help of your favourite library.

Challenge 148, page 143: The average temperature of the Earth is thus 287 K. The energy from the Sun is proportional to the fourth power of the temperature. The energy is spread (roughly) over half the Earth's surface. The same energy, at the Sun's surface, comes from a much smaller surface, given by the same angle as the Earth subtends there. We thus have $E \sim 2\pi R^2_{\text{Earth}} T^4_{\text{Earth}} = T^4_{\text{Sun}} R^2_{\text{Earth}} \alpha^2$, where α is half the angle subtended by the Sun. As a result, the temperature of the Sun is estimated to be $T_{\text{Sun}} = (T^4_{\text{Earth}}/\alpha^2)^{0.25} = 4\,\text{kK}$.

Challenge 152, page 144: Because the maximum of a spectrum in wavelengths and in frequencies is not the same, thus does *not* and cannot follow $c = f\lambda$.

Challenge 155, page 145: At high temperature, all bodies approach black bodies. The colour is more important than other colour effects. The oven and the objects have the same temperature. Thus they cannot be distinguished from each other. To do so nevertheless, illuminate the scene with powerful light and then take a picture with small sensitivity. Thus one always needs bright light to take pictures of what happens inside fires.

Challenge 156, page 146: Achieving a higher temperature would allow to break the second principle of thermodynamics. To explore this question further, read in textbooks about the so-called *Kirchhoff laws*.

Challenge 157, page 147: The effective temperature of laser light can also be described as *higher than infinite*; this allows also to heat targets to extremely high temperatures.

Challenge 160, page 151: For small mirrors or lenses, like those used in microscopes, mass production is easier for lenses. In contrast, large mirrors are much easier and cheaper to fabricate than large lenses, because mirrors use less glass, are lighter, and allow changing their shape with actuators.

Challenge 161, page 153: Syrup shows an even more beautiful effect in the following setting. Take a long transparent tube closed at one end and fill it with syrup. Shine a red helium–neon

laser into the tube from the bottom. Then introduce a linear polarizer into the beam: the light seen in the tube will form a spiral. By rotating the polarizer you can make the spiral advance or retract. This effect, called the *optical activity* of sugar, is due to the ability of sugar to rotate light polarization and to a special property of plants: they make only one of the two mirror forms of sugar.

Challenge 163, page 154: The relation, the so-called 'law' of refraction is

$$\frac{c_1}{c_2} = \frac{\sin\alpha_1}{\sin\alpha_2} . \tag{115}$$

The particular speed ratio between vacuum (or air, which is almost the same) and a material gives the *index of refraction* n of that material:

$$n = \frac{c_1}{c_0} = \frac{\sin\alpha_1}{\sin\alpha_0} \tag{116}$$

Many incorrectly call the 'law' of refraction 'Snell's law', or 'Descartes' law' even though many others found it before them (and even though the family name is 'Snel').

Challenge 164, page 156: The thin lens formula is

$$\frac{1}{d_\mathrm{o}} + \frac{1}{d_\mathrm{i}} = \frac{1}{f} . \tag{117}$$

It is valid for diverging and converging lenses, as long as their own thickness is negligible. The strength of a lens can thus be measured with the quantity $1/f$. The unit $1\ \mathrm{m}^{-1}$ is called a *diopter*; it is used especially for reading glasses. Converging lenses have positive, diverging lenses negative values.

However, the thin lens formula is only an approximation, and is never used in lens design. It is a relic of old textbooks. Modern lens designers always use Gaussian optic for calculations. (See, for example, Francis A. Jenkins & Harvey E. White, *Fundamentals of Optics*, McGraw-Hill, 1957.)

Challenge 166, page 157: A light microscope is basically made of two converging lenses. One lens – or lens system – produces an enlarged real image and the second one produces an enlarged virtual image of the previous real image. Figure 175 also shows that microscopes always turn images upside down. Due to the wavelength of light, light microscopes have a maximum resolution of about 1 μm. Note that the magnification of microscopes is unlimited; what is limited is their resolution. This is exactly the same behaviour shown by digital images. The *resolution* is simply the size of the smallest possible pixel that makes sense.

The microscope seems to have been invented by Girolamo Fracastro in 1538. The first viable microscopes were built in the Netherlands around 1590. Progress in microscopes was so slow because glass and lens production was extremely difficult at those times, especially for small lenses. Therefore, David Brewster proposed in 1819 to build a microscope using the lens of a fish eye; when this idea was realized with the lens of an eel, it resulted in a microscope with astonishing performance. To learn more about microscopes, read the beautiful text by Elizabeth M. Slater & Henry S. Slater, *Light and Electron Microscopy*, Cambridge University Press, 1993, or explore dedicated websites, such as www.mikroskopie-muenchen.de or micro.magnet.fsu.edu/primer/techniques.

Challenge 168, page 159: The dispersion at the lens leads to different apparent image positions, as shown in Figure 176. For more details on the dispersion in the human eye and the ways of using it to create three-dimensional effects, see the article by C. Ucke & R. Wolf, *Durch Farbe in die dritte Dimension*, Physik in unserer Zeit 30, pp. 50–53, 1999.

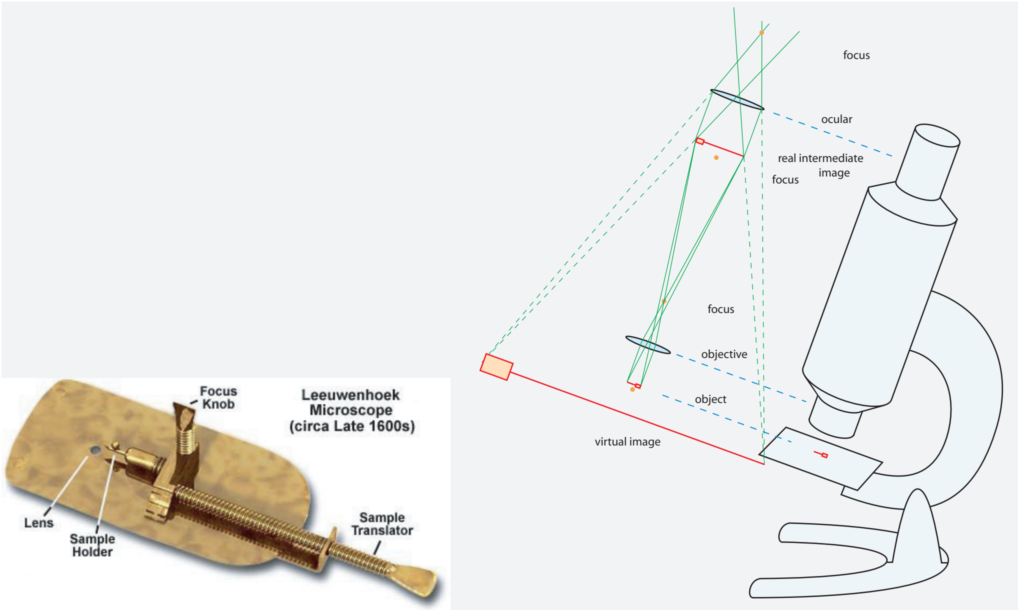

FIGURE 175 One lens made the oldest commercial microscope, from 1680 (length *c.* 8 cm, to be held close to the eye), but two converging lenses make a modern microscope (photo WikiCommons).

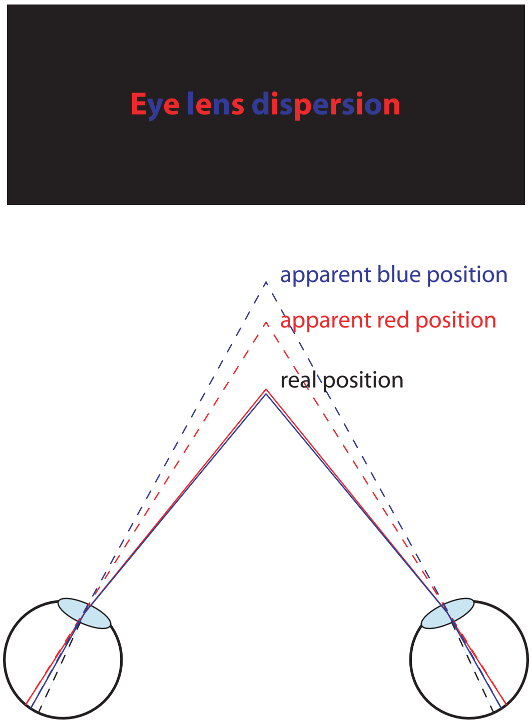

FIGURE 176 The relation between the colour depth effect and the lens dispersion of the human eye.

Challenge 169, page 162: The 1 mm beam would return 1000 times as wide as the 1 m beam. A perfect 1 m-wide beam of green light would be 209 m wide on the Moon; can you deduce this

result from the (important) formula that involves distance, wavelength, initial diameter and final diameter? Try to guess this beautiful formula first, and then deduce it. In reality, the values are a few times larger than the theoretical minimum thus calculated. See the www.csr.utexas.edu/mlrs and ilrs.gsfc.nasa.gov websites.

Challenge 170, page 162: It is often said that evolution tuned the number of cones in the eye to the maximum resolution with open pupil; the experts on the subject however maintain that there are somewhat larger numbers of cones.

Challenge 171, page 162: The answer should lie between one or two dozen kilometres, assuming ideal atmospheric circumstances.

Challenge 174, page 171: In fact, there is no way that a hologram of a person can walk around and frighten a real person. A hologram is always transparent; one can always see the background through the hologram. A hologram thus always gives an impression similar to what moving pictures usually show as ghosts. If the background is black, shine with a torch onto it to find out.

Challenge 175, page 172: The small wavelength of light probably prevents achieving this dream. For a true holographic display, the pixels need to be smaller than the wavelength of light and must be able to reproduce phase information. Thus the next question is: how much of the dream can be realized? If you find a solution, you will become rich and famous.

Challenge 177, page 178: There is a blind spot in the eye; that is a region in which images are not perceived. The brain than assumes that the image at that place is the same than at its borders. If a spot falls exactly inside it, it disappears.

Challenge 178, page 178: The mechanism that compensates the missing blue receptors in the fovea does not work so rapidly: you will see a spot due to the fovea.

Challenge 180, page 179: The eye and brain surely do not switch the up and the down direction at a certain age. Besides, where does the idea come from that babies see upside-down?

Challenge 181, page 187: The eye and vision system subtract patterns that are constant in time.

Challenge 182, page 188: Not really; a Cat's-eye uses two reflections at the sides of a cube. A living cat's eye has a large number of reflections. The end effect is the same though: light returns back to the direction it came from.

Challenge 184, page 194: Use diffraction; watch the pattern on a wall a few metres behind the hair.

Challenge 186, page 195: At 10 pc=32.6 al, the Sun would have apparent magnitude 4.7. At 20 pc=65.2 al, it would appear 4 times fainter, thus about 1.5 magnitudes more, therefore with an apparent visual magnitude of about 6.2. This is near the limit magnitude of the eye. The actual limiting magnitude of the eye is neither constant nor universal, so the distance of 50 light years is not a sharp limit. The limiting magnitude, – like the night vision, or *scotopic sensitvity* – depends on the partial pressure of oxygen in the atmosphere the observer is breathing, on the clarity of the air, on the zenith distance, and, above all, on the degree of dark adaptation. An eye exposed to the full brightness of the night sky in a very dark location far from light pollution is still not completely dark-adapted. You can easily see 7th-magnitude stars by blocking off most of the sky and just looking at a little patch of it. Some observers, under ideal conditions, have reliably reported seeing stars near 8th magnitude.

Challenge 187, page 195: The green surface seen at a low high angle is larger than when seen vertically, where the soil is also seen; the soil is covered by the green grass in low angle observation.

Challenge 188, page 195: It is indeed true. Modern telescopes have a large surface collecting light (up to 50 m^2) and have extremely sensitive detectors. The number of photons emitted by

a match lit on the moon into the direction of a large telescope (how many?) is sufficient to trigger the detector.

Challenge 189, page 197: Of course not, as the group velocity is not limited by special relativity. The energy velocity is limited, but is not changed in this experiments.

Challenge 190, page 197: He bought clothes for his mother and for himself whose colours were inappropriate.

Challenge 192, page 198: The Prussian explorer Alexander von Humboldt extensively checked this myth in the nineteenth century. He visited many mine pits and asked countless mine workers in Mexico, Peru and Siberia about their experiences. He also asked numerous chimney-sweeps. Neither him nor anybody else had ever seen the stars during the day.

Challenge 193, page 198: Watch the Sun with closed eyes, and remember the shade of red you see. Go into a closed room, turn a light bulb on, and watch it with closed eyes. Choose the distance from the bulb that yields the same shade of red. Then deduce the power of the Sun from the power of the light bulb and the inverse square dependence.

Challenge 194, page 198: If you unroll a roll of adhesive tape, in addition to light, also X-rays are emitted. This is an example of triboluminescence. See the experiment live in the film at www.youtube.com/watch?v=J3i8oRi0WNc.

Challenge 201, page 207: The human body is slightly conducting and changes the shape of the field and thus effectively short circuits it. Usually, the field cannot be used to generate energy, as the currents involved are much too small. (Lightning bolts are a different story, of course. They are due – very indirectly – to the field of the Earth, but they are too irregular to be used consistently. Franklin's lightning rod is such an example.) The fair weather field cannot be used as a power source because its internal resistance is $3\,\mathrm{G\Omega/m}$.

Challenge 202, page 207: The field at the surface of a sphere of radius r is given by $E = Q/4\pi\varepsilon_0 r^2$. Inserting $E = 200\,\mathrm{V/m}$, one gets $Q = 0.9\,\mathrm{MC}$.

Challenge 203, page 211: If you find a method that is different from the known estimates, publish it.

Challenge 209, page 215: All the illusions of the flying act look as if the magician is hanging on lines, as observed by many, including myself. (Photographic flashes are forbidden, a shimmery background is set up to render the observation of the lines difficult, no ring is ever actually pulled over the magician, the aquarium in which he floats is kept open to let the fishing lines pass through, always the same partner is 'randomly' chosen from the public, etc.) Information from eyewitnesses who have actually seen the fishing lines used by David Copperfield explains the reasons for these set-ups. The usenet news group alt.magic.secrets, in particular Tilman Hausherr, was central in clearing up this issue in all its details, including the name of the company that made the suspension mechanism.

Challenge 211, page 215: Any new one is worth a publication.

Challenge 212, page 219: Sound energy is also possible, as is mechanical work.

Challenge 213, page 222: Space-time deformation is not related to electricity; at least at everyday energies. Near Planck energies, this might be different, but nothing has been predicted yet.

Challenge 215, page 223: Ideal absorption is blackness (though it can be redness or whiteness at higher temperatures).

Challenge 216, page 223: Indeed, the Sun emits about $4 \cdot 10^{26}\,\mathrm{W}$ from its mass of $2 \cdot 10^{30}\,\mathrm{kg}$, about $0.2\,\mathrm{mW/kg}$. The adult human body (at rest) emits about $100\,\mathrm{W}$ (you can check this in bed at night), thus about $1.2\,\mathrm{W/kg}$. This is about 6000 times more than the Sun. The reason: only the very centre of the Sun actually emits energy. If that energy amount is then divided by the

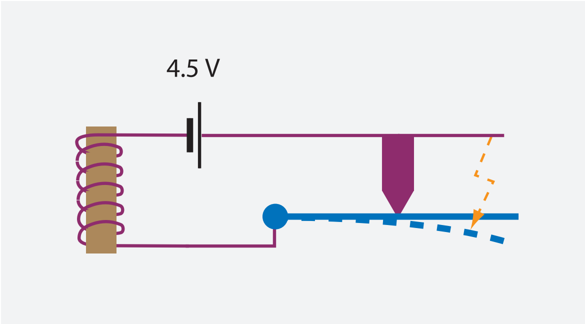

FIGURE 177 How to get electrical shocks from a 4.5 V pocket battery.

full mass, including all the mass that does not emit energy at all, one gets a small average value. By the way, any candle or, better, any laser pointer emits even more light per mass, for similar reasons.

Challenge 217, page 224: The charges on a metal box rearrange so that the field inside remains vanishing. This makes cars and aeroplanes safe against lightning. Of course, if the outside field varies so quickly that the rearrangement cannot follow, fields *can* enter the Faraday cage. (By the way, also fields with long wavelengths penetrate metals; specialized remote controls for opening security doors regularly use frequencies of 25 kHz to achieve this.) However, one should wait a bit before stepping out of a car after lightning has hit, as the car is on rubber wheels with low conduction; waiting gives the charge time to flow into the ground.

For gravity and solid cages, mass rearrangement is not possible, so that there is no gravity shield.

Mu-metal is a nickel-iron alloy, often containing traces of other metals, that has a high relative permeability μ_r in the range of 50 000 to 140 000; it is astonishingly ductile. The high permeability value effectively concentrates the magnetic fields inside the alloy and thus leads applied magnetic fields through the mu-metal and around the enclosed volume, which therefore is shielded as a result.

Challenge 221, page 225: This is a touchy topic. It is not clear whether 50 Hz fields are dangerous to humans. There is a high probability that they are not; but the issue is not settled.

Challenge 222, page 225: The number of photons times the quantum of action $\hbar$.

Challenge 223, page 226: First, Faraday could have found a superficial link using the mentioned Page 30 tube experiment. But he was looking for a possible deep connection. However, gravitation and electricity are not at all connected, as one is due to mass, the other due to charge. Much after Faraday, people discovered that gravity also includes gravitomagnetism, i.e., measurable effects Vol. II, page 165 due to moving masses – but still no relation to electromagnetism. A distant connection between gravitation and electricity will only appear in the last volume.

Challenge 224, page 226: The charging stops because a negatively charged satellite repels electrons and thus stops any electron collecting mechanism. Electrons are captured more frequently than ions because it is easier for them than for ions to have an inelastic collision with the satellite, due to their larger speed at a given temperature.

Challenge 225, page 226: Any loss mechanism will explain the loss of energy, such as electrical resistance or electromagnetic radiation. After a fraction of a second, the energy will be lost. This little problem is often discussed on the internet.

Challenge 226, page 226: Use the wire as shown in Figure 177. *If* the oscillation is properly tuned in frequency, and *if* the contact detaches properly at the tip, and *if* you touch the two contacts

with a strong grip, you will get a stronger shock than you can stand.

Challenge 228, page 227: This should be possible in the near future; but both the experiment, which will probably measure brain magnetic field details, and the precise check of its seriousness will not be simple.

Challenge 229, page 227: No, the system is not secure. In any system, the security is given by the weakest spot. And in any password system, the weakest spots are the transport of the raw data – in this case the signals from the electric cap to the computer – and the password checking system. Both are as vulnerable as any other password system. (If you want to learn about security, read the writings of Bruce Schneier, most of which are available on the internet.)

Challenge 230, page 230: The maximum electric and magnetic field values are those that exert the maximum possible force $c^4/4G$ on an elementary charge e.

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Challenge 232, page 232: See challenge 29.

Challenge 234, page 233: One can measure smallest charges, showing that they are always multiples of the same unit. This method was used by Millikan. One can also measure current fluctuations, and show that they follow from shot noise, i.e., from the flow of discrete charges.

Challenge 237, page 234: Earth's potential would be $U = -q/(4\pi\varepsilon_o R) = 60\,\mathrm{MV}$, where the number of electrons in water must be taken into account.

Challenge 238, page 234: There is always a measurement error when measuring field values, even when measuring a 'vanishing' electromagnetic field. In addition, quantum theory leads to arbitrary small charge density values through the probability density due to wave functions.

Challenge 242, page 237: The issue is: is the 'universe' a concept? In the last volume of this adventure we will show that it is not.

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Challenge 244, page 244: When thinking, physical energy, momentum and angular momentum are conserved, and thermodynamic entropy is not destroyed. Any experiment showing anything different would point to unknown processes. However, there is no evidence for such processes.

Challenge 245, page 244: The best method cannot be much shorter than what is needed to describe 1 in 6000 million, or 33 bits. The Dutch and UK post code systems (including the letters NL or UK) are not far from this value and thus can claim to be very efficient.

Challenge 246, page 245: For complex systems, when the unknowns are numerous, the advance is thus simply given by the increase in answers. For the universe as a whole, the number of open issues is quite low, as shown later on; in this topic there has not been much advance in the past years. But the advance is clearly measurable in this case as well.

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Challenge 247, page 245: Is it possible to use the term 'complete' when describing nature? Yes, it is. For a clear-cut answer, see the last volume of our adventure.

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Challenge 249, page 247: There are many baths in series: thermal baths in each light-sensitive cell of the eyes, thermal baths inside the nerves towards the brain and thermal baths inside brain cells.

Challenge 251, page 248: Yes.

Challenge 253, page 254: Chips based on trits would have to be redesigned from scratch. This would be a waste of resources and of previous work.

Challenge 255, page 259: Physicists claim that the properties of objects, of space-time and of interactions form the smallest list possible. However, this list is longer than the one found by linguists! The reason is that physicists have found primitives that do not appear in everyday life. In a sense, the aim of physicists is limited by list of unexplained questions of nature, given later on.

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Challenge 256, page 260: Neither has a defined content, clearly stated limits or a domain of application.

Challenge 257, page 261: Impossible! That would not be a concept, as it has no content. The solution to the issue must be and will be different.

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Challenge 258, page 262: To neither. This paradox shows that such a 'set of all sets' does not exist.

Challenge 259, page 263: The most famous is the class of all sets that do not contain themselves. This is not a set, but a class.

Challenge 260, page 263: Dividing cakes is difficult. A simple method that solves many – but not all – problems among N persons P1 ... PN is the following:

— P1 cuts the cake into N pieces.
— P2 to PN choose a piece.
— P1 keeps the last part.
— P2 ... PN assemble their parts back into one.
— Then P2 ... PN repeat the algorithm for one person less.

The problem is much more complex if the reassembly is not allowed. A *just* method (in finite many steps) for 3 people, using nine steps, was published in 1944 by Steinhaus, and a *fully satisfactory* method in the 1960s by John Conway. A fully satisfactory method for four persons was found only in 1995; it has 20 steps.

Challenge 261, page 264: $(x, y) := \{x, \{x, y\}\}$.

Challenge 262, page 265: Hint: show that any countable list of reals misses at least one number. This was proven for the first time by Cantor. His way was to write the list in decimal expansion and then find a number that is surely not in the list. Second hint: his world-famous trick is called the diagonal argument.

Challenge 263, page 265: Hint: all reals are limits of series of rationals.

Challenge 265, page 267: Yes, but only provided division by zero is not allowed, and numbers are restricted to the rationals and reals.

Challenge 266, page 267: There are infinitely many of them. But the smallest is already quite large: 1016949152542372881355932203389830508474576271186440677966. If the number six is changed in the puzzle, one finds that the smallest solution for 1 is 1, for 4 is 102564, for 5 is 142857, for 8 is 1012658227848, for 2 is 105263157894736842, for 7 is 1014492753623188405797, for 3 is 1034482758620689655172413793, and for 9 is 10112359550561797752808988764044943820224719. The smallest solution for 6 is the largest of this list.

Challenge 267, page 267: One way was given above: $0 := \emptyset \quad , \quad 1 := \{\emptyset\} \quad , \quad 2 := \{\{\emptyset\}\}$ etc.

Challenge 268, page 271: Subtraction is easy. Addition is not commutative only for cases when infinite numbers are involved: $\omega + 2 \neq 2 + \omega$.

Challenge 269, page 271: Examples are $1 - \varepsilon$ or $1 - 4\varepsilon^2 - 3\varepsilon^3$.

Challenge 270, page 271: The answer is 57; the cited reference gives the details.

Challenge 271, page 274: $2^{2^{22}}$ and $4^{4^{4^4}}$.

Challenge 274, page 274: The child is minus 0.75 years old, or minus 9 months old; the father is thus very near the mother.

Challenge 275, page 274: This is not an easy question. The first non-trivial numbers are 7, 23, 47, 59, 167 and 179. See Robert Matthews, *Maximally periodic reciprocals*, Bulletin of the Institute of Mathematics and its Applications 28, pp. 147–148, 1992. Matthews shows that a number

n for which $1/n$ generates the maximum of $n - 1$ decimal digits in the decimal expansion is a special sort of prime number that can be deduced from the so-called *Sophie Germain primes* S; one must have $n = 2S + 1$, where both S and $2S + 1$ must be prime and where S mod 20 must be 3, 9, or 11.

Thus the first numbers n are 7, 23, 47, 59, 167 and 179, corresponding to values for S of 3, 11, 23, 29, 83 and 89. In 1992, the largest known S that meets the criteria was

$$S = (39051 \cdot 2^{6002}) - 1 , \tag{118}$$

a 1812-digit long Sophie Germain prime number that is 3 mod 20. It was discovered by Wilfred Keller. This Sophie Germain prime leads to a prime n with a decimal expansion that is around 10^{1812} digits long before it starts repeating itself. Read your favourite book on number theory to find out more. Interestingly, the solution to this challenge is also connected to that of challenge 266. Can you find out more?

Challenge 276, page 274: Klein did not belong to either group. As a result, some of his nastier students concluded that he was not a mathematician at all.

Challenge 277, page 274: A barber cannot belong to either group; the definition of the barber is thus contradictory and has to be rejected.

Challenge 278, page 274: See the members.shaw.ca/hdhcubes/cube_basics.htm web page for more information on magic cubes.

Challenge 279, page 275: Such an expression is derived with the intermediate result $(1 - 2^2)^{-1}$. The handling of divergent series seems absurd, but mathematicians know how to give the expression a defined content. (See Godfrey H. Hardy, *Divergent Series*, Oxford University Press, 1949.) Physicists often use similar expressions without thinking about them, in quantum field theory.

Challenge 280, page 275: Try to find another one and then to prove the uniqueness of the known one.

Challenge 281, page 276: The result is related to Riemann's zeta function. For an introduction, see en.wikipedia.org/wiki/Prime_number.

Challenge 283, page 287: 'All Cretans lie' is *false*, since the opposite, namely 'some Cretans say the truth' is true in the case given. The trap is that the opposite of the original sentence is usually, but *falsely*, assumed to be 'all Cretans say the truth'.

Challenge 284, page 287: The statement cannot be false, due to the first half and the 'or' construction. Since it is true, the second half must be true and you are an angel.

Challenge 285, page 287: The terms 'circular' and 'self-referential' describe two different concepts.

Challenge 287, page 288: Extraterrestrials cannot be at the origin of crop circles because, like Father Christmas or ghosts, they do not exist on Earth.

Challenge 289, page 288: This can be debated; in any case it is definitely known that both state-
Vol. V, page 89 ments are lies, as shown later on..

Challenge 290, page 289: If this false statement were true, swimmers or divers would also die, as their skin cannot breathe either.

Challenge 291, page 289: It is equally correct to claim that the Earth was created a hundred ago, and that our environment and our memories were created in our brain to make us believe that the Earth is older. It is hard to disprove such nonsense, but it is possible. See also the next challenge.

Challenge 292, page 289: It is surprisingly hard to disprove such nonsense, if well thought through. The reason for the particular date (or for any other date) is not obvious. Neither is obvious what is meant by the term 'creation'.

Challenge 294, page 289: No. As many experiments demonstrate, homeopathy is a set of numerous lies. For example, the internet provides films of people swallowing – without any harm – hundreds of homeopathic pills at a time that are labelled as 'extremely dangerous when overdosed'.

Challenge 296, page 289: The light bulb story seems to be correct. The bulb is very weak, so that the wire is not evaporating.

Challenge 297, page 290: The origin might be the number of people present in the last supper in the New Testament; or the forgotten 13th sign of the Zodiac. There is no truth in this superstition. In fact, *every superstition is a lie.* However, beware of people who are jealous of those who do not care about superstitions, and who get violent as a reaction.

Challenge 298, page 290: Without exception so far, all those who pretend to have been stigmatized have wounds in the *palms* of their hands. However, in crucifixion, the nails are driven through the *wrist*, because nails driven through the palms cannot carry the weight of a human body: the palms would tear open.

Challenge 299, page 290: The term 'multiverse' is both a superstition and a lie. Above of all, it is nonsense. It is akin to attempting to produce a plural for the word 'everything'.

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Challenge 301, page 290: In which frame of reference? How? Beware of anybody making that statement: he is a crook.

Challenge 306, page 297: Only induction allows us to make use of similarities and thus to define concepts.

Challenge 307, page 299: This depends on the definition (of the concept) of deity used. Pantheism does not have the issue, for example.

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Challenge 308, page 299: Yes, as we shall find out.

Challenge 309, page 300: Yes, as observation implies interaction.

Challenge 310, page 300: Lack of internal contradictions means that a concept is valid as a thinking tool; as we use our thoughts to describe nature, mathematical existence is a specialized version of physical existence, as thinking is itself a natural process. Indeed, mathematical concepts are also useful for the description of the working of computers and the like.

Another way to make the point is to stress that all mathematical concepts are built from sets and relations, or some suitable generalizations of them. These basic building blocks are taken from our physical environment. Sometimes the idea is expressed differently; many mathematicians have acknowledged that certain mathematical concepts, such as natural numbers, are taken directly from experience.

Challenge 311, page 300: Examples are Achilles, Odysseus, Mickey Mouse, the gods of polytheism and spirits.

Challenge 313, page 302: Torricelli made vacuum in a U-shaped glass tube, using mercury, the same liquid metal used in thermometers. Can you imagine how? A more difficult question: where did he get mercury from?

Challenge 314, page 303: Stating that something is infinite can be allowed, if the statement is falsifiable. An example is the statement 'There are infinitely many mosquitoes.'

Other statements are not falsifiable, such as 'The universe continue without limit behind the horizon.' Such a statement is a belief, not a fact.

Challenge 315, page 305: They are not sets either and thus not collections of points.

Challenge 316, page 305: There is still no possibility to interact with all matter and energy, as this includes oneself.

Challenge 317, page 311: No. There is only a generalization encompassing the two.

Challenge 318, page 312: An explanation of the universe is not possible, as the term explanation require the possibility to talk about systems outside the one under consideration. The universe is not part of a larger set.

Challenge 319, page 312: Both can in fact be seen as two sides of the same argument: There is no other choice; there is only one possibility. Equivalently, the rest of nature shows that observations have to be the way they are, because everything depends on everything.

Challenge 321, page 328: Mass is a measure of the amount of energy. The 'square of mass' makes no sense.

Challenge 324, page 330: The formula with $n - 1$ is a better fit. Why?

Challenge 327, page 331: No! They are much too precise to make sense. They are only given as an illustration for the behaviour of the Gaussian distribution. Real measurement distributions are not Gaussian to the precision implied in these numbers.

Challenge 328, page 331: About 0.3 m/s. It is *not* 0.33 m/s, it is *not* 0.333 m/s and it is *not* any longer strings of threes!

Challenge 330, page 337: The slowdown goes *quadratically* with time, because every new slowdown adds to the old one!

Challenge 331, page 337: No, only properties of parts of the universe are listed. The universe itself has no properties, as shown in the last volume.

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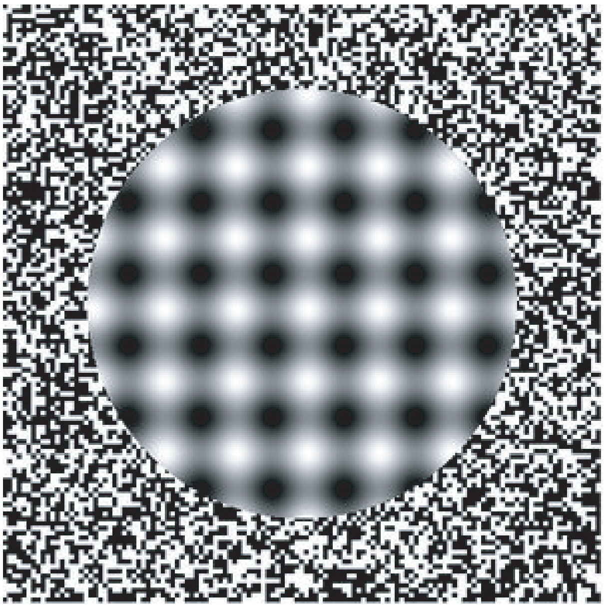

FIGURE 178 The Ouchi illusion of motion.

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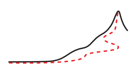

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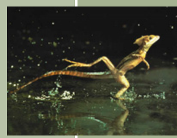

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